Estimation of Timing Delay and Frequency Offset Using a Dual-Chirp Sequence

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Abstract—A novel estimation technique of timing delay and frequency offset that takes advantage of a dual-chirp signal is proposed and investigated in this paper. The proposed technique can be implemented with low hardware or computational complexity by using fully-digital circuitry or a digital signal processor (DSP). In addition, it can achieve sufficiently low errors both on timing delay and frequency offset estimation by effectively analyzing only a single burst. Furthermore, the proposed technique enjoys higher resistance against various kinds of interference than conventional techniques. Its superiority is verified by means of simulations in conjunction with statistical analysis.

I. INTRODUCTION

Timing synchronization and frequency offset compensation are the most important for any wireless communication applications. Very often, pseudo-noise (PN) sequences are employed as training preambles for the purpose of timing synchronization in a variety of wireless transmission systems [1], [2]. Although PN sequences can provide high-resolution timing synchronization based on their good correlation properties by taking advantage of PN matched filtering (PN MF), they unfortunately cannot be applied to estimation or compensation of frequency offset [3], [4]. In addition, the cross-correlation between the received signal and a locally-generated PN sequence may be significantly destroyed due to phase rotations within the correlation/accumulation period in the PN MF in which a high frequency offset exists [3]. In order to take care of an initial frequency offset, continuous wave (CW) signals are often applied in conjunction with the PN training preamble. In a typical arrangement, a receiver can first analyze a CW signal to accurately estimate the frequency offset, and then compensate the frequency offset for the subsequent PN correlation process. However, such a conventional approach not only increases the number of preambles required by the synchronization sub-systems, but also sacrifices system performance and robustness against interference because the CW signal is vulnerable to any forms of interference. The conventional method also requires significant hardware complexity because the CW frequency offset estimation method cannot be easily performed in discrete-time form on a digital signal processor (DSP), as discussed in recent research studies [5].

A particular synchronization burst, called dual-chirp signal, was originally addressed in Geostationary Earth Orbit (GEO) Mobile Radio (GMR) mobile-satellite communications [6], [7]. The dual-chirp signal, which can be thought of as a spread spectrum (SS) waveform, can actually offer significant processing gain to combat various kinds of interference [8]. Moreover, it enables a joint estimation of timing and frequency errors. In fact, chirp signals have been commonly used in radar applications, such as pulse compression radar [8], [9]. In practice, joint estimation of timing delay and frequency offset using dual chirp in the GMR mobile-satellite communications were previously presented [10], [11]. However, the main points in [10], [11] are limited in the applicability, reducibility and implementation. Both of the time-domain (TD) and frequency-domain (FD) estimators proposed in the previous works [10], [11] jointly estimate the timing delay and frequency offset simultaneously with reduced complexity at the cost of degradation of the estimates. In other words, the TD estimator proposed in the previous works [10], [11] can accurately estimate the timing delay but can only approximately estimate the frequency offset, meanwhile the FD estimator can accurately estimate the frequency offset but can only approximately estimate the timing delay. Joint estimation simply sacrifices estimation accuracy for reducing computational complexity. In addition, performance evaluations were completely skipped in the previous works [10], [11].

II. SIGNAL MODEL AND SYSTEM DESCRIPTION

The complex baseband of the dual-chirp signal burst under investigation in this paper for estimation of timing and frequency errors can be formulated as

\[ s_C(t) = 2\Pi \left( \frac{t - \frac{T}{2}}{T} \right) \cos \left( \pi \mu \left( t - \frac{T}{2} \right)^2 \right), \] (1)

where

\[ \Pi(t) = \begin{cases} 1, & -\frac{1}{2} < t < \frac{1}{2} \\ 0, & \text{elsewhere} \end{cases} \]

is a unit rectangular pulse. An example of the dual-chirp signal is depicted in Fig. 1. It can be rewritten as

\[ s_C(t) = s_a(t) + s_d(t), \] (2)

where

\[ s_a(t) = \exp \left( j\pi\mu \left( t - \frac{T}{2} \right)^2 \right) \Pi \left( \frac{t - T/2}{T} \right), \]

\[ s_d(t) = \exp \left( -j\pi\mu \left( t - \frac{T}{2} \right)^2 \right) \Pi \left( \frac{t - T/2}{T} \right). \]
It can be seen that $s_C(t)$ is a superimposed dual-chirp burst, while $s_u(t)$ and $s_d(t)$ are the up-chirp and down-chirp bursts, respectively. The instantaneous phases and instantaneous frequencies of the up-chirp and down-chirp bursts can be expressed as

$$
\phi_i(t) = \pm \pi \mu \left( t - \frac{T}{2} \right)^2, \quad 0 < t < T;
$$

$$
f_i(t) = \frac{1}{2\pi} \frac{d}{dt} \phi_i(t) = \pm \mu \left( t - \frac{T}{2} \right), \quad 0 < t < T,
$$

respectively. The instantaneous frequency of the up-chirp burst spans from $-\frac{\mu T}{2}$ to $\frac{\mu T}{2}$, while the instantaneous frequency of the down-chirp burst spans from $\frac{\mu T}{2}$ to $-\frac{\mu T}{2}$. Therefore, the bandwidth (or frequency span) of the dual-chirp burst is $B = \mu T$ in the time interval $(0, T)$.

If the transmitted signal propagates through an AWGN channel with some frequency offset, the received signal can be written as

$$
r(t) = \sqrt{\frac{E_T}{2T}} s_C(t-\tau) \exp(j2\pi \epsilon t + \theta) + n(t), \quad (3)
$$

where $E_T$ denotes the burst energy; $\tau$ and $\epsilon$ represent the timing delay and constant frequency offset to be estimated, respectively; $\theta$ is a random phase; and

$$
n(t) = n_i(t) + jn_Q(t) \quad (4)
$$

represents complex white Gaussian noise. $n(t)$ in (4) has zero mean and an autocorrelation function of the following form

$$
R_{n,n}(\lambda) = N_0 \delta(\lambda), \quad (5)
$$

where $N_0$ is the constant power spectrum density and $\delta(\cdot)$ is the Dirac delta function.

For simplicity and with no loss of generality, the following system descriptions and derivations will be performed in their analog formulations. However, the fully-digital implementation will be presented in greater detail in the next section. At the receiving end, two matched filters (MFs), up-branch (UB) MF and down-branch (DB) MF, are employed. The impulse responses of the UB and DB MFs can be written as

$$
h_u(t) = \frac{1}{\sqrt{T}} s_u^*(T-t),
$$

$$
h_d(t) = \frac{1}{\sqrt{T}} s_d^*(T-t),
$$

respectively. Therefore, the outputs of the UB and DB MFs can, respectively, be written as

$$
y_u(t) = y_{u,u}(t) + y_{u,d}(t) + n_u(t),
$$

$$
y_d(t) = y_{d,u}(t) + y_{d,d}(t) + n_d(t),
$$

where

$$
y_{p,q}(t) = \sqrt{\frac{E_T}{2T}} \int_{-\infty}^{t} h_p(t-v)s_q(v-\tau) e^{j(2\pi \epsilon v + \theta)} dv;
$$

$$
n_p(t) = \int_{-\infty}^{t} h_p(t-v)n(v) dv, \quad p,q \in \{u,d\}. \quad (7)
$$

The components obtained in (7) require closer scrutiny. Here, $y_{u,u}(t)$ and $y_{d,d}(t)$ are the desired, signal components; $y_{u,d}(t)$ and $y_{d,u}(t)$ are the interference, crosstalk components; and $n_u(t)$ and $n_d(t)$ are the noise components obtained at the outputs of the UB and DB MFs. All of them will be described in the following subsections.

### A. Signal Components

The desired components in (7) will be considered in further detail by setting

$$
t' = \frac{t - (\tau + T)}{T} \quad \text{and} \quad \epsilon' = \frac{\epsilon}{B}. \quad (8)
$$

In (8), $t'$ and $\epsilon'$ represent the timing offset with respect to the right instant on the MF output normalized by the burst duration and the frequency offset normalized by the sweeping bandwidth, respectively. After some algebraic manipulation, the magnitudes of $y_{u,u}(t)$ and $y_{d,d}(t)$ in (7) can be reformulated as

$$
|y_{u,u}(t')| = \sqrt{\frac{E_T}{2}} \frac{\sin[\pi D (\epsilon' + t') (1 - |t'|)]}{\pi D (\epsilon' + t')},
$$

$$
|y_{d,d}(t')| = \sqrt{\frac{E_T}{2}} \frac{\sin[\pi D (\epsilon' - t') (1 - |t'|)]}{\pi D (\epsilon' - t')} \quad -1 \leq t' \leq 1, \quad (9)
$$

where $D = BT$ is the processing gain in a chirp SS communication system. Often, $T \gg \frac{1}{\pi D}$ leads to $D \gg 1$. Therefore, a larger value of $D$ provides better interference suppression. In addition, $D = BT$ is called the compression ratio in pulse-compression radar systems.

It can be easily observed that $|y_{u,u}(t')|$ and $|y_{d,d}(t')|$ in (9) are symmetric with respect to $t' = 0$. Therefore, the peaks of $|y_{u,u}(t')|$ and $|y_{d,d}(t')|$ must occur symmetrically. Examples of $|y_{u,u}(t')|$ and $|y_{d,d}(t')|$ are depicted in Fig. 2. Eventually, the peak magnitudes of $y_{u,u}(t')$ and $y_{d,d}(t')$ can be found via searches using the following formulations, and they occur at $t_u'$ and $t_d'$, respectively; i.e.,

$$
t_u' = \arg \max_t \left\{ \sqrt{\frac{E_T}{2}} \left| \frac{\sin[\pi D (\epsilon' + t') (1 - |t'|)]}{\pi D (\epsilon' + t')} \right| \right\},
$$

$$
t_d' = \arg \max_t \left\{ \sqrt{\frac{E_T}{2}} \left| \frac{\sin[\pi D (\epsilon' - t') (1 - |t'|)]}{\pi D (\epsilon' - t')} \right| \right\}. \quad (10)
From (12), it can be shown that

$$y_{d,u}(t') = \frac{1}{\sqrt{D}} \left( [C(\kappa_1) - C(\kappa_0)]^2 + [S(\kappa_1) - S(\kappa_0)]^2 \right)^{1/4},$$

where

$$\kappa_0 = \sqrt{D} \left[ -1 + |t'| - \epsilon' \right],$$

$$\kappa_1 = \sqrt{D} \left[ 1 - |t'| - \epsilon' \right],$$

$$C(\kappa) = \int_0^\infty \cos \left( \frac{\pi \alpha^2}{2} \right) d\alpha,$$

$$S(\kappa) = \int_0^\infty \sin \left( \frac{\pi \alpha^2}{2} \right) d\alpha.$$ (12)

In the above, $C(\kappa)$ and $S(\kappa)$ are the Fresnel integrals [12].

From (12), it can be shown that

$$|y_{u,d}(t')| \leq \frac{1}{\sqrt{D}} \sqrt{\frac{E_T}{2}} \left( \frac{1}{2} (\kappa_1 - \kappa_0)^3 \right)^{1/4}$$

$$= \sqrt{\frac{E_T}{4}} D^{1/4} \left[ 1 - |t'|^2 \right],$$

where $|t'| \leq 1.$ (13)

Therefore, the power ratio of the peak cross-talk and the peak desired components, which may be interpreted as the reciprocal of signal-to-interference ratio (SIR), can be obtained from

$$\frac{\max \{|y_{d,u}(t')|^2\}}{\max \{|y_{u,d}(t')|^2\}} \leq \frac{1 - |t'|}{2\sqrt{D}}, |t'| \leq 1,$$ (14)

which approaches to zero as $D$ goes to infinity. This confirms that the cross-talk effects are negligible while $D$ is sufficiently large.

C. Frequency Offset Estimation

Let us carefully consider the two signals shown below:

$$z_u(t) = r(t) \cdot \frac{1}{\sqrt{T}} s_0^*(t - \hat{\tau}) = z_u(t) + z_{u,d}(t) + n_u'(t)$$

$$z_d(t) = r(t) \cdot \frac{1}{\sqrt{T}} s_0^*(t - \hat{\tau}) = z_d(t) + z_{d,d}(t) + n_d'(t),$$ (15)

where $\hat{\tau}$ denotes the estimate of the timing delay $\tau$, and can be obtained from the timing estimate in (11); and

$$z_{p,q}(t) = \frac{1}{\sqrt{T}} s_p^*(t - \hat{\tau}) \cdot \sqrt{\frac{E_T}{2T}} s_q(t - \tau)e^{i(2\pi f t + \theta)},$$

$$n_{p}'(t) = \frac{1}{\sqrt{T}} s_p^*(t - \hat{\tau}) \cdot n(t), \quad p, q \in \{u, d\}$$

The Fourier transforms of $z_u(t)$ and $z_d(t)$ can be written as

$$Z_u(f) = Z_u(f) + Z_{u,d}(f) + N_u(f),$$

$$Z_d(f) = Z_d(f) + Z_{d,d}(f) + N_d(f).$$ (16)

After some algebraic manipulation, the magnitudes of the signal components in (16) can be derived to be

$$|Z_{u,d}(f)| = \sqrt{\frac{E_T}{2} \left( 1 - \frac{\tau_e}{T} \right)} \left| \sin \left( \left[ f - \epsilon + \mu \tau_e \right] \left[ |\tau_e| - T \right] \right) \right|;$$

$$|Z_{d,d}(f)| = \sqrt{\frac{E_T}{2} \left( 1 - \frac{\tau_e}{T} \right)} \left| \sin \left( \left[ f - \epsilon + \mu \tau_e \right] \left[ |\tau_e| - T \right] \right) \right|, \quad |\tau_e| < T.$$ (17)

where $\tau_e = \tau - \hat{\tau}$ denotes the error in the aforementioned timing delay estimate. Examples of the magnitudes of the signal components have been simulated and plotted in Fig. 3. Since the term $(|\tau_e| - T)$ in (17) is independent of $f$, it cannot

Fig. 2. The signal components on the TD MFs.

Fig. 3. The signal components on the FD MFs.
change the symmetry of \(|Z_{u,u}(f)|\) and \(|Z_{d,d}(f)|\). Therefore, the peaks can be easily obtained by
\[
\begin{align*}
\hat{f}_u &= \arg \max_f |Z_{u,u}(f)| = \epsilon + \mu \tau_e, \\
\hat{f}_d &= \arg \max_f |Z_{d,d}(f)| = \epsilon - \mu \tau_e. 
\end{align*}
\] (18)

After the peaks are located by (18), the frequency offset can be estimated by
\[
\hat{\epsilon} = \frac{\hat{f}_u + \hat{f}_d}{2}. 
\] (19)

Similarly, the power ratio of the cross-talk to the peak correlation is found to be
\[
\frac{|Z_{u,d}(f)|^2}{|Z_{u,u}(f)|^2} = \frac{|Z_{d,u}(f)|^2}{|Z_{d,d}(f)|^2} < \frac{1}{D}. 
\] (20)

Therefore, the cross-talk components are negligible for large values of \(D\) and a small residual timing error.

The autocorrelation of the two noise terms, \(N_u(f)\) and \(N_d(f)\), are shown as follows:
\[
R_{N_uN_u}(f + \Delta f) = R_{N_dN_d}(f + \Delta f) = N_0 \text{sinc}(T \cdot \Delta f) \exp \left( j2\pi \left( \frac{f}{T} + \frac{T}{2} \right) \cdot \Delta f \right). 
\] (21)

It can be seen from (21) that \(N_u(f)\) (or \(N_d(f)\)) is uncorrelated while it is sampled at equal spacings of \(\Delta f = \frac{\epsilon}{\mu} \). \(N_u(f)\) (or \(N_d(f)\)) can furthermore be considered independent because they are Gaussian distributed.

III. EXPERIMENTS

A. Fully-Digital Implementation

Based on the derivations in the previous section, the discrete-time receiver assisted by the proposed technique is depicted in Fig. 4. Due to the discrete-time correlation process conducted on a DSP, the correlation function cannot always be evaluated at the right sampling instant either in the TD or in the FD. Therefore, some performance degradation inevitably occurs in the synchronization subsystem. The synchronization performance degradation, of course, depends on the sampling rate of the incoming signal. It is obvious that the higher the sampling rate, the less severe the performance degradation. In the following experiment, the sampling rate in the TD is \(MT\) times the sweeping bandwidth \(B = \mu T\), and thus the sampling period \(T_s = T / MT\). Since there are \(N = MT \times D = MT \times B \times T\) samples in one burst, \(D\) complex multiplication operations are required per branch for each correlation update. In addition, since the correlation operation should be updated every \(T_s\) second, the number of multiplication operations per second is \(2 \cdot \frac{MB}{MTDB} = 2MTDB\) for the two branches. As a result, \(2MTDB\) multiplication operations per second are required for estimation of the timing delay.

When the timing delay estimation is completed, the frequency offset estimation begins. In order to obtain higher accuracy in the FD, over-sampling has to be performed in the FD for the purpose of minimizing the loss caused by the sampled FD correlations. This can be simply conducted by padding a sufficient amount of zeros with the TD sequence prior to the discrete Fourier transform (DFT) operation. The FD upsampling ratio used here is set to be \(M_F\). Therefore, a \((MF,D)\)-point fast Fourier transform (FFT) is employed here; requiring \(\frac{MB}{2} \log_2 (MF \cdot D)\) complex multiplication operations if the Radix-2 FFT is employed. Since \(M_F\) denotes the zero-padding ratio, the correlation is therefore up-sampled by \(M_F\) times in the FD. As a result, \(\frac{MB}{2} \log_2 (MF \cdot D)\) complex multiplication operations must be performed in one burst (i.e., \(T\)), after the timing delay estimation is completed.

B. Comparison

When the estimation of the timing delay is performed using the proposed technique, the frequency offset is still unknown and it therefore has to be considered as a nuisance parameter. On the other hand, when the frequency offset estimation begins, some residual timing error may still exist. As a result, the modified Cramer-Rao lower bounds (MCRLBs) may be the effective benchmarks for performance evaluation. Since the complexity of the MCRLB derivations baffles any reasonable efforts toward a theoretical comparison in terms of MSE, we resort a computer-aided semi-analytic approach to evaluate the estimation performance as is detailed in the following. For the purpose of theoretical comparisons, the tighten lower bound on the single-parameter estimation can be directly modified from the previous studies [5]; i.e.,
\[
\text{Var} (\hat{\epsilon}) T^2 \geq \left( \frac{2\pi^2 \frac{E_T}{N_0}}{3 \pi^2} \right)^{-1}. 
\] (22)

The parameters employed in the simulation experiment are listed in Table I. The mean-square estimation errors (MSEs) obtained using the timing delay and frequency offset estimation technique proposed in this paper on an AWGN channel with carrier frequency offset of 10 and 20 ppm are shown in Figs. 5 and 6. The MSEs of the timing delay estimation shown in Fig. 5 were normalized by the factor of \(1/B^2\). Since \(B = \frac{1}{MTDB}\), it can be seen in Fig. 5 that when a frequency offset of either 10 or 20 ppm of the carrier frequency exists, the timing delay estimator proposed here can attain a low timing error of roughly 0.6 ~ 0.7\(T_s\). The MSEs of frequency offset estimation shown in Fig. 6 were normalized by the factor of \(1/T^2\). It can be seen that the estimation error of the frequency offset estimation is very close to the CRLB derived in (22). It must be remarked that the LSI in Figs. 5 and 6

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Specification</th>
</tr>
</thead>
<tbody>
<tr>
<td>Carrier Frequency</td>
<td>(f_c = 5.8) GHz</td>
</tr>
<tr>
<td>Frequency Offset</td>
<td>(\epsilon = 10) or (20) ppm of (f_c)</td>
</tr>
<tr>
<td>Burst Duration</td>
<td>(T = 17.2) mm</td>
</tr>
<tr>
<td>Number of Samples per Burst</td>
<td>(N = 1024)</td>
</tr>
<tr>
<td>Sampling Period</td>
<td>(T_s = \frac{T}{M_F})</td>
</tr>
<tr>
<td>Sweeping Bandwidth</td>
<td>(M_F = 8)</td>
</tr>
<tr>
<td>TD Oversampling Ratio</td>
<td>(M_F = 4)</td>
</tr>
<tr>
<td>FD Oversampling Ratio</td>
<td>(B = \frac{M_F}{MTDB} = \mu T)</td>
</tr>
<tr>
<td>Processing Gain (or Compression Ratio)</td>
<td>(D = B \times T)</td>
</tr>
<tr>
<td>Normalized Frequency Offset</td>
<td>(\epsilon' = \frac{\epsilon}{\mu} &lt; 0.1)</td>
</tr>
<tr>
<td>Bandwidth of Receiving Filter</td>
<td>(\frac{1}{2MTDB})</td>
</tr>
</tbody>
</table>
stands for “least-squares interpolation”. Since the waveforms or correlations in the TD or in the FD cannot always be sampled on the peaks, the 3-point least-squares interpolation method was used for finding the true peaks in order to reduce power loss and to improve the accuracy.

IV. CONCLUSIONS

The estimation of timing delay and frequency offset using a dual-chirp sequence has been thoroughly investigated. The proposed technique can simultaneously achieve low errors in timing delay and frequency offset estimates by analyzing only a single burst. In addition, this technique exhibits better robustness with respect to various kinds of interference, and it can be digitally implemented on a DSP. The superiority has been verified through both simulations and statistical analysis.

REFERENCES