1) (S&S 5.75) Find an expression for the overall voltage gain in terms of $g_m$ and $r_o$ of Q1 and Q2.

\[ \frac{v_o}{v_i} = g_{m1} \cdot g_{m2} \cdot r_{o1} \cdot r_{o2} \]

2) (S&S 5.81) Assuming the current source I to be ideal and neglecting the body effect in Q2:
   a) Show that the short-circuit transconductance, which is the ratio of the short-circuit output signal current to the input voltage, is $g_{m1}$.
   b) Short circuit $v_i$ and use equivalent circuit models for Q1 and Q2 to show that the resistance is given by $R_o = r_{o1} + r_{o2} + g_{m2}r_{o1}r_{o2} \approx g_{m2}r_{o2}$.
   c) Use the results of a) and b) to obtain the open-circuit voltage gain $v_o/v_i$. Compare the gain realized to the gain that would be obtained without transistor Q2 (i.e. with the common-source amplifier formed by Q1 and the ideal current-source load I).
Node Eq. At D1:

\[ g_{m1} v_i = \frac{v_{gs2}}{r_{01}} + g_{m2} v_{gs2} + \frac{v_{gs2}}{r_{02}} = v_{gs2} \left( g_{m2} + \frac{1}{r_{02}} + \frac{1}{r_{01}} \right) \]  

(2)

combining equations (1) and (2):

\[ i_o = g_{m2} v_{gs2} + \frac{v_{gs2}}{r_{02}} = v_{gs2} \left( g_{m2} + \frac{1}{r_{02}} \right) \]  

(1)

for \( g_{m2} \gg 1/r_o \) (where \( r_{01} = r_{02} = r_o \)),

\[ i_o \approx g_{m1} v_i \]

Thus, the transconductance of the cascode circuit is equal to \( g_{m1} \).

b) Short-circuiting \( v_i \) and applying a signal \( v_x \) to the output node results in the equivalent circuit:
\[ i_x = g_{m2}v_{gs2} + \frac{v_x + v_{gs2}}{r_{o2}} \]  
(1)  
\[ i_x \cdot r_{o1} = -v_{gs2} \]  
(2)  

substituting for \( v_{gs} \) from (2) into (1) yields:

\[ i_x = -g_{m2}r_{o1}i_x + \frac{v_x}{r_{o2}} - \frac{i_x r_{o1}}{r_{o2}} \]
\[ i_x (1 + g_{m2}r_{o1} + \frac{r_{o1}}{r_{o2}}) = \frac{v_x}{r_{o2}} \]
\[ R_o = \frac{v_x}{i_x} = r_{o1} + \frac{r_{o1} r_{o2}}{r_{o1}} + g_{m2}r_{o1}r_{o2} \approx g_{m2}r_{o1}r_{o2} \]

(c) Combining the results of a) and b), the equivalent circuit is as follows:

Thus, \( v_o/v_i = -g_{m1}R_o = (-g_{m1}r_{o1})(g_{m2}r_{o2}) \). The common-source amplifier provides an open-circuit voltage gain of \( Av = -g_{m}R_o \). Thus, the cascode configuration increases the gain by the factor \( g_{m2}r_{o2} \). For example, if \( g_{m}r_{o} = 100 \), the common-source amplifier provides a voltage gain of 100 while the cascode amplifier provides a gain of 10,000.
3) (S&S 6.80) Consider the following MOS differential pair with the gate of Q2 grounded. Let \( \mu_n C_{ox} = 20 \mu A/V^2 \), \( V_t = 1 \) V, \( W/L = 50 \), and \( I = 40 \mu A \). Find \( V_{GS1} \), \( V_{GS2} \), \( V_S \), and \( V_{G1} \) that correspond to the following distributions of the current I between Q1 and Q2:
   a) \( i_{D1} = i_{D2} = 20 \mu A \).
   b) \( i_{D1} = 30 \mu A, i_{D2} = 10 \mu A \).
   c) \( i_{D1} = 40 \mu A, i_{D2} = 0 \) (Q2 just cuts off)

**Solution:**

\[ i_D = \frac{1}{2} \mu_n C_{ox} \cdot W/L \cdot (V_{GS} - V_t)^2 \]
\[ i_D = \frac{1}{2} \cdot 20 \cdot 50 \cdot (V_{GS} - 1)^2 \mu A \]

\[ \rightarrow i_D = 500 (V_{GS} - 1)^2 \mu A \]

\[ \rightarrow V_{GS} = \sqrt{\frac{i_D}{500}} + 1 \]

a) \( i_{D1} = i_{D2} = 20 \mu A \)
\[ V_{GS1} = V_{GS2} = 1.2 V \]
\[ V_S = -1.2 V \]
\[ V_{G1} = 0 \]

b) \( i_{D1} = 30 \mu A, i_{D2} = 10 \mu A \)
\[ V_{GS1} = \sqrt{(30/500)} + 1 = 1.24 V \]
\[ V_{GS2} = \sqrt{(10/500)} + 1 = 1.14 V \]
\[ V_S = -1.14 V \]
\[ V_{G1} = 1.24 - 1.14 = 0.10 V \]

c) \( i_{D1} = 40 \mu A, i_{D2} = 0 \) (just cuts off)
\[ V_{GS1} = \sqrt{(40/500)} + 1 = 1.28 V \]
\[ V_{GS2} = 1 V \]
\[ V_S = -1 V \]
4) (S&S 6.81) An NMOS differential amplifier utilizes a bias current of 200 µA. The devices have $V_t = 0.8$ V, $W = 100$ µm, and $L = 1.6$ µm, in a technology for which $\mu n C_{ox} = 90$ µA/V². Find $V_{GS}$, $g_m$, and the value of $v_{id}$ for full-current switching. To what value should the bias current be changed in order to double the value of $v_{id}$ for full-current switching?

Solution:

\[ I_D = \frac{1}{2} \mu n C_{ox} \cdot W/L \cdot (V_{GS} - V_t)^2 \]
\[ 200/2 = \frac{1}{2} \cdot 90 \cdot 100/1.6 \cdot (V_{GS} - 0.8)^2 \]
\[ \rightarrow V_{GS} = 0.98V \]

\[ g_m = 2 \cdot I_D / (V_{GS} - V_t) = 2 \cdot 100/(0.98 - 0.8) = 1.11 \text{ mA/V} \]

\[ v_{id}\text{full current switching} = \sqrt{2} \cdot (V_{GS} - V_t) = 0.25 \text{ V} \]

To double this value, $V_{GS} - V_t$ must be doubled which means that $I_D$ should be quadrupled (i.e. $I$ changed to 800 µA)