A MODEL FOR AIRLINE PASSENGER AND CARGO FLIGHT SCHEDULING

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Abstract: Fleet routing and flight scheduling are essential to a carrier’s profitability, level of service and market competitiveness. In this research, we develop an integrated scheduling model that combines passenger, cargo and combi flight scheduling. The objective is to maximize the operating profit, subject to the related operating constraints. We employ network flow techniques to construct the model, which must include multiple fleet-flow, passenger-flow, and cargo-flow networks. The model is formulated as an integer multiple commodity network flow problem that is characterized as NP-hard. A family of heuristics, based on Lagrangian relaxation, a subgradient method, heuristics for the upper bound solution, and a flow decomposition algorithm, is developed to solve the model. The test results, mainly using data from a major Taiwan airline’s operations, show the good performance of the model and the solution algorithms.

Key Words: Fleet routing, Flight scheduling, Time-space network, Multiple commodity network flow problem, Lagrangian relaxation

1. INTRODUCTION
In addition to the growing passenger demand, the demand for air cargo has increased drastically in Taiwan. According to the Chiang Kai-Shek (CKS) International Airport (a major airport in Taiwan) statistics, the air cargo growth rate in Taiwan was about 10.72% on average in 1991 to 2003. In such an environment, most passenger airlines have also been operating concurrent air cargo flights and here introduced combi (or combined) flights into their operations.

Unlike strictly passenger or cargo flights, these combi flights combine the transport of passengers and cargo on one aircraft at the same time. Both passenger and cargo transportation services are provided during regular operations. In particular, if an OD pair’s passenger and cargo demands are both satisfied within a certain limit, then the use of combi
flights will be of greater benefit than only passenger/cargo flights. That is, if the carrier can use combi flights to effectively handle passenger-cargo relationships, then both aircraft usage and operating performance could be improved. In terms of system optimization, it is important for carriers to consider the interrelationship between passenger, cargo and combi flight sources during fleet routing and flight scheduling.

Currently most airlines in Taiwan use a trial-and-error process scheduling passenger, cargo and combi flights separately. Such an approach, without optimization from a systemic perspective, cannot directly and effectively manage passenger, cargo and combi flights together. As a result, the obtained schedules may be inferior, especially when service networks are large. Past research on fleet routing and flight scheduling for passenger transportation has been performed by many researchers, for example, by Levin (1969, 1971), Simpson (1969), Abara (1989), Dobson and Lederer (1993), Hane et al. (1995), Clarke et al. (1996), Yan and Young (1996), Desaulniers et al. (1997), Yan and Tseng (2002), Barnhart et al. (2002), and Lohatepanont and Barnhart (2004). These models have been formulated as integer linear programs, mixed integer programs or multicommodity network flow problems. The objective has been to minimize a carrier’s operating cost or to maximize the system profit. The problems usually have been solved using exact solution methods or heuristics, such as the simplex method, the branch-and-bound technique, the cutting plane method, the Lagrangian relaxation-based algorithm, the column and row generation technique and other heuristics.

Apart from fleet routing and flight scheduling for passenger transportation, there has also been research devoted to freight transportation and fleet routing, for example, by Chan and Ponder (1979), Chestler (1985), Current et al. (1986, 1988), Powell and Sheffi (1989), Aykin (1995), and Jaillet et al. (1996). These have typically focused on airport selection and frequency planning in a service network design for long-term planning, which differs from our research, which focuses on fleet routing and flight scheduling for short-term operations. In addition, all of the aforementioned models handled passenger and freight transportation problems separately. The interrelationships that occur between passenger, cargo, and combi flights have been neglected. As a result, the schedules and fleet routes offered, may possibly decrease the system performance, or be inclined to be inaccurate and inefficient in actual operations.

To the best of the authors’ knowledge, no research has been done that integrates the passenger, the cargo, and the combi flights together to assist airlines in solving their multi-type fleet routing and flight scheduling problems. Therefore, in this research, on the basis of the carrier’s perspective, we develop an integrated scheduling model that combines passenger, cargo and combi flight schedules. The objective is to maximize the operating profit, subject to the related operating constraints. This model is capable of directly managing the interrelationship between the passenger, the cargo and the combi flight sources and is expected to be a useful planning tool for determining their suitable fleet routes and timetables in short-term operations.

We employ a network flow technique to construct the model. It includes multiple fleet-flow, passenger-flow, and cargo-flow networks. The model is formulated as an integer multiple commodity network flow problem that is characterized as NP-hard. Since the problem size is expected to be huge, the model is more difficult to solve than traditional passenger/cargo flight scheduling problems. To more efficiently solve the model given practical problems, we refer to recent efforts made to solve similar types of network flow problems (Yan and Young, 1996, and Yan and Tseng, 2002). A family of heuristics is developed based on
Lagrangian relaxation, a sub-gradient method, two heuristics for the upper bound solution and a flow decomposition algorithm. The development of an effective model, as well as an efficient solution algorithm, together are the focus of this study.

It should be mentioned that currently for this Taiwan airline, airport selection is typically part of long-term planning. It is usually performed before fleet routing and timetable setting. Thus, to facilitate problem solving, the selection at airport is not considered here. However, this model is extendable for such an application, when airport selection, fleet routing and timetable setting must be integrated together in short-term operations. In addition, the scope of this research is confined to pure fleet routing and flight scheduling. Although the scheduling process is closely related to the aircraft maintenance and the crew scheduling processes, these processes are usually separated, to facilitate problem solving (Teodorovic, 1988). Recently, Clarke et al. (1996) tried to develop a fleet assignment model that considered both maintenance and crew scheduling. However, according to the studied Taiwan airline, in practice, maintenance and crew constraints are rather flexible, due to the use of stand-by crews and a progressive maintenance policy. These activities are always planned after the fleet routes and flights schedules have been determined. To reduce the problem complexity, as in Yan and Young (1996) and Yan and Tseng (2002), we thus exclude these constraints in the modeling. The incorporation of these constraints into fleet/flight scheduling could be a topic of future research.

The rest of this paper is organized as follows: Section 2 introduces the proposed model. Section 3 develops the solution algorithms for solving the model. Section 4 describes the numerical tests. Finally, we conclude in Section 5.

2. THE MODEL
A multiple time-space network technique is applied to construct an integrated scheduling model for the purpose of maximizing the carrier profit. This model demands the optimal management of aircraft, passenger, and cargo movements within the network. The major elements in the modeling, including the fleet-flow time-space networks, the passenger-flow time-space networks, the cargo-flow time-space networks and the mathematical formulation, are described as follows.

2.1 The fleet-flow and the passenger-flow time-space networks
We adopt several time-space networks to formulate the multi-type fleet routing and flight scheduling problem. Each network indicates one specific type of potential fleet movement (i.e. passenger-fleet, cargo-fleet, or combi-fleet) within a certain time period (one week in this study) and a specific location. The vertical axis represents the time duration, while the horizontal axis indicates the airport location. A node stands for an airport at a specific time, while an arc designates an activity for an airplane. The arc flows express the flow of airplanes in the networks. There are three types of arcs: (1) Flight leg arc, (2) Ground arc and (3) Cycle arc. As well, the time-space network technique is also applied to indicate passenger movements corresponding to certain times and locations. There are three types of arcs: (1) Passenger delivery arc, (2) Passenger holding arc and (3) Passenger demand arc. To save the space, please refer to Yan and Tseng (2002) for a more detailed description about the fleet-flow and the passenger-flow time-space networks.
2.2 The cargo-flow time-space networks

The cargo-flow time-space networks shown in Figure 1 indicate cargo movements corresponding to certain times and locations. According to the time sensitivity cargos are divided into three time types, one day, four days, and one week. Therefore, unlike the fleet-flow/the passenger-flow time-space networks, each cargo-flow time-space network represents a specific OD-time-pair (e.g. OD pair 1->2 within one day). Note that the lengths of time are adjustable, according to actual cargo timeliness. For example, the time of the corresponding cargo-flow time-space network could be shorter for express deliveries, or longer for a less time sensitive cargo. These networks are designed to symmetrically correspond to the cargo-fleet-flow and the combi-fleet-flow time-space networks, so as to facilitate problem solving. Three types of arcs are described.

![Figure 1 Cargo-flow time-space networks](image)

2.2.1 Cargo delivery arc
A cargo delivery arc, marked by (1) in Figure 1, represents the transportation of cargo from one station to another on a flight leg. The transportation time is the same as the corresponding time block for the associated flight leg in the cargo-fleet-flow and the combi-fleet-flow time-space networks. The arc cost is a variable cost for handling the cargo, per unit weight, on this flight, and is, in general, very small compared to the flight cost. The arc flow’s upper bound is the aircraft’s cargo capacity (particularly for the cargo-fleet), the weight capacity for the studied airline. The arc flow’s lower bound is set to be zero, meaning that no cargo from the corresponding OD is delivered on the associated flight leg.

2.2.2 Cargo holding arc
A cargo holding arc, marked by (2) in Figure 1, indicates that the cargos are held at an airport in a time window. The arc cost for this time window is a holding (or penalty) cost. In
practice, the holding of a cargo before or after delivery is usually not decided by the airline. Therefore, similar to the passenger holding arc, if the arc just happens to connect either the departure or the arrival station of this network’s corresponding OD-time-pair, the arc cost is then zero. Certainly, it is adjustable. A suitable holding cost for special cases can be imposed. The arc flow’s upper bound is the station’s cargo service capacity (or infinity, if the capacity is relatively large), meaning that the maximum amount of cargo (in weight units appropriate for the studied airline) can be accommodated at this airport. The arc flow’s lower bound is set to be zero, indicating that no cargo from the corresponding OD-time-pair is held at the airport during the time window.

### 2.2.3 Cargo demand arc
A cargo demand arc, see (3) in Figure 1, shows the service demand for the OD-time-pair that would actually be served in the network. It connects the arrival station to the departure station of the corresponding network OD-time-pair. The time interval for the demand arcs (i.e. the arc density) is set to be the time length of the corresponding cargo-flow time-space network (i.e. one day, four days, or one week). The arc cost is the negative value of the cargo per unit weight delivered. Note that the cargo fare structure that an airline uses to charge a forwarder or a shipper is, in general, in a decreasing ladder form (i.e. a concave function in terms of accumulation of cargo). However, the cargo amount transport on a flight is far larger than the amount charged an individual forwarder or shipper. Consequently, in the flight scheduling, an average cargo fare per unit weight is usually used. The arc flow’s upper bound is the projected demand for this OD-time-pair. The aim is to maximize carrier profit, meaning that in the model for this OD-time-pair not all cargo will necessarily be served in the model. The arc flow’s lower bound is set to be zero, implying that none of the OD-time-pair’s cargos are served in the network.

### 2.3 The model formulation
In addition to the three aforementioned major elements, there are several operating constraints that need to be considered, including specifically the number of available airplanes in each fleet, the quota for each airport/airport pair, and the airplane’s capacity, respectively. As well, the same flight leg in the passenger-fleet-flow and the combi-fleet-flow networks can be served at most once. Similarly, the same flight leg in the cargo-fleet-flow and the combi-fleet-flow networks can be served at most once. Note that according to current Taiwan airline regulation, combi flights are usually incorporated into passenger flights, for determining the quota for each airport/airport pair. Moreover, in order to facilitate problem solving, we set all the arc flows in each passenger-flow and cargo-flow network to be real variables, since such simplification does not have a significant effect on the results in the planning stage (Yan and Chen, 2002). We now list the notation and symbols used in the model formulation:

\[
X_{ij}^{m}, P_{ij}^{l}, Y_{ij}^{n}: \text{arc}(i,j) \text{ flow in the } m^{\text{th}} \text{ fleet-flow, the } l^{\text{th}} \text{ passenger-flow, and the } n^{\text{th}} \text{ cargo-flow networks, respectively};
\]

\[
C_{ij}^{m}, O_{ij}^{l}, T_{ij}^{n}: \text{arc}(i,j) \text{ cost in the } m^{\text{th}} \text{ fleet-flow, the } l^{\text{th}} \text{ passenger-flow, and the } n^{\text{th}} \text{ cargo-flow networks, respectively};
\]

\[
V_i: \text{a variable cost at station } i \text{ for handling cargo per unit weight, including the loading and unloading cargo;}
\]

\[
m, M: \text{the } m^{\text{th}} \text{ type of fleet (i.e. the } m^{\text{th}} \text{ fleet-flow network) and the set of all fleets. In particular, } m=1, 2, \text{ and } 3 \text{ represent the passenger-fleet-flow, the combi-fleet-flow, and the cargo-fleet-flow networks, respectively;}
\]

\[
l, L: \text{the } l^{\text{th}} \text{ OD pair (corresponding to the } l^{\text{th}} \text{ passenger-flow network) and the set of all}
\]
ODs; 
n, N: the \( n \)th OD-time pair (i.e. the \( n \)th cargo-flow network) and the set of all OD-time pairs;

\( A_m, NF_m \): the set of all arcs and nodes in the \( m \)th fleet-flow network;

\( D_l, NG_l \): the set of all arcs and nodes in the \( l \)th passenger-flow network;

\( B_n, NP_n \): the set of all arcs and nodes in the \( n \)th cargo-flow network;

\( AF_m \): the number of available airplanes in the \( m \)th fleet-flow network;

\( FF_m, CF_m \): the set of all flight leg arcs and cycle arcs in the \( m \)th fleet-flow network, respectively;

\( BF_n \): the set of all cargo demand arcs in the \( n \)th cargo-flow network;

\( SE^{ab}, SF^{ab} \): the set of all flight leg arcs that connect the \( a \)th to the \( b \)th stations in both the passenger- and combi-fleet-flow networks, and the cargo-fleet-flow network;

\( SG^a, SH^a \): the set of all flight leg arcs associated with the \( a \)th station in both the passenger- and combi-fleet-flow networks, and the cargo-fleet-flow network;

\( QA^{ab}, QB^{ab} \): the approved passenger flight and cargo flight quotas that connect the \( a \)th to the \( b \)th stations;

\( QC^a, QD^a \): the approved passenger flight and cargo flight quotas at the \( a \)th station;

\( \alpha_j, \beta_j \): the aircraft’s passenger capacity in the passenger-fleet-flow and the combi-fleet-flow networks (a planning load factor could be used in the planning stage);

\( \gamma_j, \delta_j \): the aircraft’s cargo capacity in the combi-fleet-flow and the cargo-fleet-flow networks (a planning load factor could be used in the planning stage);

\( SA, SB \): the set of airport pairs with approved passenger flight and cargo flight quotas;

\( SC, SD \): the set of all stations in both the passenger- and combi-fleet-flow networks, and the cargo-fleet-flow network;

\( U_{ij}^m, U_{ij}^l, U_{ij}^n \): arc \((i,j)\) flow’s upper bound in the \( m \)th fleet-flow, the \( l \)th passenger-flow, and the \( n \)th cargo-flow networks, respectively.

The model formulation is shown as follows:

Model (A):

\[
\text{Min} \quad \sum_{m \in M} \sum_{j \in J} C_{ij} X_{ij}^m + \sum_{l \in L} \sum_{j \in J} O_{ij}^l P_{ij}^l + \sum_{n \in N} \sum_{j \in J} T_{ij}^n Y_{ij}^n + \sum_{n \in N} \sum_{j \in J} Y_{ij}^m (V_i + V_j)
\]

(1)

\[
\sum_{j \in J} X_{ij}^m - \sum_{k \in K_j} X_{ij}^k = 0 \quad \forall i \in NF_m, \forall m \in M
\]

(2)

\[
\sum_{j \in J} P_{ij}^l - \sum_{k \in K_j} P_{ij}^k = 0 \quad \forall i \in NG_l, \forall l \in L
\]

(3)

\[
\sum_{j \in J} Y_{ij}^n - \sum_{k \in K_j} Y_{ij}^n = 0 \quad \forall i \in NP_n, \forall n \in N
\]

(4)

\[
\sum_{j \in J} X_{ij}^m \leq AF_m \quad \forall m \in M
\]

(5)

\[
\sum_{j \in J} X_{ij}^m \leq 1 \quad \forall ij \in FF_1 \cap FF_2
\]

(6)

\[
\sum_{j \in J} X_{ij}^m \leq 1 \quad \forall ij \in FF_2 \cap FF_3
\]

(7)

\[
\sum_{m \in [1,2]} \sum_{j \in SE^{ab}} X_{ij}^m \leq QA^{ab} \quad \forall ab \in SA
\]

(8)
\[
\sum_{y \in SB} X_{ij}^a \leq QB_{ab} \quad \forall ab \in SB \tag{9}
\]
\[
\sum_{m=1}^{3} \sum_{ij \in AG} X_{ij}^m \leq QC^a \quad \forall a \in SC \tag{10}
\]
\[
\sum_{y \in SF} X_{ij}^a \leq QD^a \quad \forall a \in SD \tag{11}
\]
\[
\sum_{l \in \text{dist.}} P_{ij}^l \leq (\alpha_{ij} X_{ij}^1 + \beta_{ij} X_{ij}^2) \quad \forall ij \in FF_1 \cap FF_2 \tag{12}
\]
\[
\sum_{m \in M} Y_{ij}^m \leq (\gamma_{ij} X_{ij}^2 + \delta_{ij} X_{ij}^3) \quad \forall ij \in FF_2 \cap FF_3 \tag{13}
\]
\[
0 \leq X_{ij}^m \leq U_{ij}^m \quad \forall ij \in A_m, \forall m \in M \tag{14}
\]
\[
0 \leq P_{ij}^l \leq U_{ij}^l \quad \forall ij \in D_l, \forall l \in L \tag{15}
\]
\[
0 \leq Y_{ij}^m \leq U_{ij}^m \quad \forall ij \in B_n, \forall n \in N \tag{16}
\]
\[
X_{ij}^m \in I \quad \forall ij \in A_m, \forall m \in M \tag{17}
\]

Model (A) is formulated as a mixed integer multiple commodity network flow problem, in which the objective function (1) is to minimize the system cost. Since the revenues from the passenger-flow and the cargo-flow networks are included in the form of negative costs, this objective is equivalent to the maximization of profit. Constraints (2), (3) and (4) ensure flow conservation at every node in each fleet-flow, passenger-flow or cargo-flow network. Constraint (5) indicates that the number of airplanes used in each fleet-flow network should not exceed the number of available airplanes for that fleet. Constraint (6) denotes that the same flight leg, in the passenger-fleet-flow and the combi-fleet-flow networks, is served at most once. Constraint (7) denotes that the same flight leg, in the combi-fleet-flow and the cargo-fleet-flow networks, is served at most once. Constraints (8) and (9) respectively ensure that the sum of all flights for each airport pair does not exceed the approved passenger flight and cargo flight quotas, respectively. Constraints (10) and (11) respectively ensure that the sum of all flights at each station does not exceed its approved passenger flight and cargo flight quotas, respectively. Constraints (12) and (13) respectively keep the passenger and cargo delivery rates within the airplane’s carrying capacity. Constraints (14), (15), and (16) hold that all the arc flows are within their upper and lower bounds. Constraint (17) ensures the integrality of the airplane flows.

3. SOLUTION ALGORITHM
The model is formulated as a mixed integer program that is characterized as NP-hard. We adopt the Lagrangian relaxation technique, coupled with a sub-gradient method, to develop a family of heuristics to solve the problem. The solution processes of the heuristic methods are the same. We first relax the side constraints (constraints (5) to (13)) to construct a Lagrangian problem, and then solve it, to procure the lower bound of the optimal solution. Secondly, two heuristics, UP1 and UP2, are developed to solve for the upper bound of the optimal solution. Then, a sub-gradient method for revising the Lagrangian multipliers is utilized to iterate the lower and upper bounds, until an acceptable convergence result is reached, or until the number of iterations exceeds a preset number. For ease of writing, we first define the family of heuristics. The main differences are as follows:

1. UP1_Z: The upper bound of the optimal solution is found using UP1 and the initial Lagrangian multipliers are set to be zero.
2. UP2_Z: The upper bound of the optimal solution is found using UP2 and the initial Lagrangian multipliers are set to be zero.
3. UP1_O: The upper bound of the optimal solution is found using UP1 and the initial Lagrangian multipliers are set to be the extra volume of the violated constraints for the optimal solution of the linear program (by relaxing the integer constraints (17)).
4. UP2_O: The upper bound of the optimal solution is found using UP2 and the initial Lagrangian multipliers are set to be the extra volume of the violated constraints for the optimal solution of the linear program (by relaxing the integer constraints (17)).

3.1 UP1_Z
The major parts of UP1_Z, including the lower bound of the optimal solution, the upper bound of the optimal solution, the sub-gradient method and the solution process, are addressed as follows:

3.1.1 The lower bound of the optimal solution
The steps for searching for the lower bound are listed below:

Step 1: Side constraints (5) to (13) are relaxed with the corresponding non-negative Lagrangian multiplier sets, $\mu_5$ to $\mu_{13}$, and are added to the objective function of Model (A), resulting in Model (B). The optimal objective value for Model (B) becomes the lower bound of Model (A).

Model (B):
\[
\text{Min } \sum_{m} \sum_{ij} C_{ij}^m X_{ij}^m + \sum_{i} \sum_{j} O_i^j P_i^j + \sum_{n} \sum_{i} \sum_{j} T_{ij}^n Y_{ij}^n + \sum_{n} \sum_{i} \sum_{j} Y_{ij}^n \left( V_i + V_j \right) \\
+ \sum_{m} \sum_{ij} \mu_5^m \left( \sum_{ij} X_{ij}^m - AF_m \right) + \sum_{ij} \mu_6^j \left( \sum_{ij} X_{ij}^m - 1 \right) + \sum_{ij} \mu_7^j \left( \sum_{ij} X_{ij}^m - 1 \right) \\
+ \sum_{ab} \mu_8_{ab} \left( \sum_{ij} X_{ij}^m - QA_{ab} \right) + \sum_{ab} \mu_9_{ab} \left( \sum_{ij} X_{ij}^m - QB_{ab} \right) \\
+ \sum_{ij} \mu_10^j \left( \sum_{ij} X_{ij}^m - QC_{ij}^a \right) + \sum_{ij} \mu_11_{ij} \left( \sum_{ij} X_{ij}^m - QC_{ij}^b \right) \\
+ \sum_{ij} \mu_12_{ij} \left( \sum_{ij} \left( P_i^j - (\alpha_y X_i^1 + \beta_y X_i^2) \right) \right) + \sum_{ij} \mu_13_{ij} \left( \sum_{ij} \left( Y_{ij}^n - (\gamma_y X_i^2 + \delta_y X_i^3) \right) \right)
\]
subject to constraints (2), (3), (4), (14), (15), (16), and (17).

Step 2: Model (B) is Decomposed, omitting the constant terms (e.g. $-\mu_5^m \cdot AF_m$), into five independent groups of networks, such as the passenger-fleet-flow network, the combi-fleet-flow network, the cargo-fleet-flow network, the passenger-flow networks and the cargo-flow networks.

Step 3: The five networks are pure network flow problems, and are also characterized as minimum cost network flow problems, which can be solved directly using the mathematical programming solver, CPLEX.

Step 4: The lower bound of the optimal solution is obtained by summing up all five network costs and the constant terms.

3.1.2 The upper bound of the optimal solution
UP1 is used to find an upper bound (a feasible solution). The steps are listed below. For ease of introduction, we first define the symbols that are used in the heuristic as follows:

PFFN, CFFN, CAFFN: the passenger-fleet-flow network, the combi-fleet-flow network
and the cargo-fleet-flow network.

FFNS, PFNS, CFNS: the fleet-flow networks, the passenger-flow networks and the cargo-flow networks. In particular, FFNS includes PFFN, CFFN and CAFFN.

PFN, CFN: a passenger-flow network and a cargo-flow network.

MFFNS, MPFNs, MCFNS: the modified fleet-flow networks, the modified passenger-flow networks and the modified cargo-flow networks.

$pff_i^a, pff_i^b$: the passenger-fleet flows in the $i^{th}$ iteration, where subscripts a and b indicate whether the flows are infeasible or feasible.

cff_i^a, cff_i^b: the combi-fleet flows in the $i^{th}$ iteration, where subscripts a and b indicate whether the flows are infeasible or feasible.

caff_i^a, caff_i^b: the cargo-fleet flows in the $i^{th}$ iteration, where subscripts a and b indicate whether the flows are infeasible or feasible.

$ff_i^a, ff_i^b$: the fleet flows in the $i^{th}$ iteration, where subscripts a and b denote whether the flows are infeasible or feasible. $ff_i^a$ includes $pff_i^a, cff_i^a$ and $caff_i^a$, while $ff_i^b$ includes $pff_i^b, cff_i^b$ and $caff_i^b$.

$pflow_i^a, pflow_i^b$: the passenger flows in the $i^{th}$ iteration, where subscripts a and b indicate whether the flows are infeasible or feasible.

cflow_i^a, cflow_i^b: the cargo flows in the $i^{th}$ iteration, where subscripts a and b indicate whether the flows are infeasible or feasible.

$\Delta pflow_i^a, \Delta pflow_i^b$: the increased passenger flows for $sol_i$, where subscripts a and b indicate whether the increased flows do not or do assure that the passenger delivery volume is within the aircraft capacity.

$\Delta cflow_i^a, \Delta cflow_i^b$: the increased cargo flows for $sol_i$, where subscripts a and b indicate whether the increased flows do not or do assure that the cargo delivery volume is within the aircraft capacity.

$sol_i$: the feasible upper bound solution in the $i^{th}$ iteration, including $ff_i^b$, $pflow_i^b$, and $cflow_i^b$.

$obj_i$: the objective value of $sol_i$.

npflow_{i+1}^b, ncflow_{i+1}^b$: the new feasible passenger and cargo flows in the $(i+1)^{th}$ iteration.

nsol_{i+1}: the new feasible upper bound solution in the $(i+1)^{th}$ iteration.

nobj_{i+1}: the objective value of $nsol_{i+1}$.

The steps of UP1 are listed below:

Step1: Let the passenger and the cargo flows for the initial lower bound solution be $pflow_1^a$ and $cflow_1^a$, respectively. Solve $ff_1^b$ (including $pff_1^b, cff_1^b$ and $caff_1^b$), based on $pflow_1^a$ and $cflow_1^a$ as follows. First, the modified fleet-flow network (MFFNS) are constructed the same as FFNS (including the fleet size and other related constraints), except for the cost of each flight leg arc. In particular, if the flight leg arc is in PFFN/CAFFN, then the arc cost must include the original operating cost plus the sum of all profits obtained from the corresponding passenger/cargo delivery arc flows of $pflow_1^a / cflow_1^a$. If the flight leg arc is in CFFN, then the arc cost includes the original operating cost plus the sum of all profits obtained from both the corresponding passenger and cargo delivery arc flows of...
Step 2: Solve $pflow^b_i$ based on $pff^b_i$ and $eff^b_i$. We construct the modified passenger-flow networks (MPFNS) which are the same as PFNS, except that the passenger delivery arc flows in MPFNS are restricted by the passenger loading constraint (12), based on $pff^b_i$ and $eff^b_i$. Then, solve MPFNS, using CPLEX, to find $pflow^b_i$.

Step 3: Find $cflow^b_i$ based on $eff^b_i$ and $caff^b_i$. We construct the modified cargo-flow networks (MCFNS) which are the same as CFNS, except that the cargo delivery arc flows in MCFNS are restricted by the cargo loading constraint (13), based on $eff^b_i$ and $caff^b_i$. Then, solve MCFNS to find $cflow^b_i$ using CPLEX.

Step 4: Achieve an initial feasible solution $sol_{i_1}$, and its objective value, $obj_{i_1}$ by combining $ff^b_i$, $pflow^b_i$, and $cflow^b_i$.

Step 5: Solve $\Delta pflow^a_i$ based on $sol_{i_1}$ to increase unserved passengers as follows. First, for every passenger delivery arc in each PFN, the flow upper bound is reset to be the capacity of the aircraft associated with the flight leg arc in $pff^b_i$ or $eff^b_i$, minus its arc flow in $pflow^b_i$. For every passenger demand arc in each PFN, recalculate the residual demand as the flow upper bound, which is equal to the projected passenger demand minus its arc flow in $pflow^b_i$. Then, solve PFNS to find $\Delta pflow^a_i$ using CPLEX based on the new flow upper bounds of the passenger delivery and the passenger demand arcs (all other parameters remain the same). Finally, in the same way, update the flow upper bounds of every passenger delivery and passenger demand arcs.

Step 6: Add up $pflow^b_i$ and $\Delta pflow^a_i$ to form $pflow_{i_1}^a$, which, together with $pff^b_i$ and $eff^b_i$, usually violates the passenger loading constraint (12).

Step 7: Using the same technique as in Step 5, solve $\Delta cflow^a_i$ based on $sol_{i_1}$ to increase the unserved cargos.

Step 8: Add up $cflow^b_i$ and $\Delta cflow^a_i$ to form $cflow_{i_1}^a$, which, along with $eff^b_i$ and $caff^b_i$, usually violates the cargo loading constraint (13).

Step 9: Referring to Step 1, solve $ff^b_i$ (including $pff^b_{i_1}$, $eff^b_{i_1}$, and $caff^b_{i_1}$) based on $pflow_{i_1}^a$ and $cflow_{i_1}^a$ obtained from Steps 6 and 8.

Step 10: Referring to Step 2, solve $pflow_{i_1}^b$ based on $pff_{i_1}^b$ and $eff_{i_1}^b$.

Step 11: Referring to Step 3, solve $cflow_{i_1}^b$ based on $eff_{i_1}^b$ and $caff_{i_1}^b$.

Step 12: By combining $ff^b_i$, $pflow^b_i$, and $cflow^b_i$, we find the feasible solution, $sol_{i_1}$, and its objective value, $obj_{i_1}$.

Step 13: If $obj_{i_1}$ is better than $obj_{i_1}$, then go to Step 5; else, go to Step 14.

Step 14: Solve $\Delta pflow^b_i$ based on $sol_{i_1}$ as follows. The method is similar to Step 5, except that the recalculation of the flow upper bound of every passenger delivery arc is different. To increase the passenger demand without violating constraint (12), we do not allow any flow to be augmented into the passenger delivery arcs.
when the corresponding flight leg arc flows in both $pff^b_{r_{i+1}}$ and $cff^b_{r_{i+1}}$ are zero. In other words, for every passenger delivery arc, if the associated flight leg arcs in $pff^b_{r_{i+1}}$ and $cff^b_{r_{i+1}}$ both equal zero, then its flow upper bound is set as zero.

Using the same technique as in Step 5, we can solve $\Delta pflow^b$.

Step 15: Add up $pflow^b_{r_{i+1}}$ and $\Delta pflow^b$ to form a new $pflow^b_{r_{i+1}}$ ($npflow^b_{r_{i+1}}$).

Step 16: Using the same technique as in Step 14, solve $\Delta cflow^b$ based on $sol_{r_{i+1}}$.

Step 17: Add up $cflow^b_{r_{i+1}}$ and $\Delta cflow^b$ to form a new $cflow^b_{r_{i+1}}$ ($ncflow^b_{r_{i+1}}$).

Step 18: Find a new $ff^b_{r_{i+1}}$ ($nff^b_{r_{i+1}}$) based on $npflow^b_{r_{i+1}}$ and $ncflow^b_{r_{i+1}}$. To do this, we first fix the $npflow^b_{r_{i+1}}$ and $ncflow^b_{r_{i+1}}$ variables in the objective function (1), constraints (3), (4), (12), (13), (15), and (16), and then solve the rest of Model (A). Finally, by combining $nff^b_{r_{i+1}}$, $npflow^b_{r_{i+1}}$, and $ncflow^b_{r_{i+1}}$, we find a new feasible solution and its objective value, that is, $nsol_{r_{i+1}}$ and $nobj_{r_{i+1}}$.

Step 19: If $nobj_{r_{i+1}}$ is better than $obj_{r_{i+1}}$, then update $sol_{r_{i+1}}$ and $obj_{r_{i+1}}$, and go to Step 14; else, we find the final feasible solution, $sol_{r_{i+1}}$.

3.1.3 The subgradient method and the solution process

Yan and Young’s (1996) sub-gradient method, for adjusting Lagrangian multipliers is applied in this research, so as to obtain good convergence in the iteration results. The steps of UP1_Z, a Lagrangian relaxation-based algorithm, are shown as follows:

Step 1: Set iteration $i = 0$ and the initial Lagrangian multiplier $\mu^i$ to be 0.

Step 2: Use CPLEX to solve Models (C), (D), (E), (F), and (G) to get a lower bound, $Z^L(\mu^i)$. If the solution is feasible and also satisfies the complementary slackness condition, then we have found an optimal solution and can stop the solution process. Otherwise, update the lower bound, $Z^L$.

Step 3: Apply the UP1 to find an upper bound, $Z^U(\mu^i)$, and update the upper bound, $Z^U$.

Step 4: If the gap between the lower bound, $Z^L$, and the upper bound, $Z^U$, falls within a specified tolerance, $\theta$ (i.e. $|Z^U - Z^L|/Z^U \leq \theta$), or the number of iterations reaches a preset limit, stop the algorithm.

Step 5: Adjust $\mu^i$ to help improve the convergence by applying the sub-gradient method developed in Yan and Young (1996).

Step 6: Set $i = i + 1$. Go to Step 2.

3.2 UP2_Z, UP1_O and UP2_O

3.2.1 UP2_Z

In this heuristic, we apply UP2 to find an upper bound. The other major parts of UP2_Z, including the lower bound of the optimal solution, the sub-gradient method and the solution process, are all the same as for UP1_Z. The idea behind UP2 is first to solve $pff^b_1$, $cff^b_1$ and $pflow^b$, then to solve $caff^b_1$ as well as $cflow^b$, based on the obtained $cff^b_1$. In particular, the UP2 steps in UP2_Z are the same as for UP1 in UP1_Z, except for steps 1 to 4, which are outlined below:

Step1: Let the cargo flows for the initial lower bound solution be $cflow^a_1$. 

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Step 2: Solve $p_{ff}^1$, $c_{ff}^1$, and $p_{flow}^1$ based on $c_{flow}^1$ as follows. First, a modified combi-fleet-flow network (MCFFN) is constructed, the same as CFFN, except that the cost of each flight leg arc in MCFFN includes the original operating cost plus the sum of all profits obtained from the corresponding cargo delivery arc flows for $c_{flow}^1$. Note that the sum of the profits is set to be at most the aircraft capacity. All other operating constraints, such as the fleet size, the available airport/airport pair quota, and the loading constraints, are the same as for model (A). Then, use CPLEX to solve PFFN, MCFFN, and PFNS together, to find $p_{ff}^1$, $c_{ff}^1$, and $p_{flow}^1$.

Step 3: Solve $c_{aff}^1$ and $c_{flow}^1$ based on $c_{ff}^1$. To do this, we fix the $c_{ff}^1$ variables in CFFN and then, use CPLEX to solve CFFN, CAFFN, and CFN together. Note that all the operating constraints are the same as in model (A). We have found $c_{aff}^1$ and $c_{flow}^1$.

Step 4: By combining $p_{ff}^1$, $c_{ff}^1$, $p_{flow}^1$, $c_{aff}^1$, and $c_{flow}^1$, we achieve an initial feasible solution $sol_1$, and its objective value, $obj_1$.

3.2.2 UP1_O and the UP2_O
To find a better initial lower bound, unlike UP1_Z/UP2_Z, we relax the integer constraint (17) in Model (A) and using CPLEX solve for the optimal solution of this linear program. We calculate the violated volume for every side constraint (i.e. constraints (5) to (13)), based on the linear optimal solution. Then, the initial Lagrangian multipliers in UP1_O/UP2_O are set to be the violated volume of the corresponding side constraints. All the major parts of UP1_Z/UP2_Z, including the lower bound of the optimal solution, the upper bound of the optimal solution, the sub-gradient method and the solution process, are all the same as for UP1_Z/UP2_Z.

It should be mentioned that the fleet flows obtained from the above process cannot yet be directly put into practice without identifying the route of each airplane. A flow decomposition method (Yan and Young, 1996) is applied to decompose the arc flows into arc chains. Each arc chain denotes an airplane’s route. Note that these arc chains may not be unique, while the objective values for different arc chain patterns are the same, since the fleet flows do not make any difference. In practice, carriers could apply several arc chain patterns provided they meet aircraft maintenance and crew scheduling constraints.

4. NUMERICAL TESTS
To test how well the model and the solution algorithms may be applied in the real world, we performed numerical tests using operational data from a major Taiwan airline, with reasonable assumptions. We used the C computer language, coupled with the mathematical programming solver, CPLEX 8.1, to develop all the necessary programs. The tests were performed on a Pentium 4 – 3.0GHz with 2Gb of RAM in the environment of Microsoft Windows 2000.

The numerical tests were mainly based on data obtained from a major Taiwan airline’s operations in Asia during 2002. There were 12 and 10 cities served by passenger and cargo services, respectively. Three types of aircraft were used, including 5 B767-300 passenger aircraft (226 seats each), 8 B747-400 comi aircraft (272 seats and 35 metric tons each), and 6 MD-11F cargo aircraft (80 metric tons each). The planning passenger and cargo load factors for each flight were set not to exceed 0.7 and 0.8, respectively. All the cost parameters and
other fleet-flow, passenger-flow and cargo-flow time-space network inputs were set primarily based on the actual operating data and Taiwan government regulations, with reasonable simplifications. Altogether, the test contained three layers of fleet-flow time-space networks, twenty-two layers of passenger-flow time-space networks, and twenty-eight layers of cargo-flow time-space networks, involving 82,243 nodes and 443,251 arcs (variables). The model, which was substantially large in terms of combinatorial optimization, included 158,246 constraints, in which 82,243 constraints were for ensuring flow conservation and the other 76,003 constraints were side constraints.

To evaluate the feasibility of applying the exact solution method to the model (A), we used CPLEX to solve model (A) for different limited computational times. For ease of writing, we denoted that OBJ represents the objective function value obtained. LB represents the best objective function value of all the unexplored nodes in the branch-and-bound tree obtained by using CPLEX, which can serve as a lower bound of the problem. LBG (%) denotes the gap between OBJ and LB. We found that the LBG was about 27.08% when the computational time was limited to 12 hours. Even with a computational time of up to 72 hours, the LBG was about 15.01%. These results show that it is difficult to optimally solve the model simply by using a commercial optimization solver, such as CPLEX.

For ease of comparison, the OBJs were obtained by the 4 heuristic methods within a limited computational time of 12 hours. The percentage of improvement obtained using CPLEX is defined as follows:

$$IPC (%) = \left( \frac{OBJ_{UP1\_O\_UP1\_O} - OBJ_{CPLEX}}{OBJ_{CPLEX}} \right) \times 100\%$$  \hspace{1cm} (19)$$

As shown in Table 1, in general, UP2_O/UP2_Z could provide a better result than UP1_O/UP1_Z. However, the OBJs obtained by the 4 heuristic methods were all better than those obtained by CPLEX. In particular, for UP2_Z/UP2_O, the IPC was about 26.71/27.53%. Moreover, the LBGs of the 4 heuristic methods were all better than those of CPLEX. In particular, for UP2_Z/UP2_O, compared with CPLEX, the LBG was improved from 27.08% to 7.61/7.01%. These results indicate that our heuristics are significant improvement over CPLEX, and have the potential to solve huge scheduling problems. Moreover, we also found that in UP2_O (with the best OBJ), 8 airplanes were used to provide 272 flights/a week. The average passenger and cargo load factors in UP2_O were 66.76% and 69.82%, respectively. The passenger and cargo service rates of UP2_O were both the highest, 97.23% and 95.81%, respectively. These results show that UP2_O could use the resources more efficiently than could the other heuristics.

It should be mentioned that in UP1/UP2 we have also tried different approaches to solve the problem. In particular, we randomly generated the cost of each flight leg arc, instead of using that obtained in Step 1 of UP1, and then solved MFFNs. This process was repeated to find the various $f^b_{1\_i}$s. Then, we selected the best $f^b_{1\_i}$ to be the feasible fleet flow in Step1 of UP1. In addition, compared with UP2, we also tried to solve $cf^b_{1\_i}$, $caff^b_{1\_i}$, and $cflow^b_{1\_i}$ first, then to solve $pff^b_{1\_i}$ as well as $pflow^b_{1\_i}$, based on the obtained $cf^b_{1\_i}$. However, the obtained results were all inferior to that obtained by UP2_Z/UP2_O. That is, the LBGs of these approaches were all greater than 7.0%. These results indicate that it is practically difficult to optimally solve such a huge scheduling problem,
with more than 443,000 variables and 158,000 constraints. The heuristics, UP2_Z and UP2_O, have the potential to be useful for the efficient solution of such problems.

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5. CONCLUSIONS
In this research, on the basis of the carrier’s perspective, we develop an integrated scheduling model that combines passenger, cargo and combi flight schedules. The model is capable of directly managing passenger, cargo and combi flight source interrelationships. A family of heuristics, based on Lagrangian relaxation, a sub-gradient method, four self-developed upper bound heuristics and a flow decomposition algorithm, is developed to efficiently solve the problem. Numerical tests, utilizing data from a major Taiwan airline’s operations, were performed to evaluate the model and the heuristics. The test results show that the model and the solution methods have the potential to be useful for solving such huge scheduling problems and could be useful for carriers in real operations.

Finally, other operating constraints (e.g. airport selection, maintenance and crew scheduling) or other objectives may be incorporated into the model. Furthermore, for practical large-scale problems, the heuristic method could be further improved. For example, the upper bound heuristic or the sub-gradient method may be improved. As well, other useful algorithms or modern meta-heuristic techniques, for example, column generation, genetic algorithms, tabu search methods or threshold accepting methods, may be developed or incorporated into the algorithm to help solve the problem. All of these model and solution algorithm developments could be a direction for future research.

ACKNOWLEDGEMENTS
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