OPTIMAL SCHEDULING OF HIGHWAY EMERGENCY REHABILITATIONS AND RELIEF DISTRIBUTIONS AFTER A MAJOR DISASTER

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Abstract: In this research we develop a model with the objective of minimizing the time length of both emergency rehabilitation and relief distribution, subject to related operating constraints. The model is expected to help the decision maker to efficiently set the emergency rehabilitation and the relief distribution schedules within a limited time. We employ network flow techniques to construct emergency rehabilitation and relief distribution time-space networks that can formulate the flows of work teams and relief commodities in both time and space dimensions. A number of side constraints are set between these two networks according to real constraints. The model is formulated as a multi-objective, mixed-integer, multiple commodity network flow problem that is characterized as NP-hard. Since the problem size could be huge in practice, we develop a heuristic algorithm, with the assistance of a mathematical programming solver, to solve the problem. To evaluate the model and the solution algorithm, we perform a case study using data referring to the 1999 Chi-chi earthquake in Taiwan. The results are good, showing that the model and the solution algorithm could be useful in practice.

Key Words: emergency rehabilitation, relief distribution, time-space network, multi-objective, mixed-integer multiple commodity network flow problem.

1. INTRODUCTION

Natural disasters, such as earthquakes, typhoons, floods, volcanic eruptions, mudflows and landslides, inevitably occur in daily life and have significant devastating effects in terms of human injuries and property damages. In addition, Taiwan is located in the circum-pacific seismic zone which is one of the places that has the most earthquakes in the world. Earthquake damage can break down the traffic and lifeline systems and obstruct the operation of rescue machines, rescue cars, ambulances and relief work. That could not only affect the rescue efficiency but also increase human injuries. Public officials were faced with many critical questions after the occurrence of the Chi-chi earthquake in 1999. The importance of these questions is related to how to respond to these emergencies in the most efficient way, to minimize the loss of life and maximize the efficiency of the rescue operations.

In the past, emergency rehabilitation and relief distribution have usually been handled manually and separately by decision makers, based on their experience, omitting any thought of the interrelation between emergency rehabilitation and relief distribution from the system perspective. Consequently, the resultant solution could very possibly be inferior. In this research, we studied both the emergency rehabilitation problem and the relief distribution problem in the same framework. To the authors’ knowledge, there has not yet been research considering these two problems simultaneously. In the past, emergency rehabilitation and
Relief distribution have commonly been studied separately.

Research on emergency rehabilitations has appeared in the literature. For example, Tamura et al. (1994) used a genetic algorithm to solve the urban road improvement priority problem. Sato and Ichii (1996) developed the model built by Tamura et al. (1994) with the addition of a work team assignment. The model was solved using a single populated genetic algorithm. Arimura et al. (1999) also used a genetic algorithm to solve the road investment of restoration planning. Chen and Tzeng (1999) developed a fuzzy multi-objective model and a genetic algorithm for solving the reconstruction order and the assignment of reconstruction work to relevant work-troops. Fiedrich et al. (2000) developed a dynamic optimization model to find the best assignment of available resources to operational areas, so as to minimize the total number of fatalities in the initial search-and-rescue period after the occurrence of strong earthquakes.

Relief distributions have also been extensively researched. For example, Haghani and Oh (1996) presented a model for solving a logistical disaster relief management problem. The model was formulated as a large-scale multi-commodity, multi-modal network flow problem with time windows. Barbarosoglu et al. (2002) developed a two-level multi-objective model for helicopter mission planning during a disaster relief operation.

Other related research includes work on emergency response management (e.g., see Brown and Vassiliou, 1993) and logistics management following urban disasters (e.g., see Ardekani, 1992). However, the above research is not the same as ours.

To sum up, our model differs from the past in at least three ways:
1. In the past, most of the emergency rehabilitation models have been formulated as vehicle routing problems, which included many sub-tour constraints and were difficult to optimally solve. These constraints, however, were generally unnecessary in practice. Moreover, these models did not incorporate time variables into the decision, which resulted in incomplete vehicle scheduling decisions in real operations.
2. Some emergency rehabilitation models, not formulated as vehicle routing problems, can be divided into two types. The first type did not incorporate time variables into the decision. Although the second type considered time variables in the decision, these time variables were incorporated with nonlinear logic constraints. Consequently, the resulting problem could neither be solved using a mathematical programming solver, nor could its optimal/near optimal solution be ensured.
3. In past research on relief distribution, different aspects have commonly been studied independently, without detailed consideration of the emergency rehabilitation of segments of highway that have been damaged. Relief distribution may be planned, excluding highway with damage, or including these segments with damage and the set of times within that they can be repaired. However, the interrelationship between relief distribution and emergency rehabilitation was not analyzed systematically, which might possibly result in inferior decisions.

In the past, the emergency rehabilitation and relief distribution models were formulated independently. To remedy this, in this research, we employ network flow techniques in order to develop a multi-objective integer programming model, with the objective of minimizing the time length of emergency rehabilitation and relief distribution, subject to the related operating constraints. The model is formulated as a multi-objective, mixed-integer, multiple commodity network flow problem, which is characterized as NP-hard and is thus in practice difficult to optimally solve, especially for large-scale problems. Therefore, we
develop a heuristic algorithm for efficiently solving the problem. The model is expected to help the decision maker to efficiently set the emergency rehabilitation and the relief distribution schedules, within a limited time. The rest of the paper is organized as follows. First, we introduce our model. Then we formulate the model as a multi-objective mixed-integer program. A solution algorithm is developed thereafter. Finally, we perform a case study to test the model in the real world.

2. THE MODEL

In order to reasonably reflect reality and to facilitate problem solving, the following information or assumptions, based on real practice, are given:

1. The disaster information is given, including the locations of every damaged point and the rehabilitation time needed for every damaged point.
2. The resource information is given, including the work station location and the number of work teams at each work station.
3. The highway-network, including the average travel time for every segment, is given.
4. The work teams do not need to return to their work stations. The rescue machinery, fuel and other resources will be supplied to the work teams. In other words, the resulting routing problem for the work teams is different from the conventional vehicle routing problem, with its many sub-tour constraints.
5. Since two work teams at most can enter a highway segment from either end to repair the segment, it is assumed there are at most two damaged points on the segment between two intersections. However, since a highway segment, from a distribution center to an intersection, or from an intersection to a commodity demand point, is shorter than that between two intersections containing this highway segment, we assume that it contains at most one damaged point.
6. After the disaster, some work teams may have to be assigned to some damaged points for the first time. Their assignment is made outside the model and is used as model input.
7. Damaged points will obstruct the road.
8. Because the area of the intersections is much less than the area of the segments of highway, the probability that damage will occur at the intersections is much smaller than on the highway. Thus for simplicity of modeling, damaged points are assumed to occur on highway segments. However, the model can be further modified to consider damage that might occur in intersections.
9. Every damaged point needs a work team in order to repair it for rehabilitation.
10. All different kinds of commodity are transformed into the same equivalent unit, for simplicity of modeling.
11. The total commodity demand is greater than or equal to the total commodity supply, under emergency conditions. The commodity demand for each demand point, is greater than or equal to the supply amount. As well, based on fairness for the relief commodity distribution, we assume that each demand point must be meet with a minimum percentage of its commodity demand. For this assumption, the total commodity supply is assumed to be greater than or equal to the sum of the minimum amount distributed to each demand point.

2.1 Time-space network

A time-space network technique is applied for constructing an integrated model that combines emergency rehabilitation and relief distribution in order to efficiently set the emergency rehabilitation and the relief distribution schedules. The major elements in the modeling,
which include the emergency rehabilitation time-space network, the relief distribution

time-space network, and the mathematical formulation, are described below.

### 2.1.1 Emergency rehabilitation time-space network

A time-space network, shown in Figure 1, is established for work team routing within a

specified time period and at specified locations. The horizontal axis represents the work

stations, the intersections and the damaged points; the vertical axis stands for the time
duration. “Nodes” and “arcs” are the two major components in the network.

A node, expect for the collection node, represents a work station, an intersection, or a
damaged point at a specific time. In particular, a node associated with a work station

represents a supply node that supplies a number of work teams. The collection node is used
to ensure the flow conservation.

The arc flows express the flow of work teams in the network. The arc flow’s lower/upper

bound is defined as the minimum/maximum number of flow units allowable in an arc. Three
types of arcs are defined:

1. **Work team arc**
   - A work team arc is shown as (1)-(5) in Figure 1, respectively representing a work team trip
     (1) from a work station to an intersection, (2) from a work station to a damaged point, (3)
     from an intersection to another intersection, (4) from an intersection to a damaged point or
     vice versa, and (5) from a damaged point to another damaged point. Each work team arc
     contains information about the departure time, the departure node, the arrival time and the
     arrival node. The time block for a work team work trip is calculated from the time when
     the team starts to prepare for this work to the time when this work is finished. Basically, it
     includes the travel time and the rehabilitation time (and a buffer time to absorb any minor
     delays that occur in real operations). Note that, the arc flow’s upper bound, from an
     intersection to another intersection, shown as (3) in Figure 1, is the road capacity (or infinity
     if the capacity is large), indicating the maximum number of work teams that can travel on
     the associated highway segment during a specific time window. The arc flow’s upper
     bound, from a work station to an intersection, shown as (1) in Figure 1, is the number of
     work teams located at this work station at the beginning. The other arc flow’s upper bound
     is one, implying that at most one work team is assigned from/to a damaged point. The arc
     flow’s lower bound is zero.

2. **Holding arc**
   - A holding arc, shown as (6)-(8) in Figure 1, represents the holding of work teams (6) at a
     work station, (7) at an intersection, and (8) at a damaged point in a time window. The arc
     flow, a non-negative integer variable, denotes the number of work teams held at a work
     station, an intersection, or a damaged point in a time window. The arc flow’s upper bound
     is, for (6), the work station capacity, for (7) it is infinity, and for (8) it is one, indicating the
     maximum number of work teams that can be held at a work station, an intersection, and a
     damaged point, respectively, during a specific time in this time window. The arc flow’s
     lower bound is zero, indicating that no work team is held at a work station, an intersection,
     or a damaged point in this time window.

3. **Collection arc**
   - A collection arc, shown as (9) in Figure 1, connecting the last node associated with every
     work station and the node associated with every damaged point to the collection node, is
used to ensure flow conservation at the last time associated with the work stations and damaged points. The arc flow’s upper bound is infinity, indicating the maximum number of work teams that leave the system. The arc flow’s lower bound is zero.

For every collection arc, from a damaged point node to the collection node, the arc cost is set as the time associated with the damaged point node. For the other arcs, their cost is zero. The objective of emergency rehabilitation is to minimize the length of time for repairing all damaged points, which is equivalent to the minimization of the last collection arc cost, from all damaged point nodes to the collection node. In other words, the objective function can be formulated as the Minimax function, which will be addressed later.

It should be noted that, in practice, the importance of different points of damage to the system may be different. A time limit to rehabilitation may be required of the decision maker for some damaged points. For this, time windows can be incorporated into the construction of the network. For example, as shown in Figure 1, if the 2nd damaged point has to be repaired before the 10th time, then a cut-off line is made at the 10th time. All arcs after the 10th time for the 2nd damaged point are removed.

2.1.2 The relief distribution time-space network

The time-space network is also applied to formulating relief movements corresponding to certain times and stations. As shown in Figure 2, a relief distribution time-space network represents the supplies and the demands. This network is similar to the emergency rehabilitation time-space network. The horizontal axis represents the distribution centers, intersections and demand points; the vertical axis stands for time duration. The “nodes” and “arcs” are the two major components in the network.

A node, expect for the collection node and the artificial node, represent a distribution center, an intersection, or a demand point at a specific time. In particular, a distribution center represents a supply node with an amount of relief commodity to supply. Suppose that there are m distribution centers and the i-th distribution center has cs_i supply, then the total relief
commodity in the system equals \( cs_1 + cs_2 + \ldots + cs_m = \sum_{k=1}^{m} cs_k \). Suppose that there are \( n \) demand points and the \( i^{th} \) demand point has \( cd_i \) demand, then the total demand in the system equals \( cd_1 + cd_2 + \ldots + cd_n = \sum_{k=1}^{n} cd_k \). The collection node and the artificial node are used for ensuring network flow conservation. The collection node demand is set as \(- \sum_{k=1}^{n} cd_k\) and the artificial node demand is set as \( \sum_{k=1}^{n} cd_k - \sum_{k=1}^{m} cs_k \).

The arc flows express the flow of relief commodity in the network. The arc flow’s lower/upper bound is defined as the minimum/maximum flow units allowable in an arc. Four types of arcs are defined below:

1. Commodity arc
   A commodity arc is shown as (1)-(4) in Figure 2. They represent, respecting, the commodity trip (1) from a distribution center to an intersection, (2) from an intersection to another intersection, (3) from an intersection to a demand point, or (4) from a distribution center to a demand point. Each commodity arc contains information about the departure time, the departure node, the arrival time and the arrival node. The time block for a trip is calculated as from the time when the relief commodity is transported to the time when the commodity arrives. Basically, it includes the travel time and a buffer time used for absorbing minor delays occurring in real operations. Note that, the arc flow’s upper bound, from a distribution center to an intersection or a demand point, is the amount of its commodity supply, indicating the maximum amount of commodity to be transported from the associated distribution center. The other arc flow’s upper bound is infinity, denoting that an unlimited amount of commodity can be transported. The arc flow’s lower bound is zero.

2. Holding arc
   A holding arc, shown as (5)-(7) in Figure 2, represents the holding of commodities (5) at a distribution center, (6) at an intersection, or (7) at a demand node, in a time window. The arc flow, a non-negative variable, denotes the amount of commodity at a distribution center, an intersection, or a demand point, in a time window. The arc flow’s upper bound is the amount of commodity supply for a distribution center, infinity for an intersection, and the demand amount for a demand point, indicating the maximum number of commodities that can be held at a distribution center, an intersection, or a demand point, respectively, in this time window. The arc flow’s lower bound is zero, meaning that no commodity is held at a distribution center, an intersection, or a demand point, in this time window.

3. Collection arc
   A collection arc, shown as (8) in Figure 2, connects the last node associated with every distribution center and the node associated with every demand point to the collection node, and is used to ensure flow conservation at the last time associated with the distribution centers and the demand points. The arc flow’s upper bound is infinity, indicating the maximum amount of relief commodity that leaves the system. The arc flow’s lower bound is zero.

4. Artificial arc
An artificial arc, shown as (9) in Figure 2, connecting the artificial node to the collection node, is used to ensure network flow conservation. The arc flow’s upper bound is \( \sum_{k=1}^{m} c_k = \sum_{k=1}^{n} c_{sk} \), and the lower bound is zero.

The arc cost of a collection arc, from a demand point to node the collection node, is set as the time associated with the demand point node. For the collection arc, from the last node associated with a distribution center to the collection node, the arc cost is set as the last time. For the other arcs, their cost is zero. Since property damage is far more important than the relief distribution cost after a serious disaster, the decision makers typically aim to transport the relief commodities to demand points as early as possible. Therefore, similar to the objective of emergency rehabilitation, the objective of relief distribution is set to minimize the length of time necessary for sending relief supplies to all demand points, which is equivalent to the minimization of the last collection arc cost, from all demand points to the collection node. The objective function can be formulated as a Minimax function, which will be addressed later. Note that, time windows, similar to those in the emergency rehabilitation time-space network, may be incorporated into the construction of this network.

2.2 The side constraints

In addition to the emergency rehabilitation and the relief distribution time-space networks, there are seven types of side constraints that should be considered, in correspondence with actual practices:

1. If a work team/commodity arc between two highway segment intersections with two damaged points has a positive flow, then these two damaged points have been repaired.
2. The sum of all work team arc flows pointing to the nodes associated with each damaged point should be equal to one, meaning that exactly one work team is required to repair a damaged point.
3. The sum of all commodity arc flows pointing to the nodes associated with each demand point should not be greater than its commodity demand, meaning that the commodity supply sent to each demand point may not be more than its commodity demand in emergency conditions. On the other hand, based on fairness for relief commodity distribution, each demand point must be sent a specified minimum percentage of its commodity demand.
4. If a work team arc between two intersections of a highway segment with a damaged point has a positive flow, then it means that this damaged point has been repaired.
5. If a work team arc between two intersections of a highway segment with two damaged points has a positive flow, then it means that these two damaged points have been repaired.
6. The commodity flow on a highway segment with damaged points is allowed, provided that the damaged points are repaired. Specifically, if a commodity arc from a distribution center to an intersection, or from an intersection to a commodity demand, for a highway segment with damaged points, has a positive flow, then these damaged points have been repaired. Consequently, the relief distribution time-space network flow is based on the emergency rehabilitation time-space network flow.
7. To transform the two minimax objective functions into linear function form we design additional side constraints for the transformation.

2.3 The Formulation

Before introducing the model formulation, we list the notations for the symbols that are used
in the model formulation as follows:

The decision variables:

\( x_{ij} \): the arc \((i,j)\) flow in the emergency rehabilitation network (unit: work team);

\( m_{ij} \): a binary variable which indicates if the arc \((i,j)\), in the emergency rehabilitation network, has flows. If \( m_{ij}=1 \), then \( x_{ij} \geq 0 \); if \( m_{ij}=0 \), then \( x_{ij} = 0 \);

\( y_{ij} \): the arc \((i,j)\) flow in the relief distribution network (unit: commodity equivalency);

\( n_{ij} \): a binary variable which indicates if the arc \((i,j)\), in the relief distribution network, has flows. If \( n_{ij}=1 \), then \( y_{ij} \geq 0 \); if \( n_{ij}=0 \), then \( y_{ij} = 0 \);

\( er, rd \): variables for transforming the minmax functions, associated with the emergency rehabilitation network and the relief distribution network, into linear functions

The parameters:

\( ce_{ij} \): the arc \((i,j)\) cost in the emergency rehabilitation network;

\( cr_{ij} \): the arc \((i,j)\) cost in the relief distribution network;

\( ue_{ij} \): the arc \((i,j)\) flow’s upper bound in the emergency rehabilitation network;

\( ur_{ij} \): the arc \((i,j)\) flow’s upper bound in the relief distribution network;

\( AE, NE \): the set of all arcs and nodes in the emergency rehabilitation network;

\( AR, NR \): the set of all arcs and nodes in the relief distribution network;

\( WS \): the set of all work stations. \( WS=\{1,2,\ldots,n_{WS}\} \), where \( n_{WS} \) is the total number of work stations;

\( IN \): the set of all intersections. \( IN=\{1,2,\ldots,n_{IN}\} \), where \( n_{IN} \) is the total number of intersections;

\( DA \): the set of all damaged points. \( DA=\{1,2,\ldots,n_{DA}\} \), where \( n_{DA} \) is the total number of damaged points;

\( DC \): the set of all distribution centers. \( DC=\{1,2,\ldots,n_{DC}\} \), where \( n_{DC} \) is the total number of distribution centers;

\( DE \): the set of all demand points. \( DE=\{1,2,\ldots,n_{DE}\} \), where \( n_{DE} \) is the total number of demand points;

\( cd_{n} \): the commodity demand of the \( n \)th demand point. (unit: commodity equivalency);

\( cs_{m} \): the commodity supply of the \( m \)th distribution center;

\( AD^{d} \): the set of all work team arcs pointing to the nodes associated with the \( d \)th damaged point;

\( AH^{d} \): the set of all work team arcs between two intersections of a highway segment with the \( d \)th damaged point;

\( AH^{uv} \): the set of all work team arcs between two intersections of a highway segment with the \( u \)th and \( v \)th damaged points;

\( t(i) \): the function that determines the time associated with the \( i \)th node;

\( tr(i,q) \): the function that determines the travel time from the \( i \)th node to the \( q \)th node;

\( O_{t(i)+tr(i,q)}^{e(i)+tr(e(i),q)} \): the set of all work team arcs from the \( e \)th intersection to the \( d \)th damaged point before the \( [t(q)] \)th time, where \( t(q)=t(i)+tr(i,q) \);

\( I_{t(i)+tr(i,q)}^{u(i)+tr(u(i),q)} \): the set of all work team arcs from the \( u \)th to the \( v \)th damaged point before the \( [t(q')] \)th time, where \( t(q')=t(i)+tr(i,q') \);

\( AD^{d} \): the set of all commodity arcs associated with a highway segment, from a distribution center to an intersection, that contains the \( d \)th damaged point;

\( AI^{d} \): the set of all commodity arcs associated with a highway segment, from an intersection to a commodity demand point, that contains the \( d \)th damaged point;

\( ADE^{n} \): the set of all commodity arcs pointing to the nodes associated with the \( n \)th demand point;

\( AEC \): the set of all collection arcs in the emergency rehabilitation network;

\( ARC \): the set of all collection arcs in the relief distribution network;
The minimum percentage of the commodity demand to be sent to the $n^{th}$ demand point;

The $i^{th}$ node supply in the emergency rehabilitation network. If the $i^{th}$ node is a supply node, then $a_i$ equals its supply of work teams; else if the $i^{th}$ node is a collection node, then $a_i$ equals the negative value of total work teams; otherwise, $a_i$ equals zero;

The $i^{th}$ node supply in the relief distribution network. If the $i^{th}$ node is a supply node, then $b_i$ equals its supply of relief commodity (for distribution centers) or the total demand minus the total relief commodity (for the artificial node); else if the $i^{th}$ node is the collection node, then $b_i$ equals the negative value of total relief commodity; otherwise, $b_i = 0$.

The model is formulated as follows:

Min

$Z_1 = er$,  

$Z_2 = rd$.  

Subject to

$ce_i x_{ij} \leq er$,  \hspace{1cm} \forall ij \in AEC$;  

$cr_j n_j \leq rd$,  \hspace{1cm} \forall ij \in ARC$;  

$x_{ij} \leq u e_j m_j$,  \hspace{1cm} \forall ij \in AH^{vu}, \forall u, v \in DA$;  

$y_{ij} \leq u r_j n_j$,  \hspace{1cm} \forall ij \in ARC \cup AH^{vu}, \forall u, v \in DA$;  

$\sum_{j \in NE} x_{ij} - \sum_{k \in NE} x_{ki} = a_i$,  \hspace{1cm} \forall i \in NE$;  

$\sum_{j \in NR} y_{ij} - \sum_{k \in NR} y_{ki} = b_i$,  \hspace{1cm} \forall i \in NR$;  

$\sum_{y \in ADE^d} x_{y} = 1$, \hspace{1cm} \forall d \in DA$;  

$\sum_{y \in ADE^v} y_{y} \leq cd_n$, \hspace{1cm} \forall n \in DE$;  

$p_n \times cd_n \leq \sum_{y \in ADE^v} y_{y}$, \hspace{1cm} \forall n \in DE$;  

$x_{ij} \leq u e_g \left( \sum_{q \in \Omega(q_{ij}, w_{ij}, v_{ij})} x_{f_q} + \sum_{q \in \Omega(q_{ij}, w_{ij}, v_{ij})} x_{f_q} \right)$,  \hspace{1cm} \forall ij \in AH^d, \forall i, i' \in NE^e, \forall j, j' \in NE^w, \forall e, w \in IN$,  

$\forall q \in NE^d, \forall d \in DA, t(q) = t(i) + tr(i, q)$;  

$m_{ij} \leq \frac{1}{2} \left( \sum_{q \in \Omega(q_{ij}, w_{ij}, v_{ij})} x_{f_q} + \sum_{q' \in \Omega(q_{ij}, w_{ij}, v_{ij})} x_{f_q'} + \sum_{q'' \in \Omega(q_{ij}, w_{ij}, v_{ij})} x_{f_q''} + \sum_{q''' \in \Omega(q_{ij}, w_{ij}, v_{ij})} x_{f_q'''} \right)$, \hspace{1cm} \forall ij \in AH^v, \forall i, i' \in NE^e$, \hspace{1cm} \forall j, j' \in NE^w, \forall e, w \in IN, \forall q' \in NE^w, \forall q'' \in NE^w, \forall q''' \in NE^w, \forall u, v \in DA, t(q') = t(i) + tr(i, q')$, \hspace{1cm} \forall q'' = t(i) + tr(i, q'');  

$y_{ij} \leq ur_j \left( \sum_{q \in \Omega(q_{ij}, w_{ij}, v_{ij})} x_{f_q} + \sum_{q \in \Omega(q_{ij}, w_{ij}, v_{ij})} x_{f_q} \right)$, \hspace{1cm} \forall ij \in AH^d, \forall q \in NE^d, \forall d \in DA, \forall i \in NR^e / NE^e$, \hspace{1cm} \forall i \in NE^e, \forall j \in NR^e / NE^w, \forall j' \in NE^w, \forall e, w \in IN, t(q) = t(i) + tr(i, q)$;
\[ n_y \leq \frac{1}{2} \left( \sum_{q' \in Q^{\text{AR}}(i,j)} x_{ij} + \sum_{q'' \in Q^{\text{AE}}(i,j)} x_{ij} + \sum_{q''' \in Q^{\text{AR}}(i,j)} x_{ij} + \sum_{q'''' \in Q^{\text{AE}}(i,j)} x_{ij} \right), \quad \forall ij \in AH^{\text{AE}}, \forall i \in NR^e / NE^e, \]
\[ \forall i' \in NE^w, \forall j \in NR^w / NE^w, \forall q' \in NE^w, \forall e,w \in IN, \forall q'' \in NE^w, \forall q''' \in NE^w, \forall u,v \in DA, \]
\[ t(q') = t(i) + tr(i,q'), t(q'') = t(i) + tr(i,q''); \]
\[ y_y \leq u_y \left( \sum_{q' \in Q^{\text{AR}}(i,j)} x_{ij} + \sum_{q'' \in Q^{\text{AE}}(i,j)} x_{ij} \right), \quad \forall ij \in ADL^d, \forall i \in NR^e, \forall e' \in DC, \forall j \in NR^w, \forall i' \in NE^e, \]
\[ \forall j' \in NE^w, \forall e,w \in IN, \forall q \in NE^d, \forall d \in DA, t(q) = t(i) + tr(i,q); \]
\[ m_y = 0 \quad \text{or} \quad 1, \quad \forall ij \in AE; \quad \text{(18)} \]
\[ n_y = 0 \quad \text{or} \quad 1, \quad \forall ij \in AR; \quad \text{(19)} \]
\[ 0 \leq x_y \leq u_e x_y, \quad \forall ij \in AE; \quad \text{(20)} \]
\[ 0 \leq y_y \leq u_y, \quad \forall ij \in AR; \quad \text{(21)} \]
\[ x_y \in I, \quad \forall ij \in AE; \quad \text{(22)} \]
\[ \text{where} \quad er = \text{Max} \quad ce_y x_y, \quad \forall ij \in AEC; \quad \text{rd} = \text{Max} \quad cr_y n_y, \quad \forall ij \in ARC. \]

The model is formulated as a multi-objective mixed integer multiple commodity network flow problem, in which both objective functions are to minimize the time length of the emergency rehabilitation and the relief distribution.  

Objective functions (1) and (2), coupled with constraints (3) and (4), are used to transfer the minimax objective functions into linear function form.  
Constraints (5) and (6) are used to determine whether each arc, in both networks, has a positive flow or not.  
Constraints (7) and (8) are the flow conservation constraints at every node in the emergency rehabilitation/relief distribution networks, respectively.  
Constraint (9) indicates that every damaged point is repaired by exactly one work team.  
Constraints (10) and (11) denote that the upper and lower limits of relief commodity to be sent to each demand point in the relief distribution network.  
Constraint (12) formulates whether every highway segment with a damaged point is passable or not.  
Similar to constraint (12), constraint (13) determines whether each highway segment with two damaged points is passable or not.  
Constraints (14) and (15) represent that the commodity flow on a highway segment with one/two damaged points is allowed, given that the damaged points are repaired.  
These two constraints are similar to constraints (12) and (13) and are not addressed in detail.  
Similar to constraint (14), constraints (16) and (17) determine whether every highway segment, from a distribution center to an intersection, or from an intersection to a demand point, that contains a damaged point is passable, or not.  
Constraints (18) and (19) indicate that all the decision variables are either zero or one.  
Constraints (20) and (21) hold all the arc flows within their bounds, and constraint (22) ensures the integrality of the work team flows.  

Since the government decision maker can always specify a preference in advance, we suggest using the weighting method to handle problems in this research.  
Using the weighting method, the two objective functions are transformed into a weighted objective function as
shown in equation (23).

\[ \min \alpha \cdot er + \beta \cdot rd \]  

(23)

Where, \( \alpha + \beta = 1 \). The user can set various weighting vectors, then find the optimal solution for each one. The optimal solution for the weighting vector will be the non-dominated solution of the multiple-objective problem.

3. SOLUTION ALGORITHMS

The model is formulated as a multi-objective mixed integer multiple commodity network flow problem that is characterized as NP-hard and is difficult to optimally solve for realistically large problems under time constraints, especially for emergency conditions. For example, referring to the 1999 Chi-chi earthquake in Taiwan, the model will include 30579 nodes, 112837 arcs, and 54140 constraints (excluding variable upper or lower bound constraints), in which 30579 constraints ensure flow conservation and 23561 side constraints. It is almost impossible to optimally solve such a large problem within a limited time. Therefore, we develop a heuristic algorithm to efficiently solve this problem. To solve this problem, we first generate an initial solution and then improve the initial solution to find a near-optimal solution. The solution algorithm is outlined as follows:

3.1 The initial solution method

To efficiently generate initial solutions, focusing on an emergency rehabilitation network, we first decompose the originally large problem into several smaller sub-problems, each one associated with a work station. Then, we solve these sub-problems independently, using a mathematical programming solver, CPLEX. Finally, we combine the emergency rehabilitation network sub-problem solutions, then add the relief distribution network, to solve for a feasible solution. This feasible solution serves as the initial solution. Note that the initial solution method will always yield a feasible solution and can also help us to reduce the analyzed time length for both networks, as well as the problem size.

3.2 The improvement solution method

We classify the damaged points into three types, as shown in Figure 3. 1. Restricted damaged points: damaged points that are enclosed by the other damaged points (called obstructive damaged points) and cannot be reached by work teams, unless these obstructive damaged points are repaired. 2. Obstructive damaged points: damaged points that block the restricted damaged points from being reached by work teams. 3. Free damaged points: damaged points that do not influence other damaged points in terms of rehabilitation.
The solution method is described as follows: First, focusing on the emergency rehabilitation network, we solve the problem with only a few damaged points. Then, we fix the obtained solution and add some other damaged points, and solve the problem again. The process is repeated until all damaged points are added and repaired. We thus obtain the emergency rehabilitation network flow. Given the emergency rehabilitation network flow, we then solve the relief distribution network solution flow.

To decide on the order by which the damaged points that are added into the problem, we use the above three types of damaged point defined to set several orders, then choose the best one. Note that there are at most six different orders that can be set with three types of damaged points. However, some of these intuitively yield inferior solutions. Therefore, we choose the following three orders, which are intuitively better than others: 1. free damaged points → obstructive damaged points → restricted damaged points; 2. obstructive damaged points → restricted damaged points → free damaged points; and 3. restricted damaged points → obstructive damaged points → free damaged points.

Note that, after preliminary tests, if all the damaged points for a type are placed in the problem at the beginning or later, then it would be too time-consuming for the heuristic algorithm to solve the problem. Therefore, we gradually add a number of damaged points into the problem, to improve solution efficiency. The steps of the solution method are listed as follows:

Step 0: Focusing on the emergency rehabilitation network, we first decompose the originally large problem into several smaller sub-problems, each one associated with a workstation. Then, we solve these sub-problems independently using CPLEX. Finally, we combine the sub-problem solutions of the emergency rehabilitation network, and add the relief distribution network, to solve for a feasible solution. We use this initial solution as the incumbent solution to set the time length in the next step. Note that this solution, a feasible solution, is an upper bound solution for the original problem.

Step 1: Use the first order of damaged points (free damaged points → obstructive damaged points → restricted damaged points) to solve the problem, in which a series of sub-problems are solved using CPLEX in a pre-set time.

Step 1.1: Focusing on the emergency rehabilitation network, we first solve the problem with 2 free damaged points. Then we fix the obtained solution and add 3 free damaged points to solve the problem. The process is repeated until all free damaged points are added and repaired.

Step 1.2: Based on the above solution, we add 2 obstructive damaged points to solve the problem. The process is repeated until all obstructive damaged points are added and repaired.

Step 1.3: Based on the above solution, we add all restricted damaged points to solve the problem. We find a complete emergency rehabilitation network flow.

Step 1.4: Given the emergency rehabilitation network flow, we solve the relief distribution network solution flow.

Step 1.5: If the obtained solution is feasible, then update the incumbent (and the time length).

Step 2: Similar to Step 1, solve the problem with the second order of damaged points (obstructive damaged points → restricted damaged points → free damaged points), in which a series of sub-problems are solved using CPLEX in a pre-set time. If we get
Step 3: Similar to Step 1, solve the problem with the second order of damaged points (restricted damaged points → obstructive damaged points → free damaged points), in which a series of sub-problems are solved using CPLEX in a pre-set time. Note that to solve the problem with only restricted damaged points, constraint (9) is relaxed by modifying the “equality” to be “less than or equal to” for the obstructive damaged points. If we get a feasible solution, then update the incumbent solution (and the time length).

Step 4: Use the incumbent solution (the best solution obtained during the process) as the final solution.

It should be noted that the above heuristic rules were set based on a preliminary test. To apply the model and the solution algorithm in practice in the future, more tests should be performed by the user to set up suitable rules for his/her own operations. Moreover, a set of suitable heuristic rules may also be used for solving for better solutions, with parallel computations, in order to save time.

4. NUMERICAL TESTS

To test how well the model may be applied in the real world, we performed a case study on the 1999 Chi-chi earthquake in Taiwan. To build and solve the model, we used the C computer language to write the necessary programs, coupled with the CPLEX 7.1 mathematical programming solver, to solve the problem. The tests were performed on an Intel Pentium 4 2GHz with 1GB RAM under Microsoft Windows 2000. We used the Nantou County Chi-chi earthquake data to build the mathematical model, and then applied the solution algorithm to solve the problem.

The input data includes the highway-network information, the emergency rehabilitation resources and the commodity supplies and demands. The highway-network information includes the highway segments and intersections in Nantou County, the location of damaged points and work stations and the location of supply and demand points. The emergency rehabilitation resources include the work teams in each station and the average time for a work team to repair each damaged point. The commodity supplies/demands include amount of commodities provided/required for each supply/demand point. These commodities include, for example, sleeping bags, camps, bottled water, canned food, rice, sanitary utensils and batteries; where the amounts are calculated in a unit of commodity equivalency. For ease of testing, the demand and supply data from the operating statistics are used to determine the projected demand and supply commodities. To assure that each demand point is supplied, assume that, to be fair, the minimum amount of commodity required for supplying every demand point is set to be 70% multiplied by its commodity demand, meaning that the \( p_o \) of constraint (11) is equal to 0.7. Thus there are 55 intersections, 24 damaged points, 8 demand nodes, 9 work stations, 24 work teams, 5 distribution centers, 196 unit time length, and the weighting vector, (0.5, 0.5) that are used in the tests. All the problems dealt with by the heuristic algorithm were solved using CPLEX to within an error gap of 0.05. The maximum computation time allowed for solving each problem in the heuristic algorithm is limited to 1800 seconds.

To test whether CPLEX is applicable or not for directly solving the problem, we first solved the problem using CPLEX. Table 1 shows the test results. As shown in Table 1, it took a
time of 172851 seconds (about 2 days), to find the first feasible solution with the multi-objective value of 37 and a convergence gap of 68%. It took 384899 seconds (about 4.5 days) to finally find the optimal solution with the multi-objective value of 28.

<table>
<thead>
<tr>
<th>Table 1 Results Using CPLEX to Directly Solve the Problem</th>
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<tbody>
<tr>
<td><strong>Objective</strong></td>
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<td>er</td>
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<tr>
<td>First solution</td>
</tr>
<tr>
<td>Final solution</td>
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</tbody>
</table>

Due to the fact that in emergency conditions the decision is typically time constrained, using CPLEX to directly solve this problem is too time-consuming and is not practical. Therefore, we developed a heuristic algorithm, with the assistance of CPLEX, to solve the problem. The problem size handled in each step is shown in Table 2. In particular, the number of variables in step 0 is 65,460, which is significantly fewer than the 112,837 of the original problem. The number of variables is further decreased to 12,216 in Steps 1, 2, and 3. Similarly, the numbers of flow conservation constraints and side constraints in step 0 are 15485 and 16701, respectively, which are significantly fewer than 30579 and 23561, respectively, of the original problem. The numbers of flow conservation constraints and side constraints are decreased to 3082 and 2657, respectively, in Steps 1, 2, and 3. Therefore, the algorithm efficiency can be enhanced.

<table>
<thead>
<tr>
<th>Table 2 Problem Size Handled in Each Step</th>
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<tbody>
<tr>
<td><strong>Network</strong></td>
</tr>
<tr>
<td>Problem size</td>
</tr>
<tr>
<td>Original problem size</td>
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<tr>
<td>Step 0</td>
</tr>
<tr>
<td># Time-space network</td>
</tr>
<tr>
<td># Nodes</td>
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<tr>
<td># Arcs</td>
</tr>
<tr>
<td><strong>Model</strong></td>
</tr>
<tr>
<td>Problem size</td>
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<tr>
<td>Original problem size</td>
</tr>
<tr>
<td># Variables</td>
</tr>
<tr>
<td>112837</td>
</tr>
<tr>
<td># Flow conservation constraints</td>
</tr>
<tr>
<td>30579</td>
</tr>
<tr>
<td># Side constraints</td>
</tr>
<tr>
<td>23561</td>
</tr>
<tr>
<td># Arcs</td>
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<tr>
<td>112837</td>
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<tr>
<td># Nodes</td>
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<tr>
<td>30579</td>
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<td># Time-space network</td>
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<td>2</td>
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<tr>
<td># Nodes</td>
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<td>30579</td>
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<td># Arcs</td>
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<td>112837</td>
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The test results in each step and of the final solution are shown in Table 3. It took 1904.86 seconds (about 31.74 minutes) in total to solve the problem within an error gap of 1.79% (= (28.5-28)/28). The objective values in Steps (0), (2) and (3) are the same, 28.5, implying that the initial solution method (in Step 0) yields a near-optimal solution.

<table>
<thead>
<tr>
<th>Table 3 Test Results of the Heuristic Algorithm</th>
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<tbody>
<tr>
<td><strong>Step</strong></td>
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<tr>
<td></td>
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<tr>
<td>0</td>
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<tr>
<td>1</td>
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<td>2</td>
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<tr>
<td>3</td>
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<tr>
<td>Final solution</td>
</tr>
</tbody>
</table>

*: No feasible solution is found in the preset time limit.
5. CONCLUSIONS

There has not yet been a model that could formulate both emergency rehabilitation and relief distribution in the same framework. In this research we develop a model with the objective of minimizing the time length of both emergency rehabilitation and relief distribution, subject to related operating constraints. Moreover, the time-space network technique is, to the first time, being applied in this research to the emergency rehabilitation problem. This model can be used to find optimal work team routes/schedules and relief commodity transportation plans, in the same framework, so that the user can efficiently and effectively adjust both the work team movements and the relief commodity flows after a major disaster.

Mathematically, the model is formulated as a multi-objective, mixed-integer, multiple commodity network flow problem. We develop a heuristic algorithm, with the assistance of a mathematical programming solver, CPLEX, to solve the problem. To test how well the model and the heuristic algorithm may be applied to actual operations, we perform a case study on the 1999 Chi-chi earthquake in Taiwan. Many problem scenarios were tested, with a substantial problem size of up to 112,837 variables and 54,140 constraints. The model and the algorithm developed in this research performed well. The results are good, showing that the model and the solution algorithm could be used as a reference for both decision makers and researchers.

6. REFERENCES