Lecture 13:

MOS Circuits

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Overview

- **Reading**
  - S&S: Chapter 5.5~5.7

- **Background**
  - We continue our discussion of MOSFETs by looking at its operation as an amplifier. Like we saw for the BJT, there are three basic single-stage amplifier configurations – common-source, common-gate, and common-drain amplifiers. In order to operate as an amplifier, the MOSFET must first be biased in the saturation region. Once biased, we can again analyze the circuit with small-signal equivalent models.
DC Bias

- First bias MOSFET in saturation region (equivalent to active region in BJTs) to operate as an amplifier
  - set $v_{gs} = 0$ and find $I_D$ (for now, assume $\lambda=0$)
    \[ I_D = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_{GS} - V_t)^2 \]
  - To be in saturation,
    \[ V_D > V_{GS} - V_t \]
  - Apply a small signal, $v_{gs}$, to the gate
    \[ v_{GS} = V_{GS} + v_{gs} \]
    \[ i_D = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_{GS} + v_{gs} - V_t)^2 \]
    \[ i_D = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_{GS} - V_t)^2 + \mu_n C_{ox} \frac{W}{L} (V_{GS} - V_t)v_{gs} + \frac{1}{2} \mu_n C_{ox} \frac{W}{L} v_{gs}^2 \]
Three components of $i_D$

- First term = DC current
- Second term = current proportional to $v_{gs}$
- Third term = undesired nonlinear distortion

Make $v_{gs}$ small to reduce effect of third term

$$\frac{1}{2} \mu_n C_{ox} \frac{W}{L} v_{gs}^2 \ll \mu_n C_{ox} \frac{W}{L} (V_{GS} - V_t) v_{gs}$$

This is the small-signal condition and let's us use the following approximation

$$i_D \approx I_D + i_d \text{ where } i_d = \mu_n C_{ox} \frac{W}{L} (V_{GS} - V_t) v_{gs}$$

and we can again relate $i_d$ to $v_{gs}$ with a transconductance

$$g_m = \frac{i_d}{v_{gs}} = \mu_n C_{ox} \frac{W}{L} (V_{GS} - V_t)$$
- **Voltage Gain**

\[
v_D = V_{DD} - R_D i_D \quad \Rightarrow \quad v_D = V_{DD} - R_D (I_D + i_d) \quad \Rightarrow \quad v_D = V_D - R_D i_d
\]

\[
v_d = -R_D i_d = -g_m R_D v_{gs} \quad \Rightarrow \quad \frac{v_d}{v_{gs}} = -g_m R_D
\]

- This gain equation hold for small signals
- Notice that the output is 180° out of phase w.r.t the input

- Again, we can separate out the DC bias conditions and the small-signal operation of the circuit
  - Look at the small-signal equivalent circuit for a MOSFET biased in the saturation region
Small-Signal Equivalent Circuit

- A MOSFET operates like a voltage-controlled current source (for small signals)

\[ V_A \approx \frac{1}{\lambda} \]

- Like the Early effect in the BJT, channel length modulation results in an output resistance, \( r_o \)

\[ r_o \approx \frac{V_A}{I_D} \]

- where \( V_A = \frac{1}{\lambda} \)
Transconductance

• Let’s take a closer look at transconductance, $g_m$

$$g_m = \frac{i_d}{V_{gs}} = \mu_n C_{ox} \frac{W}{L} (V_{GS} - V_t)$$

• Depends on
  – process technology – $\mu_n C_{ox}$
  – physical geometry – W/L
    • make short and wide for high $g_m$
  – DC bias – $V_{GS}$
    • making $V_{GS}$ large increases $g_m$, but can limit voltage range on drain

• Another way to write $g_m$ …

$$g_m = \sqrt{2\mu_n C_{ox}} \sqrt{W/L} \sqrt{I_D}$$

  – $g_m$ is proportional to the square root of the DC bias current
  – $g_m$ is proportional to sqrt(W/L)
**T-Model and Body Effect**

- **T-Model**
  - Resistance looking into the source is $1/g_m$
  - Resistance looking into $G$ is still $\infty$ since $i_g=0$

- **Body Effect**
  - We saw that the substrate bias $V_{BS}$ affects $V_t$ which has the effect of influencing current like another gate

\[
\begin{align*}
    g_{mb} &= \frac{\partial i_D}{\partial V_{GS}} \bigg|_{V_{GS}=\text{const}, V_{DS}=\text{const}} \\
    g_{mb} &= \frac{\gamma}{2\sqrt{2\phi_f + V_{SB}}} g_m
\end{align*}
\]
There are a variety of ways to bias the MOSFET amplifier. Instead, we will rely on a current source for active biasing and avoid using resistors which may not be available in some processes or discouraged.
The following circuits is called a current mirror b/c $Q_2$ mirrors the current flowing in $Q_1$

- If $\lambda = 0$,
  
  $$I_O = \frac{1}{2} \mu_n C_{ox} \frac{W_2}{L_2} (V_{GS} - V_t)^2$$

  $$I_{REF} = I_1 = \frac{1}{2} \mu_n C_{ox} \frac{W_1}{L_1} (V_{GS} - V_t)^2$$

  $$\frac{I_O}{I_{REF}} = \frac{W_2/L_2}{W_1/L_1}$$

- If $\lambda \neq 0$, $I_O$ varies with respect to $V_O$ due to channel length modulation

Can have multiple devices connected to the diode connected $Q_1$ in order to generate multiple currents based on a single reference
Common-Source Amplifier

- Active load – uses current source instead of load resistor
  - Biasing so that $Q_2$ in saturation and $i$ output resistance is the effective resistor load for $Q_1$
- Combine the I-V curves →
• Look at the Voltage Transfer Characteristics (VTC) of the circuit
  – Operates like a high-gain amplifier (steep slope) in region III
CS Amplifier low-frequency small-signal model

\[ v_o/v_i = -g_m (r_{o1} || r_{o2}) \]

\[ g_{m1} = \sqrt{2\mu_n C_{ox} (W_1/L_1) I_{REF}} \]

\[ r_{o1} = \frac{|V_{A1}|}{I_{REF}} \quad r_{o2} = \frac{|V_{A2}|}{I_{REF}} \quad V_{A1} \approx V_{A2} \]

\[ A_v = \frac{v_o}{v_i} \approx -\sqrt{\frac{1}{2} \mu_n C_{ox} (W_1/L_1)} \frac{V_A}{\sqrt{I_{REF}}} \]
Common-Gate Amplifier

- Bias the gate with a DC voltage and drive the source
  - small signal into the gate is effectively grounded
  - Need to consider body effect
• Small-signal model needs to include body effect
• Node equation at the output \( v_o \) can be written to calculate the voltage gain

\[
v_{bs1} = -v_i
\]

\[
\frac{v_i - v_o}{r_{o1}} + (g_{m1} + g_{mb1})v_i = \frac{v_o}{r_{o2}}
\]

\[
A_v = \frac{v_o}{v_i} = \left( g_{m1} + g_{mb1} + \frac{1}{r_{o1}} \right) r_{o1} \parallel r_{o2}
\]

\[
A_v = \frac{v_o}{v_i} \approx (g_{m1} + g_{mb1}) r_{o1} \parallel r_{o2}
\]

• To find the input resistance…

\[
R_{in} = \frac{v_i}{i_i} \approx \frac{1}{g_{m1} + g_{mb1}} \left( 1 + \frac{r_{o2}}{r_{o1}} \right)
\]

– Input resistance increases \( \sim 2x \) due to \( r_o \)
Common-Source Amplifier

- Keep the source biased with a current source and drive the gate
  - Also called a source follower
- Basic characteristics
  - gain is less than unity
  - high input impedance
  - low output resistance (can drive low-impedance loads with low loss of gain)
  - Will see it has ability to extend high-frequency response by its impedance buffering action
- Small-signal model
  - replace $Q_1$ with small-signal model that includes body effect
  - $Q_2$ can be modeled by the output resistance
• Model $Q_2$ with its output resistance and include back body effect for $Q_1$
• Can simplify the model
  – $v_{bs1} = -v_o$ so the VCCS becomes a resistor $1/g_{mb1}$
  – Use a single lumped resistor $R_s$
• What is the voltage gain?

$$v_o = v_{s1} = g_{m1}R_s v_{gs1}$$
$$v_i = v_{gs1} + v_{s1} = v_{gs1} + g_{m1}R_s v_{gs1}$$

$$A_v = \frac{v_o}{v_i} = \frac{g_{m1}R_s}{1 + g_{m1}R_s}$$

– Can also be rewritten

$$A_v = \frac{g_{m1}}{g_{m1} + g_{mb1} + \frac{1}{r_{o1}} + \frac{1}{r_{o2}}} \approx \frac{g_{m1}}{g_{m1} + g_{mb1}}$$

$$R_S = \frac{1}{\frac{1}{g_{mb1}} || r_{o1} || r_{o2}}$$
• CD output resistance
  – Short the input to ground
  – Apply a test voltage to the output
  – Look to see $R_o$

  \[ \frac{1}{g_{mb1}} \quad \frac{1}{g_{m1}} \quad r_{o1} \quad r_{o2} \quad V_x \]

  – The parallel combination results in a small $R_o$