1. S&S 6.85

\[ V_{gs} - V_t = 0.5 \]

\[ V_{OS1(\Delta R_D)} = \frac{1}{2} \left( V_{gs} - V_t \right) \frac{\Delta R_D}{R_D} = \frac{1}{2} \times 0.5 \times 0.04 = 10 \text{ mV} \]

\[ V_{OS2(\Delta W/L)} = \frac{1}{2} \left( V_{gs} - V_t \right) \Delta (W/L) = \frac{1}{2} \times 0.5 \times 0.04 = 10 \text{ mV} \]

\[ V_{OS3(\Delta V_t)} = 0.04 \times 1 = 40 \text{ mV} \]

Since these effects are uncorrelated

\[ V_{OS} = \sqrt{V_{OS1}^2 + V_{OS2}^2 + V_{OS3}^2} \]

\[ V_{OS} = \sqrt{10^2 + 10^2 + 40^2} = 42.4 \text{ mV} \]

The major contribution is due to the threshold mismatch.

To find the required mismatch, \( \Delta R_D \) that can correct for \( V_{OS} \)

\[ 42.4 = \frac{V_{gs} - V_t}{2} \frac{\Delta R_D}{R_D} \Rightarrow \frac{\Delta R_D}{R_D} = 0.17 = 17\% \]

If \( \Delta V_t \) is reduced to 4 mV, \( V_{OS} \) is reduced to

\[ V_{OS} = \sqrt{10^2 + 10^2 + 4^2} = 14.4 \text{ mV} \]

\[ \frac{\Delta R_D}{R_D} = \frac{2 \times 0.0144}{0.5} = 5.8\% \]

However, treating these as uncorrelated statistical variation is not exactly the same as the absolute worst case, in which they all add, giving an offset of 60 mV. Repeating the procedure with this value yields answers of a mismatch of 24%, reduced to 9.6% when reducing the threshold offset. Note also that we used 0.04 for a 4% mismatch and not 2%, as the worst case scenario involves the transistors in a differential amplifier moving in opposite directions of the +/- 2%.
2. S&S 7.27

(a)

\[ V_o = -g_m R_i V_{gs} \]
\[ V_{gs} = V_i - g_m R_s V_{gs} \]
\[ V_i = V_{gs}(1 + g_m R_s) \]
\[ \frac{V_o}{V_i} = -\frac{g_m R_i}{1 + g_m R_s} \]

(b)

\[ I_x = \frac{(g_m V_x - I_x)R_i + V_x}{R} \]
\[ I_x (R + R_s) = (1 + g_m R_s)V_x \]
\[ R_{gs} \equiv \frac{V_x}{I_x} = \frac{R + R_s}{1 + g_m R_s} \]
\[
R_{gd} = \frac{-I_x R - V_{gs}}{-1} \Rightarrow V_{gs} = -\frac{I_x R}{1 + g_m R_s}
\]

At D:

\[
I_x = g_m V_{gs} + \frac{V_x - I_x R}{R_L}
\]

\[
I_x = -\frac{g_m I_x R}{1 + g_m R_s} - \frac{I_x R}{R_L} + \frac{V_x}{R_L}
\]

\[
I_x \left[ 1 + \frac{R}{R_L} + \frac{g_m R}{1 + g_m R_s} \right] = \frac{V_x}{R_L}
\]

\[
R_{gd} = R_L + R + \frac{g_m R R_L}{1 + g_m R_s}
\]

(c)

Rs=0

\[
\frac{V_o}{V_i} = -\frac{4 \times 5}{1} = -20
\]

\[
R_{gs} = R = 100k\Omega
\]

\[
R_{gd} = R_L + R + g_m R_L R = 5 + 100 + 4 \times 5 \times 100 = 2105k\Omega
\]

\[
\omega_H = \frac{1}{C_{gs} R_{gs} + C_{gd} R_{gd}} = \frac{1}{10^{-12} \times 100 \times 10^3 + 10^{-12} \times 2105 \times 10^3}
\]

\[
= \frac{10^9}{100 + 2105} = 453.5 \text{ krad/s}
\]

\[
GBW = 20 \times 453.5 = 9.07 \text{ Mrad/s}
\]

Rs=100
\[
\frac{V_o}{V_i} = \frac{-4*5}{1+4x0.1} = -14.3
\]
\[
R_{gs} = \frac{100k\Omega + 0.1}{1+4*0.1} = 72.1k\Omega
\]
\[
R_{gd} = 5 + 100 + \frac{4*5*100}{1+4*0.1} = 1529.6k\Omega
\]
\[
\omega_H = \frac{1}{C_{gs}R_{gs} + C_{gd}R_{gd}} = \frac{1}{10^{-12}x72.1x10^3 + 10^{-12}x1529.6x10^3}
\]
\[
= 624.3 \text{ krad/s}
\]
\[
GBW = 14.3 \times 624.3 = 8.93 \text{ Mrad/s}
\]

Rs=250
\[
\frac{V_o}{V_i} = \frac{-4*5}{1+4x0.25} = -10
\]
\[
R_{gs} = \frac{100k\Omega + 0.25}{1+4*0.25} = 50.1k\Omega
\]
\[
R_{gd} = 5 + 100 + \frac{4*5*100}{1+4*0.25} = 1105k\Omega
\]
\[
\omega_H = \frac{1}{C_{gs}R_{gs} + C_{gd}R_{gd}} = \frac{1}{10^{-12}x50.1x10^3 + 10^{-12}x1105x10^3}
\]
\[
= 865.7 \text{ krad/s}
\]
\[
GBW = 8.66 \text{ Mrad/s}
\]

The gain-bandwidth product is approximately constant.

3. S&S 7.49
\[ g_{m1} = \sqrt{2\mu_n C_{ox} (W/L)} I_D = \sqrt{2 \times 90 \times (100/1.6) \times 100} = 1.06mA/V \]

\[ r_{01} = \frac{V_{A1}}{I_{D1}} = \frac{12.8}{0.1} = 128k\Omega \]

\[ r_{02} = \frac{V_{A2}}{I_{D2}} = 192k\Omega \]

\[ DCgain = -g_{m1} (r_{01} \parallel r_{02}) = -1.06(128 \parallel 192) = -81.4 \]

Total capacitance between output node and ground

\[ = C_{gd2} + C_{dh2} + C_{dh1} + C_{par} = 0.015 + 0.036 + 0.020 + .3 = 0.371pF \equiv C_L \]

Node equation at output

\[ sC_{gd1}(V_i - V_o) = g_{m1}V_i + \frac{V_o}{r_{01}} + \frac{V_o}{r_{02}} + sC_L V_o \]

\[ V_o \frac{\ }{V_i} = -\frac{g_{m1} - sC_{gd1}}{r_{01} + \frac{1}{r_{02}}} + s(C_L + C_{gd1}) \]

\[ f_z = \frac{1}{2\pi C_{gd1}} = \frac{1}{2\pi} \frac{1.06 \times 10^{-3}}{0.015 \times 10^{-12}} = 11.1GHz \]

\[ f_p = \frac{1}{2\pi} \frac{1}{C_L + C_{gd1}} \left( \frac{1}{r_{01}} + \frac{1}{r_{02}} \right) = \frac{1}{2\pi} \frac{1}{0.071 + 0.015} \times 10^{-12} \left( \frac{1}{128} + \frac{1}{192} \right)^3 = 5.37MHz \]

\[ C_L \text{ should be } 0.371 \text{ pF making } f_p 5.37 \text{ MHz} \]

4. S&S 7.63

Neglecting \( r_{02} \) and employing the source-absorption theorem results in the following equivalent circuit:
(b) The dc gain is

\[ A_0 = g_{m1} \left( r_{01} \parallel 1/g_{m2} \right) x - g_{m2} r_{03} \approx -g_{m1} \frac{1}{g_{m2}} g_{m2} r_{03} = -g_{m1} r_{03} \quad \text{for} \quad r_{01} \gg \frac{1}{g_{m2}} \]

The frequency of the input pole is

\[ f_{p2} = \frac{1}{2 \pi r_{03} C_2} \]

and the frequency of the input pole is found as follows:

A node equation at d1 yields

\[ s C_{gd1} (V_{i} + V_{g2}) = g_{m1} V_{i} - \left( \frac{1}{r_{01}} + g_{m2} s C_1 \right) V_{g2} \]

\[ \Rightarrow \frac{V_{g2}}{V_{i}} = \frac{g_{m1} - s C_{gd1}}{\frac{1}{r_{01}} + g_{m2} + s (C_1 + C_{gd1})} \]

The input pole is at

\[ f_{p1} = \frac{1}{2 \pi} \frac{r_{01} + g_{m2}}{C_1 + C_{gd1}} = \frac{g_{m2}}{2 \pi (C_1 + C_{gd1})} \]

The zero has a frequency of

\[ f_z = \frac{g_{m1}}{2 \pi C_{gd1}} \]
(c) 

\[ g_{m1} = g_{m2} = \sqrt{2\mu_n C_{ox}(W/L)I_D} = \sqrt{2 \times 20 \times 1 \times 50} = 44.7 \mu A/V \]

\[ r_{03} = \frac{V_A}{I_D} = \frac{1}{\lambda_0 I_D} = \frac{1}{0.02 \times 50 \times 10^{-6}} = 1 \Omega \]

\[ A_0 = -44.7 \times 1 = 44.7 \]

\[ C_{gd1} = 3.5 \text{fF} \]

\[ C_1 = C_{db1} + C_{sb2} + C_{gr2} = 24 + 24 + 30 = 78 \text{fF} \]

\[ C_2 = C_{db2} + C_{db3} + C_{gd2} + C_{gd3} + C_L = 24 + 12 + 3.5 + 3.5 + 100 = 145 \text{fF} \]

\[ f_p2 = \frac{1}{2\pi \times 10^6 \times 143 \times 10^{-15}} = 1.11 \text{MHz} \]

\[ f_p1 = \frac{44.7 \times 10^{-6}}{2\pi \times 81.5 \times 10^{-15}} = 87.3 \text{MHz} \]

\[ f_z = \frac{44.7 \times 10^{-6}}{2\pi \times 3.5 \times 10^{-15}} = 2.03 \text{GHz} \]

5. S&S 7.64

\[ \frac{V_{C1}}{V_i} = -g_{m1} \left( r_{01} || \frac{1}{g_{m2}} \right) \approx -\frac{g_{m1}}{g_{m2}} = -1 \]

\[ \frac{V_0}{V_{C1}} = g_{m2} (R_{02} || R_{0bias}) = g_{m2} (R_{02} || R_{03}) = \frac{1}{2} g_{m2} R_{02} \]

But \( R_{02} \), the output resistance of the cascode is given by

(Equation 6.116, p. 534)

\[ R_{02} = g_{m2} r_{02} r_{01} \Rightarrow \frac{V_{01}}{V_i} = -1 + \frac{1}{2} g_{m2} g_{m3} r_{02} r_{01} = -\frac{1}{2} (g_m r_0)^2 \]
(b) At the output node the total capacitance is $C_L$ and the total resistance is $R_{02}/2 = 0.5 * g_m r_0^2$.

$$\omega_H = \frac{1}{C_L * 0.5 * g_m r_0^2} = \frac{2}{C_L * g_m r_0^2}$$

(c) $A_M = -\frac{1}{2} (0.5 * 100)^2 = -1250$

$$f_H = \omega_H / 2\pi = 63.7 kHz$$

6. S&S 7.78

DC gain

$$V_{g4} = -g_m V_{g1} \frac{1}{g_m} = -\frac{g_{m1}}{g_m} \left( \frac{V_i}{2} \right)$$

$$V_o = -g_m V_{g4} + g_m V_{g2} \left( r_{02} \parallel r_{04} \right) = -\left( -g_m \frac{g_{m1}}{g_m} \left( \frac{V_i}{2} \right) - g_m \left( \frac{V_i}{2} \right) \right) \left( r_{02} \parallel r_{04} \right)$$

$$= \left( \frac{V_i}{2} \right) \left( g_m \frac{g_{m1}}{g_m} + g_m \right) \left( r_{02} \parallel r_{04} \right)$$

$$A_0 = \frac{V_o}{V_i} = \frac{1}{2} \left( g_m \frac{g_{m1}}{g_m} + g_m \right) \left( r_{02} \parallel r_{04} \right)$$

For $g_{m1} = g_m = g_{m3} = g_{m4}$ and $r_{02} = r_{04} = r_0$

$$A_0 = \frac{g_m r_0}{2}$$

For the equivalent circuit, two poles are obvious

$$f_{p1} = \frac{1}{2\pi C_1 (1/g_m)} = \frac{g_m}{2\pi C_1}$$

$$f_{p2} = \frac{1}{2\pi C_2 (r_{02} \parallel r_{04})}$$

Since $C_1$ and $C_2$ are of the same order of magnitude and $g_m3 >> 1/(r_{02} \parallel r_{04})$, we see that $f_{p2} << f_{p1}$. $f_{p2}$ is dominant.