NEW FULL-RESPONSE $Q^2$PSK SIGNAL FORMAT

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ABSTRACT
Based on a novel class of orthogonal pulse pairs, new full-response quadrature-quadrature phase-shift keying ($Q^2$PSK) signal format is proposed in the paper. The proposed full-response $Q^2$PSK signals can provide higher spectral compactness than conventional full-response $Q^2$PSK signal, minimum-shift keying signal and quadrature phase-shift keying signal.

KEY WORDS
Quadrature-quadrature phase-shift keying, minimum-shift keying, full-response signaling, pulse shaping.

1 Introduction
Quadrature-quadrature phase-shift keying ($Q^2$PSK) is a four-dimensional linear memoryless modulation signal which is constructed by an orthogonal multiplexing of two quadrature phase-shift keying ($QPSK$) component signals, both modulated on the same quadrature carrier but shaped by distinct orthogonal pulses [1]-[11]. Because two spectrally overlapping component QPSK signals are orthogonally multiplexed, $Q^2$PSK provides the same error performance as QPSK over the ideal additive white Gaussian noise channel, while yielding better spectral efficiency when orthogonal pulses of finite duration are used [1], [4], [10]. In [1], $Q^2$PSK made its debut in terms of an orthogonal pair of sinusoidal and cosinusoidal pulses that extend over one symbol time (i.e., full-response), and was shown to use the available signal space more efficiently than the conventional two-dimensional QPSK and minimum-shift keying (MSK) modulation schemes, while at sacrifice of envelope constancy. To achieve even higher spectral efficiency, smoother pulses of infinite duration [1] or finite duration [4] were also suggested at the cost of higher implementation complexity and longer demodulation delay. Motivated by the spectral prevalence, $Q^2$PSK has been studied extensively in various issues of practical interest, including the signal synchronization [1], [8]-[9] and the error performance characteristics over bandlimited channels [1]-[3], nonlinear satellite channels [4]-[5], coded channels [5]-[6], and differentially coherent channels [6], [11].

In this paper, a novel class of orthogonal pulse pairs is proposed for constructing new full-response $Q^2$PSK signals. Section II gives the proposed full-response $Q^2$PSK. Analytically, the power spectrum of the proposed $Q^2$PSK signal with a pulse order $N$ is shown to decay asymptotically with $f^{-2N-2}$. Numerical results also show that the proposed full-response $Q^2$PSK can provide higher spectral compactness than conventional full-response $Q^2$PSK, MSK and QPSK. Section III concludes the paper.

2 New Full-Response $Q^2$PSK Signals
In the nominal symbol interval $|t| \leq T/2$, the proposed (unit-power) memoryless full-response $Q^2$PSK signal is expressed as

$$s(t) = [a_I p(t) \cos(2\pi f_c t) + a_Q p(t) \sin(2\pi f_c t)] + [b_I q(t) \cos(2\pi f_c t) + b_Q q(t) \sin(2\pi f_c t)].$$

Here, \((a_I, a_Q)\) and \((b_I, b_Q)\) are two QPSK data phasors where all data components are independent and identically distributed random variables and take value from \(\{-1/\sqrt{2}, 1/\sqrt{2}\}\) with equal probability. \(p(t)\) and \(q(t)\) are mutually orthogonal pulse shapes defined on $|t| \leq T/2$ and, for notational simplicity, normalized to have energy two. Assuming that the carrier frequency $f_c$ is much larger than the bandwidth of $p(t)$ and $q(t)$, \(\{p(t) \cos(2\pi f_c t), q(t) \cos(2\pi f_c t), p(t) \sin(2\pi f_c t), q(t) \sin(2\pi f_c t)\}\) forms a set of basis waveforms which have unit energy and are orthogonal (i.e., orthonormal) over $|t| \leq T/2$. Within the modeling, the lowpass equivalent power spectral density (PSD) of $s(t)$ is given by

$$S(f) = \frac{1}{4} |P(f)|^2 + \frac{1}{4} |Q(f)|^2$$

where $P(f)$ and $Q(f)$ denote the Fourier transforms of $p(t)$ and $q(t)$, respectively. Because $P(f)$ and $Q(f)$ contribute equally to $S(f)$, the pulse properties of $p(t)$ and $q(t)$ are...
both of paramount importance to the spectral property of full-response Q\textsuperscript{2}PSK.

In [1], \( p(t) \) and \( q(t) \) are orthogonal half-period sinusoids \( p(t) = 2T^{-1/2} \cos(\pi t/T)u(t) \) and \( q(t) = 2T^{-1/2} \sin(\pi t/T)u(t) \) where \( u(t) \) is the rectangular window with \( u(t) = 1 \) if \( |t| \leq T/2 \) and \( u(t) = 0 \) otherwise. This defines the conventional full-response Q\textsuperscript{2}PSK [1] which has the lowpass equivalent PSD

\[
S(f) = \frac{\sin^2(\pi fT + \pi/2)}{2(\pi fT + \pi/2)^2} + \frac{\sin^2(\pi fT - \pi/2)}{2(\pi fT - \pi/2)^2}
\] (3)

Notably, \( S(f) \) decays asymptotically as \( f^{-2} \), primarily due to the pulse property that \( q(t) \) is discontinuous at \( t = \pm T/2 \) [12]. Such a decay rate resembles that of rectangularly pulsed QPSK, but is slower than that of MSK in [13] which has \( S(f) \) decaying asymptotically as \( f^{-4} \). Hence, the conventional Q\textsuperscript{2}PSK is apt to cause interchannel interference in bandlimited applications [3].

In this paper, a class of pulse pairs is proposed for \( p(t) \) and \( q(t) \) in a way that \( S(f) \) decays asymptotically with a rate faster than \( f^{-2} \). Specifically, we define

\[
p(t) = 2\alpha \sum_{n=0}^{N} \binom{N}{n} \cos(2n\pi t/T)u(t) \] (4)

\[
q(t) = 2\beta \sum_{n=1}^{N} \binom{N}{n} \sin(2n\pi t/T)u(t) \] (5)

where the integer \( N \) denotes the pulse order, and \( \alpha \) and \( \beta \) are gain factors given by \( \alpha = [T(2N)/N]^{-1/2} \) and \( \beta = [T(2N)/N - 1]^{-1/2} \). The transmitter model is shown in Fig. 1. With a little effort, it is straightforwardly shown that both pulse functions in (4) and (5) have all derivatives, up to the \( (N-1) \)-th order, equal to zero at \( t = \pm T/2 \). As such, both \( |P(f)|^2 \) and \( |Q(f)|^2 \) decay asymptotically as \( f^{-2N-2} \) [12], and so does \( S(f) \). This spectral property can be further verified as follows.

The lowpass equivalent PSD of Q\textsuperscript{2}PSK using (4) and (5) is given by

\[
S(f) = \frac{\sin^2(\pi fT)}{\pi^2 f^2} \cdot \left\{ \alpha^2 \sum_{n=0}^{N} \binom{N}{n} (-1)^n \left( 1 - \frac{n^2}{f^2 T^2} \right)^{-1} \right\}^2 + \beta^2 \left\{ \sum_{n=1}^{N} \binom{N}{n} (-1)^n \left( 1 - \frac{n^2}{f^2 T^2} \right)^{-1} \right\}^2 \] (6)

where \( \{|x|\} \) represents the smallest integer that is larger than \( x \). It is noted from (7) that \( S(f) \) decays asymptotically with \( |fT|^{-2N-2} \) as \( |fT| \) approaches to infinity. This implies that the signal spectrum for the proposed full-response Q\textsuperscript{2}PSK exhibits faster rolloff at spectral sidelobes when \( N \) is increased.

\[14, \text{ eq. 1.184.3}\] as

\[
S(f) = \frac{\sin^2(\pi fT)}{\pi^2 f^2} \cdot \left\{ \alpha^2 \sum_{m=\{|\frac{N}{T}\}+1}^{N} \binom{N}{m} (-1)^m \left( \frac{m}{fT} \right)^{2m} \right\}^2 + \beta^2 \left\{ \sum_{m=\{|\frac{N}{T}\}}^{N} \binom{N}{m} (-1)^m \left( \frac{m}{fT} \right)^{2m+1} \right\}^2 \] (7)

3 Numerical Results

Fig. 2 compares the spectral compactness characteristics of the proposed Q\textsuperscript{2}PSK with conventional full-response Q\textsuperscript{2}PSK in [1], MSK in [13], and rectangularly pulsed QPSK. The spectral compactness is characterized by the fraction of total power \( \eta \) that is not captured within the normalized frequency band \([-BT_b/2, BT_b/2]\), where \( B \) and \( T_b \) denote the bandwidth and the bit time, respectively. Therefore, to achieve a fixed \( \eta \) value, denoted by \( \eta_0 \), the signal requiring a smaller \( BT_b \) exhibits higher spectral compactness. The ideal performance of Nyquist-pulsed QPSK is also plotted to serve as a benchmark. As indicated, the spectral compactness of the proposed Q\textsuperscript{2}PSK depends on the pulse order \( N \). For example, the highest compactness can be achieved by the proposed Q\textsuperscript{2}PSK.
with $N = 1, 2, 3$ and $4$ when the required $\eta_0$ falls in the decibel ranges $[0, -25.1], [-25.1, -45.7], [-45.7, -61.5]$, and $[-61.5, -75.9]$, respectively. Particularly, the proposed $Q^2$PSK with $N = 1$ outperforms MSK uniformly for all $\eta_0$ values and conventional $Q^2$PSK when $\eta_0$ is smaller than $-11.1$ dB. Moreover, the proposed $Q^2$PSK with $N = 1$ provides the highest spectral compactness among the demonstrated full-response modulation signals when $\eta_0$ is around $-20$ dB (the measure of practical interest).

4 Conclusion

In the paper, new full-response $Q^2$PSK signals are constructed from a novel class of orthogonal pulse pairs. The proposed full-response $Q^2$PSK signal with a pulse order $N$ can provide the compact power spectrum which decays asymptotically with $f^{-2N-2}$ and significantly outperform conventional full-response $Q^2$PSK, MSK and QPSK when the required fraction of out-of-band power is small. In particular, the proposed full-response $Q^2$PSK with $N = 1$ outperforms MSK uniformly in spectral compactness.

References


