Lecture 18:

Feedback

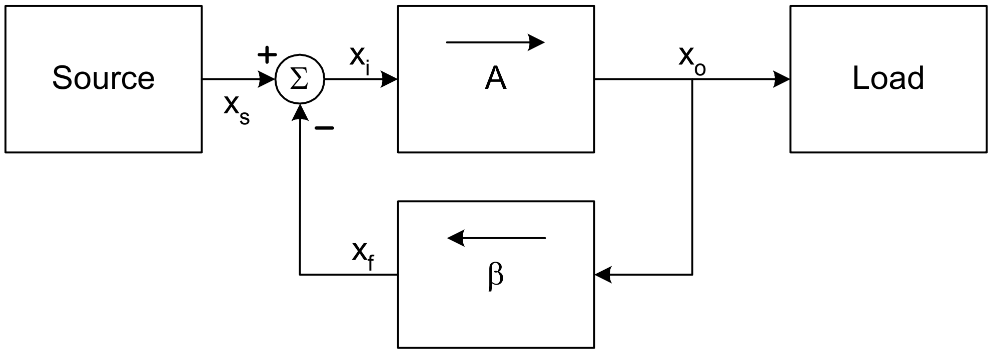
Gu-Yeon Wei
Division of Engineering and Applied Sciences
Harvard University
guyeon@eecs.harvard.edu

Overview

- **Reading**
  - S&S: Chapter 8.1~8.8
  - S&S: Appendix B (Two-Port Network Parameters)
- **Background**
  - Negative feedback for amplifiers was invented in 1927 by Harold Black to stabilize the gain and correct the distortion of amplifiers used in long-distance telephone networks. Negative feedback (as well as positive feedback) is widely used in analog circuits today. In fact, we used negative feedback when we constructed op amps with gain set (fixed) using resistors. Throughout the next lecture, we will investigate the general theory of feedback and look at four basic feedback topologies for four types of amplifier topologies. We will return to concepts of feedback covered in Chapter 8 of S&S after taking investigating how to build single- and two-stage Op amps. We will return to material in Chapter 8 to study stability of amplifiers, and see how "compensation" can be utilized.
  - S&S has several examples of circuits with feedback using BJTs. These lecture notes give MOS circuit examples (many from Razavi's textbook).

General Feedback Structure

Let's start with the basic structure of a feedback amplifier. To make it general, the figure shows signal flow as opposed to voltages or currents (i.e., signals can be either current or voltage).



The open-loop amplifier has gain A → $x_o = A*x_i$

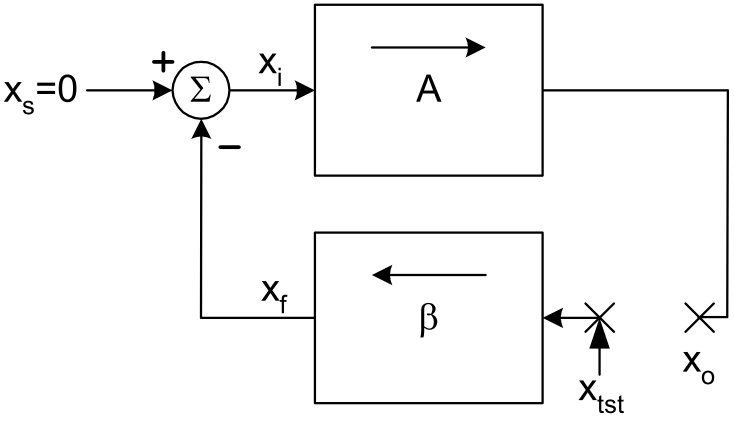
- Output is fed back through a feedback network which produces a sample (x_f) of the output (x_o) → $x_f = \beta x_o$
  - Where β is called the feedback factor
- The input to the amplifier is $x_i = x_s - x_f$ (the subtraction makes feedback negative)
  - Implicit to the above analysis is that neither the feedback block nor the load affect the amplifier's gain (A). This not generally true and so we will later see how to deal with it.
- The overall gain (closed-loop gain) can be solved to be:

$$A_f \equiv \frac{x_o}{x_s} = \frac{A}{1 + A\beta}$$

- $A\beta$ is called the loop gain and $1+A\beta$ is called the "amount of feedback"

Finding Loop Gain

- Generally, we can find the loop gain with the following steps:
  - Break the feedback loop anywhere (at the output in the ex. below)
  - Zero out the input signal x_s
  - Apply a test signal to the input of the feedback circuit
  - Solve for the resulting signal x_o at the output
    - If x_o is a voltage signal, x_{tst} is a voltage and measure the open-circuit voltage
    - If x_o is a current signal, x_{tst} is a current and measure the short-circuit current



$$x_f = \beta x_{tst}$$

$$x_i = 0 - x_f$$

$$x_o = Ax_i = -Ax_f = -\beta A x_{tst}$$

$$\text{loop gain} = -\frac{x_o}{x_{tst}} = \beta A$$

  - The negative sign comes from the fact that we are apply negative feedback

Negative Feedback Properties

- Negative feedback takes a sample of the output signal and applies it to the input to get several desirable properties. In amplifiers, negative feedback can be applied to get the following properties
  - Desensitized gain – gain less sensitive to circuit component variations
  - Reduce nonlinear distortion – output proportional to input (constant gain independent of signal level)
  - Reduce effect of noise
  - Control input and output impedances – by applying appropriate feedback topologies
  - Extend bandwidth of amplifier
- All of these properties can be achieved by trading off gain

- Let's investigate a couple of these properties in a little more detail using the general structure described in the previous slide

Gain Desensitivity

- Feedback can be used to desensitize the closed-loop gain to variations in the basic amplifier. Let's see how.
- Assume β is constant. Taking differentials of the closed-loop gain equation gives…

$$A_f = \frac{A}{1 + A\beta} \quad \Rightarrow \quad dA_f = \frac{dA}{(1 + A\beta)^2}$$

Take derivative of both sides

  – Divide by A_f

$$\frac{dA_f}{A_f} = \frac{dA}{(1 + A\beta)^2} \frac{1 + A\beta}{A} = \frac{1}{1 + A\beta} \frac{dA}{A}$$

- This result shows the effects of variations in A on A_f is mitigated by the feedback amount. $1+A\beta$ is also called the desensitivity amount

- We will see through examples that feedback also affects the input and resistance of the amplifier (increases R_i and decreases R_o by $1+A\beta$ factor)

Bandwidth Extension

- We've mentioned several times in the past that we can trade gain for bandwidth. Finally, we see how to do so with feedback…

  Consider an amplifier with a high-frequency response characterized by a single pole and the expression:

$$A(s) = \frac{A_M}{1 + s/\omega_H}$$

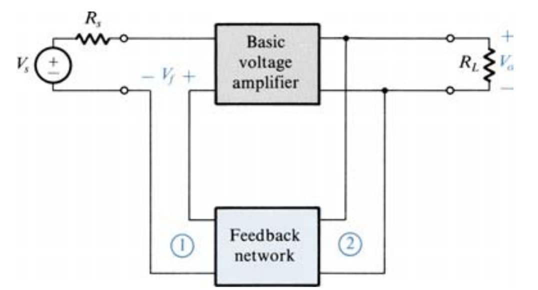
  Apply negative feedback β and the resulting closed-loop gain is:

$$A_f(s) = \frac{A(s)}{1 + \beta A(s)} = \frac{A_M/(1 + A_M\beta)}{1 + s/\omega_H(1 + A_M\beta)}$$

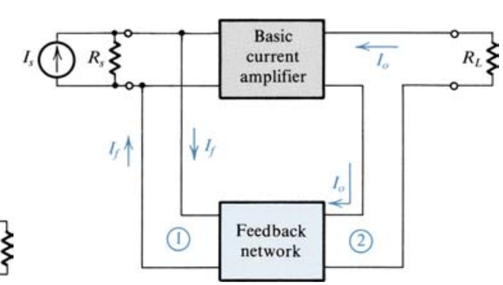
- Notice that the midband gain reduces by $(1+A_M\beta)$ while the 3-dB roll-off frequency increases by $(1+A_M\beta)$

Basic Feedback Topologies

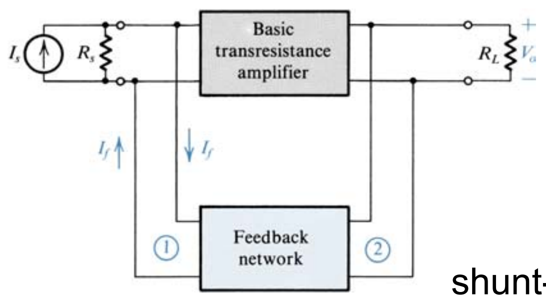
- Depending on the input signal (voltage or current) to be amplified and form of the output (voltage or current), amplifiers can be classified into four categories. Depending on the amplifier category, one of four types of feedback structures should be used (series-shunt, series-series, shunt-shunt, or shunt-series)
  - Voltage amplifier – voltage-controlled voltage source
    - Requires high input impedance, low output impedance
    - Use series-shunt feedback (voltage-voltage feedback)
  - Current amplifier – current-controlled current source
    - Use shunt-series feedback (current-current feedback)
  - Transconductance amplifier – voltage-controlled current source
    - Use series-series feedback (current-voltage feedback)
  - Transimpedance amplifier – current-controlled voltage source
    - Use shunt-shunt feedback (voltage-current feedback)



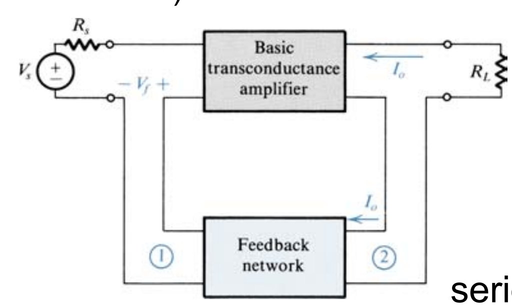
series-shunt



shunt-series

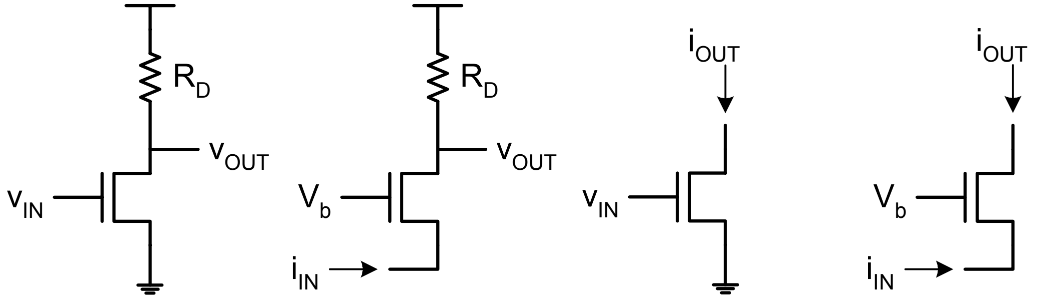


shunt-shunt



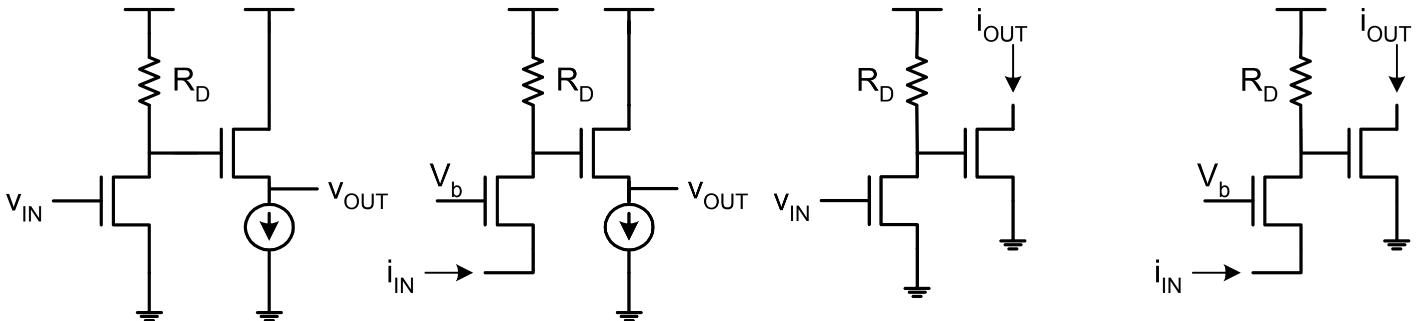
series-series

Examples of the Four Types of Amplifiers



Voltage Amp | Transimpedance Amp | Transconductance Amp | Current Amp

- Shown above are simple examples of the four types of amplifiers. Often, these amplifiers alone do not have good performance (high output impedance, low gain, etc.) and are augmented by additional amplifier stages (see below) or different configurations (e.g., cascoding).

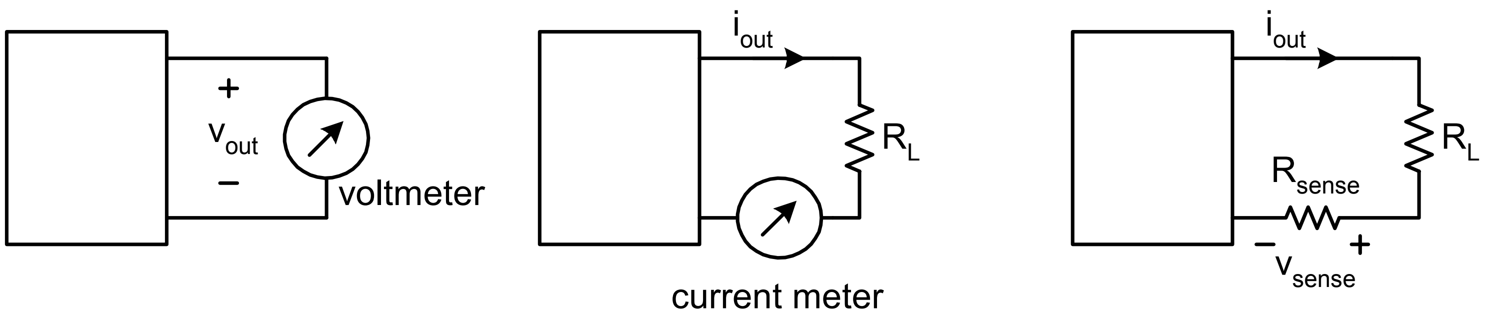


lower Z_{out} | lower Z_{out} | higher gain | higher gain

Sense and Return Mechanisms

- Adding a feedback loop consists of sensing the output signal and returning (a fraction) of the result to the summing node at the input.
  - Given the inputs and outputs can be either voltages or currents, there are four types of feedback: voltage-voltage, voltage-current, current-voltage, and current-current
    - where the first part denotes the quantity sensed at the output and the second denotes the type of signal returned

- Examples of sensing voltage and current:

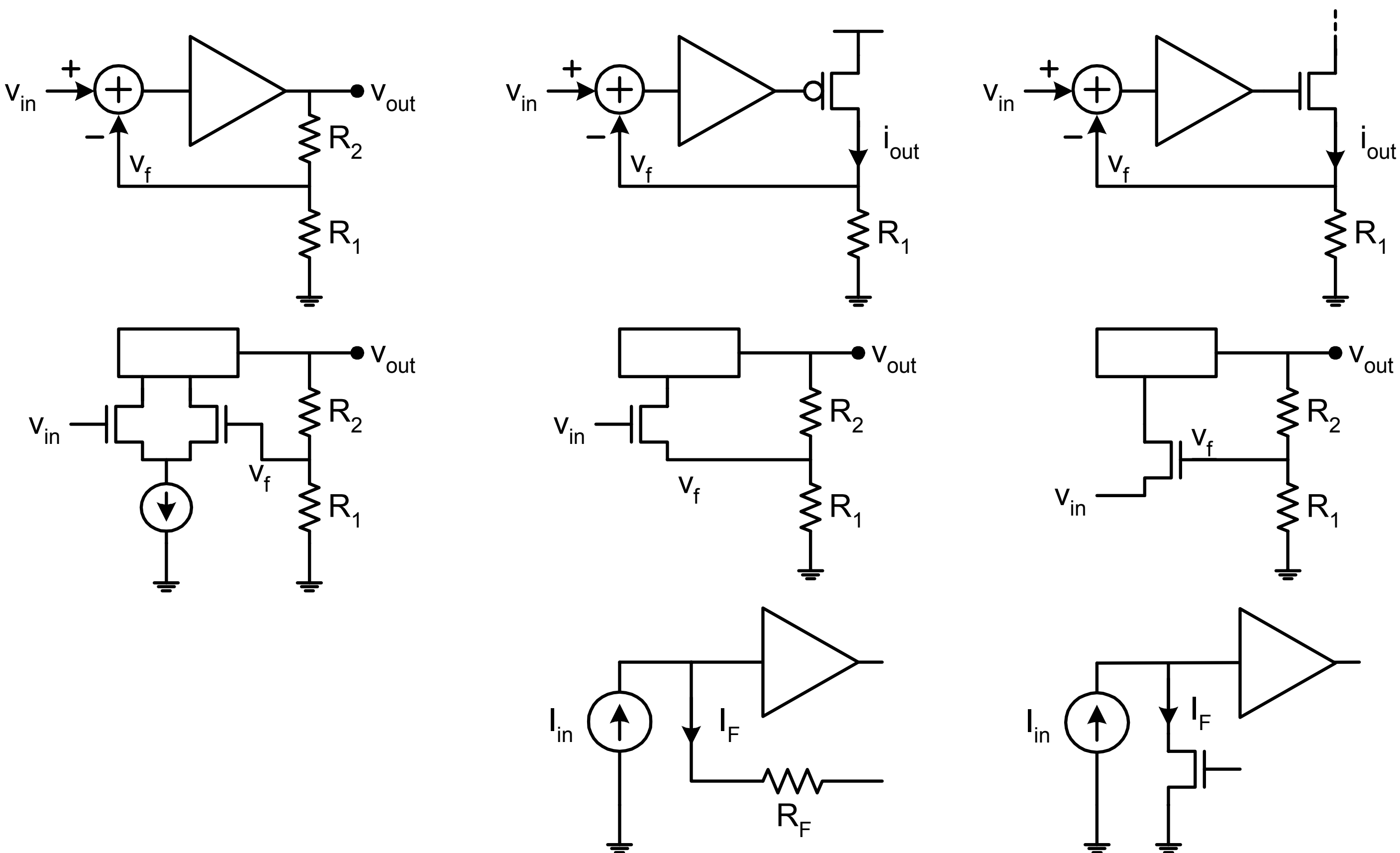


Sense and Return Cont'd

- Here are some circuit examples of sensing and returning voltages and currents:

Voltage return

Current return



Series-Shunt Feedback Amplifier (Voltage-Voltage Feedback)

- Samples the output voltage and returns a feedback voltage signal
  - Ideal feedback network has infinite input impedance and zero output resistance
- Find the closed-loop gain and input resistance

$$V_f = \beta V_o$$

$$V_i = V_s - V_f$$

$$V_o = A(V_s - \beta V_o)$$

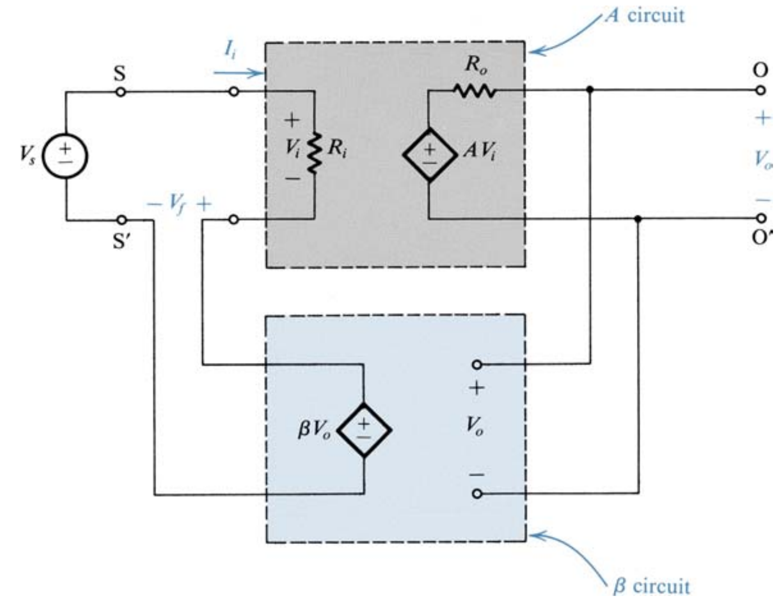
$$A_f \equiv \frac{V_o}{V_s} = \frac{A}{1+\beta A}$$

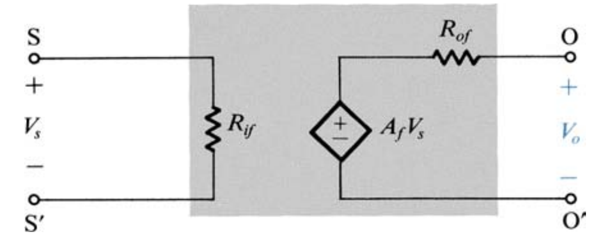
$$R_{if} = \frac{V_s}{I_i} = \frac{V_s}{V_i/R_i} = R_i \frac{V_s}{V_i} = R_i \frac{V_i + \beta A V_i}{V_i} = R_i(1+A\beta)$$

  - The output resistance can be found by applying a test voltage to the output

$$R_{of} = \frac{R_o}{1+\beta A}$$

- So, increases input resistance and reduces output resistance → makes amplifier closer to ideal VCVS

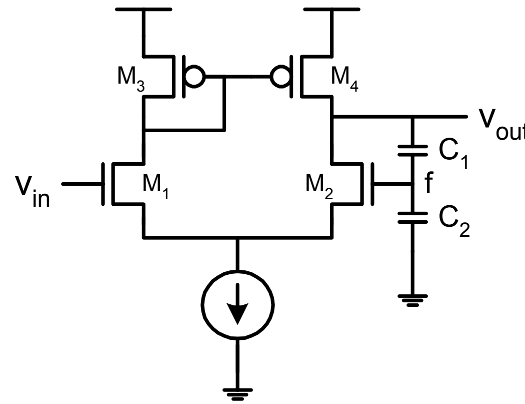




Circuit Example

- Feedback can be constructed out of capacitors
- Find the open-loop gain
  - At low frequency, cap loads negligible
  - Break feedback and zero out input to feedback
  - Assume V_f is at some DC bias equal to the DC value of v_{in}

$$A = g_m\left(r_{o2} \parallel r_{o4}\right)$$



- Find the loop gain, closed-loop gain, and closed-loop R_{out}

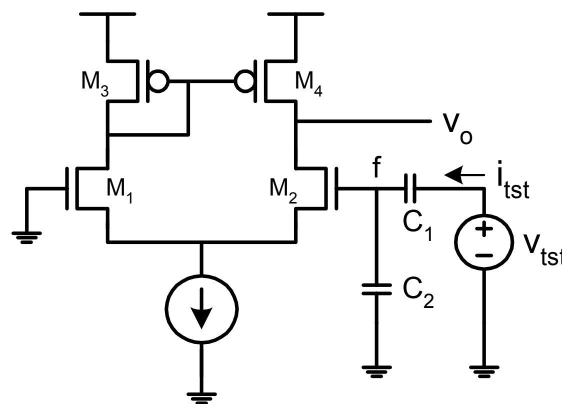
Note correction: v_o and not v_f

$$v_o = -\frac{C_1}{C_1 + C_2} g_{m1}\left(r_{o2} \parallel r_{o4}\right) v_{tst}$$

$$\beta A = -\frac{v_o}{v_{tst}} = \frac{C_1}{C_1 + C_2} g_{m1}\left(r_{o2} \parallel r_{o4}\right)$$

$$A_f = \frac{A}{1+\beta A} = \frac{g_{m1}\left(r_{o2} \parallel r_{o4}\right)}{1 + \frac{C_1}{C_1 + C_2} g_{m1}\left(r_{o2} \parallel r_{o4}\right)}$$

$$R_{out,closed} = \frac{r_{o2} \parallel r_{o4}}{1 + \frac{C_1}{C_1 + C_2} g_{m1}\left(r_{o2} \parallel r_{o4}\right)} \approx \left(1 + \frac{C_1}{C_1 + C_2}\right)\frac{1}{g_{m1}}$$

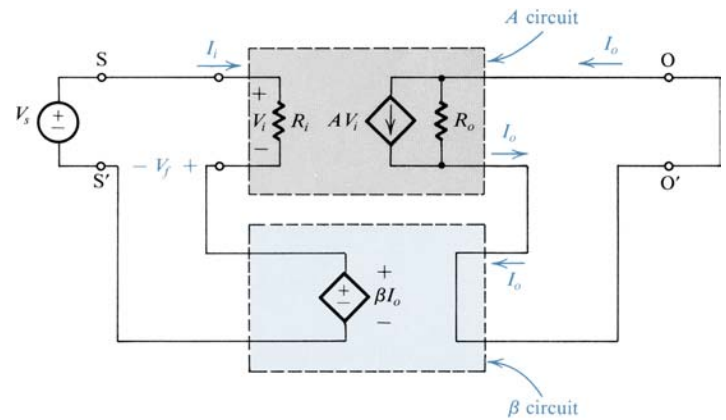


Circuit to solve loop gain

Series-Series Feedback Amplifier (Current-Voltage FB)

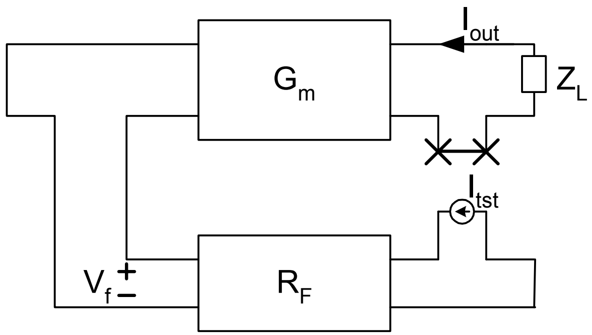
- For a transconductance amplifier (voltage input, current output), we must apply the appropriate feedback circuit
  - Sense the output current and feedback a voltage signal. So, the feedback current is a transimpedance block that converts the current signal into a voltage.

$$A_f \equiv \frac{I_o}{V_s} = \frac{A}{1+A\beta} \qquad A \equiv \frac{I_o}{V_i} \text{ (also called } G_m\text{)}$$



- To solve for the loop gain:
  - Break the feedback, short out the break in the current sense and applying a test current

$$\text{Loop Gain} = A\beta = -\frac{I_{out}}{I_{tst}} = G_m R_f$$

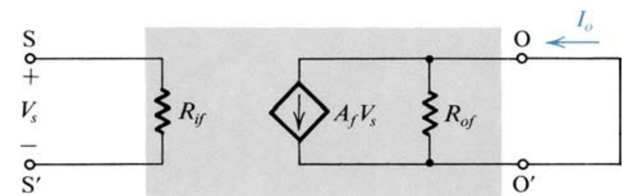


- To solve for R_{if} and R_{of}

$$R_{if} = \frac{V_s}{I_i} = \frac{V_i + V_f}{I_i} = \frac{R_i I_i + \beta I_o}{I_i} = \frac{R_i I_i + A\beta V_i}{I_i} = R_i(1+A\beta)$$

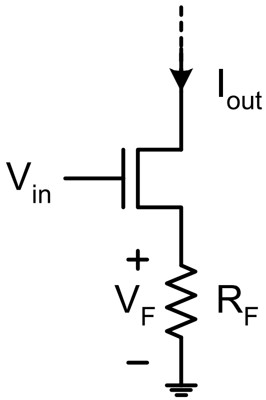
  - Apply a test voltage V_{tst} across O and O'

$$R_{of} = \frac{V_{tst}}{I_{tst}} = \frac{(I_{tst} - AV_i)R_o}{I_{tst}} = \frac{(I_{tst} + A\beta I_{tst})R_o}{I_{tst}} = (1+A\beta)R_o$$



Circuit Example

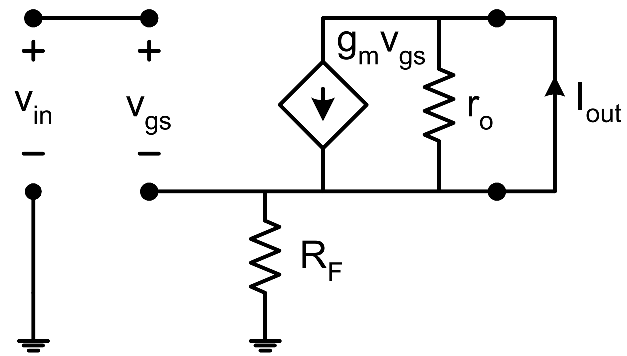
- Source degeneration (with R_F) is a form of current-voltage FB
  - Voltage across the resistor is the feedback voltage
    - V_F subtracts from V_{in} to reduce V_{gs} of the nMOS
  - Without the feedback R, $i_{out}/v_{in} = g_m \rightarrow A = g_m$
  - R_F is the feedback circuit that senses the output current and subtracts a voltage from the input
    - $V_F = \beta I_{out} \rightarrow \beta = R_F$



$$i_{out} = g_m v_{gs} = g_m\left(v_{in} - i_{out}R_F\right)$$

$$i_{out}\left(1 + g_m R_F\right) = g_m v_{in}$$

$$A_f \equiv \frac{i_{out}}{v_{in}} = \frac{A}{1 + A\beta} = \frac{g_m}{1 + g_m R_F}$$



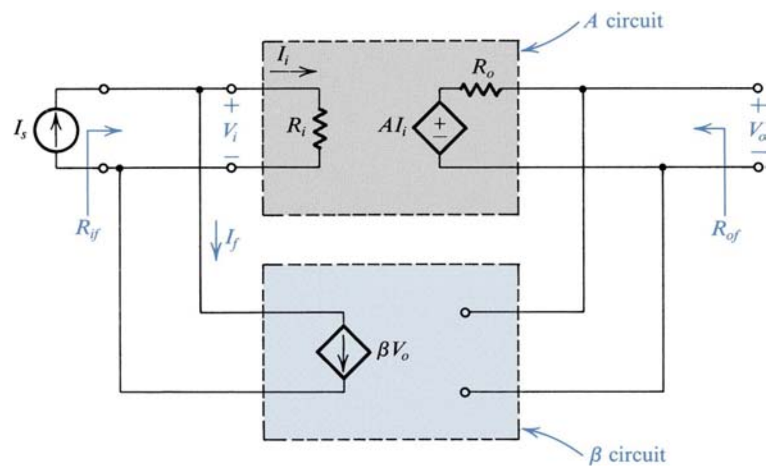
Shunt-Shunt Feedback Amplifier (Voltage-Current FB)

- When voltage-current FB is applied to a transimpedance amplifier, output voltage is sensed and current is subtracted from the input
  - The gain stage has some resistance
  - The feedback stage is a tranconductor

$$A = \frac{V_o}{I_i}$$

$$I_s = I_i + I_f = \frac{V_o}{A} + \beta V_o$$

$$A_f \equiv \frac{V_o}{I_s} = \frac{A}{1 + A\beta}$$



  - Input and output resistances (R_{if} and R_{of}) follow the same form as before based on values for A and β

$$R_{if} = R_i(1 + A\beta)$$

$$R_{of} = \frac{R_o}{(1 + A\beta)}$$

Circuit Example

- Transimpedance gain stage is through M2 (a common-base amplifier) and feedback is thru the capacitor divider and M_1 (transconductor). Solve for A, Aβ, and A_f.
  - v_{gs} is whatever voltage necessary for i_{in}

$$A = \frac{v_{out}}{i_{in}} = \frac{i_{in} R_D}{i_{in}} = R_D$$

  - Loop gain is found by breaking loop and applying V_{tst}

$$v_F = \frac{C_1}{C_1 + C_2} v_{tst}$$

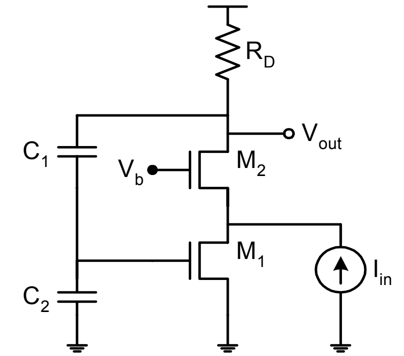
$$i_1 = g_m v_F$$

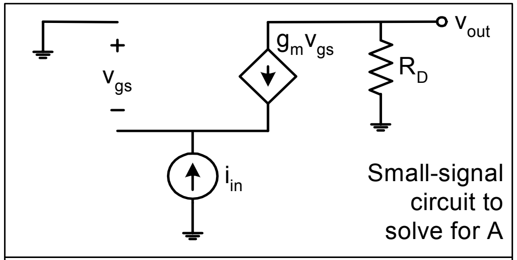
$$v_{out} = -i_1 R_D$$

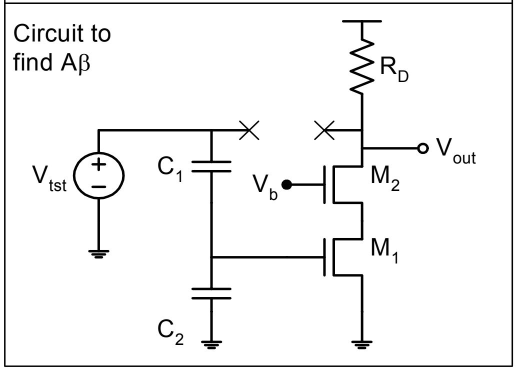
$$A\beta = -\frac{v_{out}}{v_{tst}} = -\frac{-g_m R_D\, C_1/(C_1 + C_2) v_{tst}}{v_{tst}} = g_m R_D \frac{C_1}{C_1 + C_2}$$

  - Closed-loop gain is just

$$A_f \equiv \frac{v_{out}}{i_{in}} = \frac{A}{1 + A\beta} = \frac{R_D}{1 + g_m R_D \dfrac{C_1}{C_1 + C_2}}$$

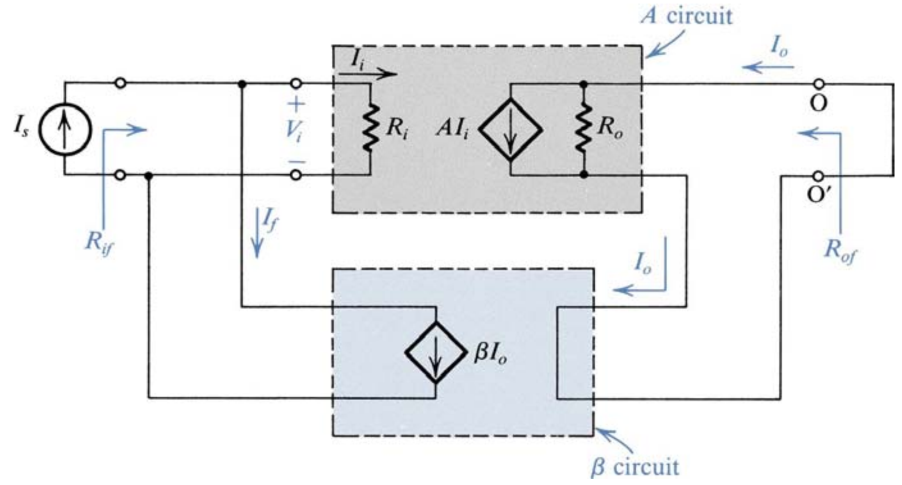




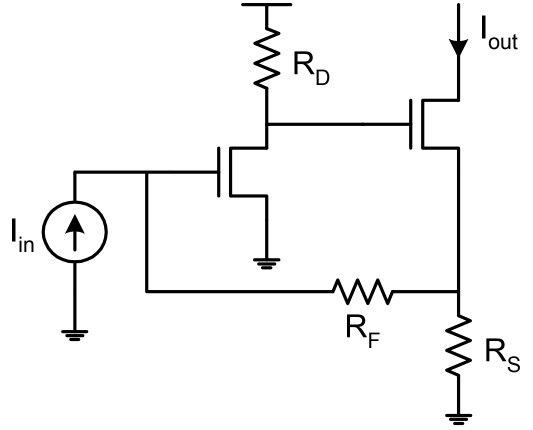


Shunt-Series Feedback Amplifier (Current-Current FB)

- A current-current FB circuit is used for current amplifiers
  - For the β circuit – input resistance should be low and output resistance be high

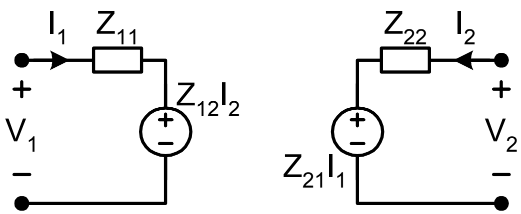


- A circuit example is shown
  - R_S and R_F constitute the FB circuit
    - R_S should be small and R_F large
  - The same steps can be taken to solve for A, $A\beta$, Af, R_{if}, and R_{of}
    - Remember that both A and β circuits are current controlled current sources

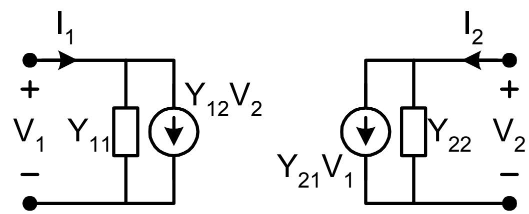


Effects of Loading

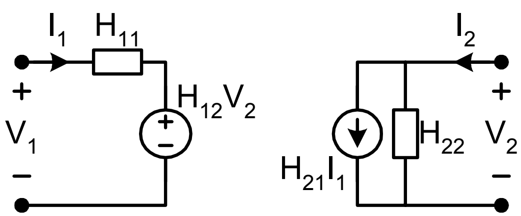
- So far, we have assumed that the feedback circuit does not load the feedforward (open-loop) amplifier at the input or the output. In reality, this is not the case. Loading may not be negligible and complicate analysis.
- In order to properly model loading effects, we should first review models of two-port networks. The feedback circuit can be considered to be a two-port network that senses and produces currents or voltages. The following are four types of linear two-port networks using Z, Y, or H or G (impedance, admittance, or hybrid) models



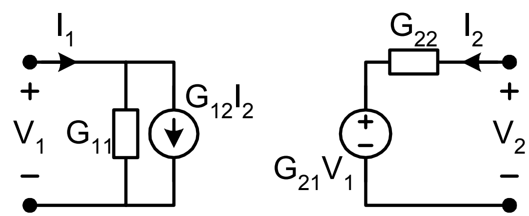
$$V_1 = Z_{11} I_1 + Z_{12} I_2$$
$$V_2 = Z_{21} I_1 + Z_{22} I_2$$



$$I_1 = Y_{11} V_1 + Y_{12} V_2$$
$$I_2 = Y_{21} V_1 + Y_{22} V_2$$

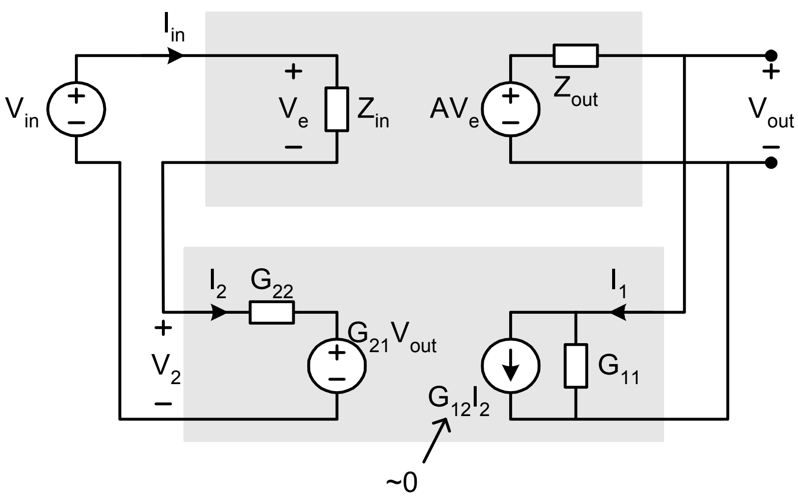


$$V_1 = H_{11} I_1 + H_{12} V_2$$
$$I_2 = H_{21} I_1 + H_{22} V_2$$



$$I_1 = G_{11} V_1 + G_{12} I_2$$
$$V_2 = G_{21} V_1 + G_{22} I_2$$

Loading in Series-Shunt (Voltage-Voltage) FB



- Which is the best model to use?
  - Ideally, want infinite input resistance and zero output resistance → G-model
  - $G_{12}I_2$ contribution to V_{out} is small compared to amplification *A* so ignore G_{12} current source
    - reverse transmission thru the FB circuit is made purposely small
- Computer the closed-loop gain…

$$V_e = (V_{in} - G_{21}V_{out})\frac{Z_{in}}{Z_{in}+G_{22}}$$

$$V_{out} = (V_{in} - G_{21}V_{out})\frac{Z_{in}}{Z_{in}+G_{22}} A \frac{G_{11}^{-1}}{G_{11}^{-1}+Z_{out}} \quad \Rightarrow \quad \frac{V_{out}}{V_{in}} = \frac{A\dfrac{Z_{in}}{Z_{in}+G_{22}}\dfrac{G_{11}^{-1}}{G_{11}^{-1}+Z_{out}}}{1+A\dfrac{Z_{in}}{Z_{in}+G_{22}}\dfrac{G_{11}^{-1}}{G_{11}^{-1}+Z_{out}}G_{21}}$$

- In the limit (ideal case), $1/G_{11}=\infty$ and $G_{22}=0$ then equation reduces to…

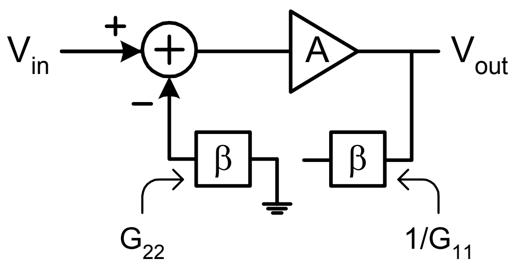
$$A_f = \frac{A}{1+A\beta} = \frac{A}{1+AG_{21}}$$

- We can retain this form for closed-loop gain by solving for the open-loop gain that takes loading into account

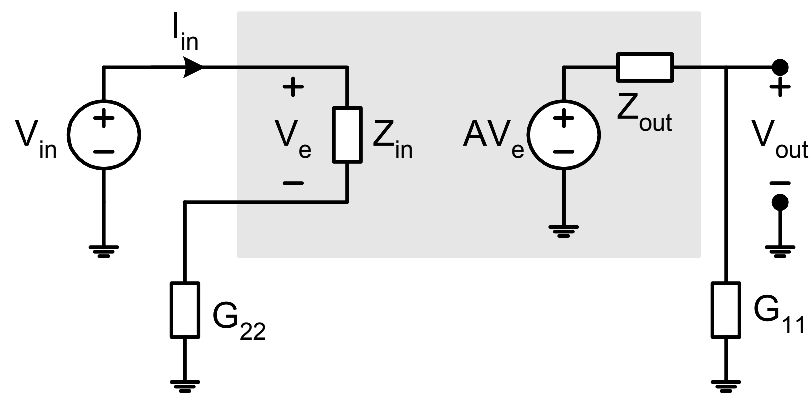
Open-Loop Gain with Loading

- For the previous example, proper method for including loading to calculate the open-loop gain is shown

- Conceptually, the approach is…



  – add load both sides (input and output) with the feedback circuit
  – short the input of the β circuit that loads the input of the amplifier
  – leave the output unconnected for the β circuit that loads the output of the amplifier
  – Calculate the open-loop gain



$$G_{11} = \left.\frac{I_1}{V_1}\right|_{I_2=0} \text{ and } G_{22} = \left.\frac{V_2}{I_2}\right|_{V_1=0}$$

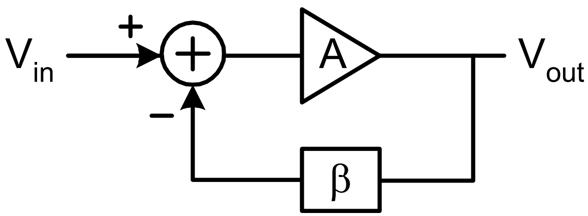
Summary of FB Types and 2-Port Models

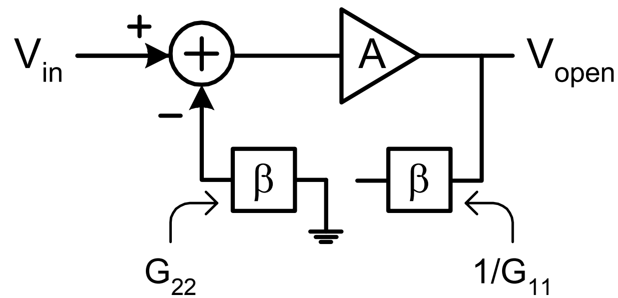
- There is an appropriate 2-port model to use for each of the feedback types (in table)
- A conceptual summary of solving for the open-loop gain with feedback loading effects are as follows
  - Attach separate feedback circuits to both input and output (individually) as it would normally be connected
  - For the feedback circuit attached to the input (of the amplifier), zero out the other side of the feedback circuit (input port)
    - If input port to β circuit is a voltage, attach to a 0-Volt source
    - If input port to β circuit is a current, attach to a 0-Amp source
  - For the feedback circuit attached to the output (of the amplifier), leave the other side disconnected
    - If output port of β circuit is a voltage, leave as open circuit
    - If output port of β circuit is a current, short output to ground

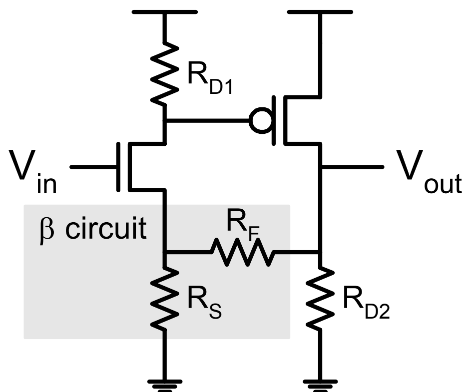
- Let's look at some examples of how to break the feedback and determine the open-loop gain that includes the effects of feedback.

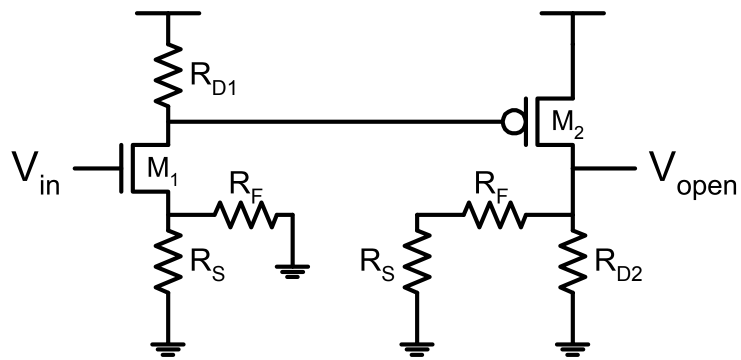
Feedback Type	2-port Model
Series-Shunt (Voltage-Voltage)	G Model
Series-Series (Current-Voltage)	Z Model
Shunt-Shunt (Voltage-Current)	Y Model
Shunt-Series (Current-Current)	H Model

Loading in Series-Shunt (Voltage-Voltage) FB Example





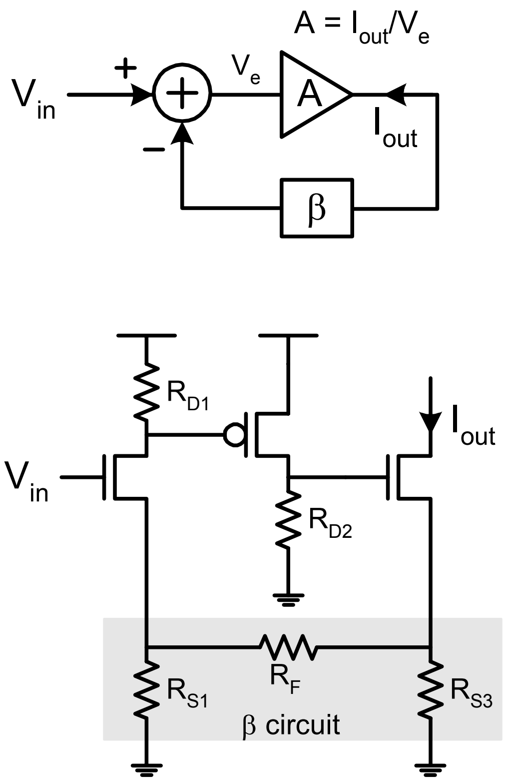


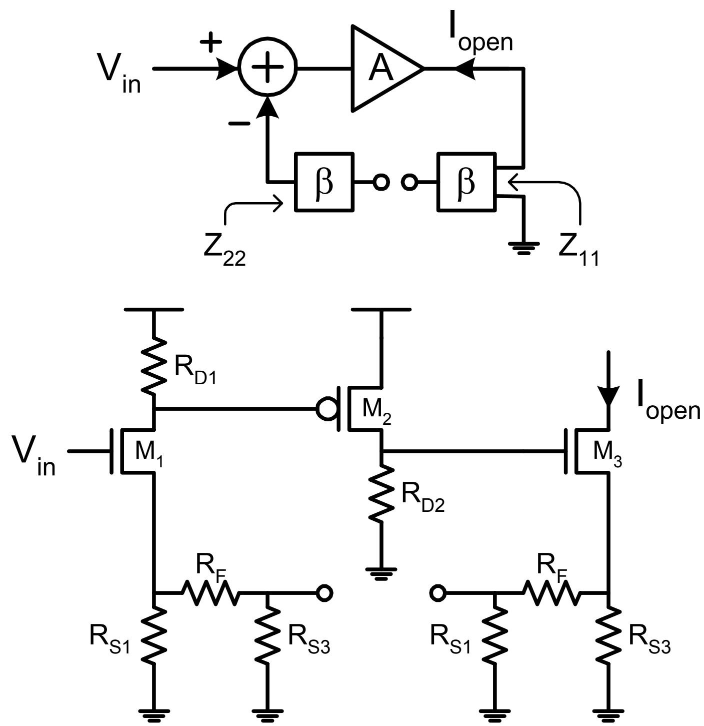


- The open-loop gain (with loading) can be calculated to be

$$A_{open-loop} = \frac{V_{open}}{V_{in}} = \frac{-R_{D1}}{R_F \parallel R_S + 1/g_{m1}} \left\{ -g_{m2} \left[R_{D2} \parallel \left(R_F + R_S \right) \right] \right\}$$

Loading in Series-Series (Current-Voltage) FB Example

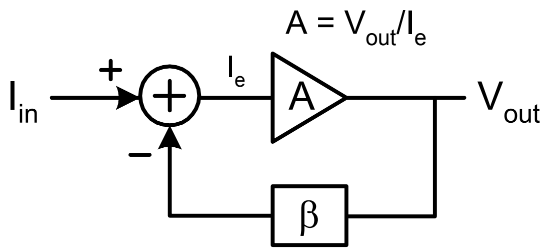


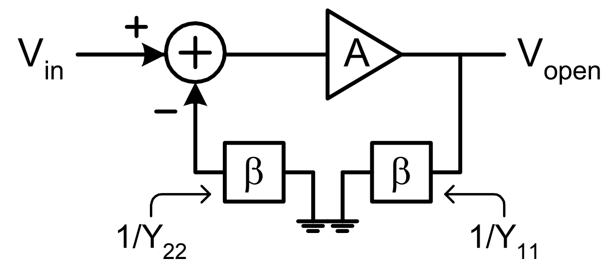


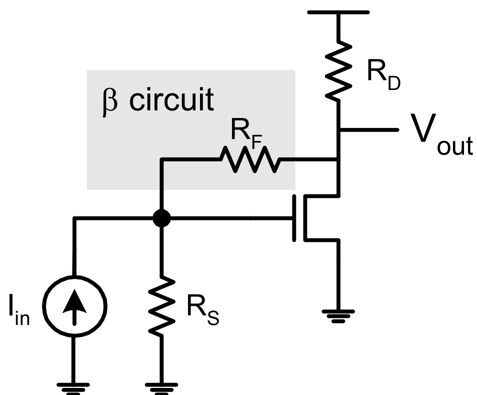
- The open-loop gain (transconductance) can be solved to be:

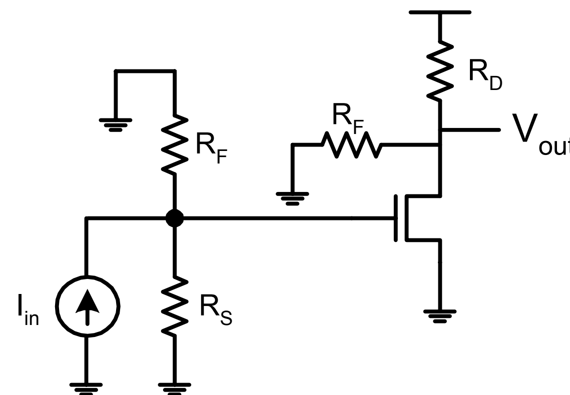
$$A_{open-loop} = \frac{I_{open}}{V_{in}} = \frac{-R_{D1}}{(R_F + R_{S3}) \| R_{S1} + 1/g_{m1}} \frac{-g_{m2} R_{D2}}{(R_F + R_{S1}) \| R_{S3} + 1/g_{m3}}$$

Loading in Shunt-Shunt (Voltage-Current) FB Example





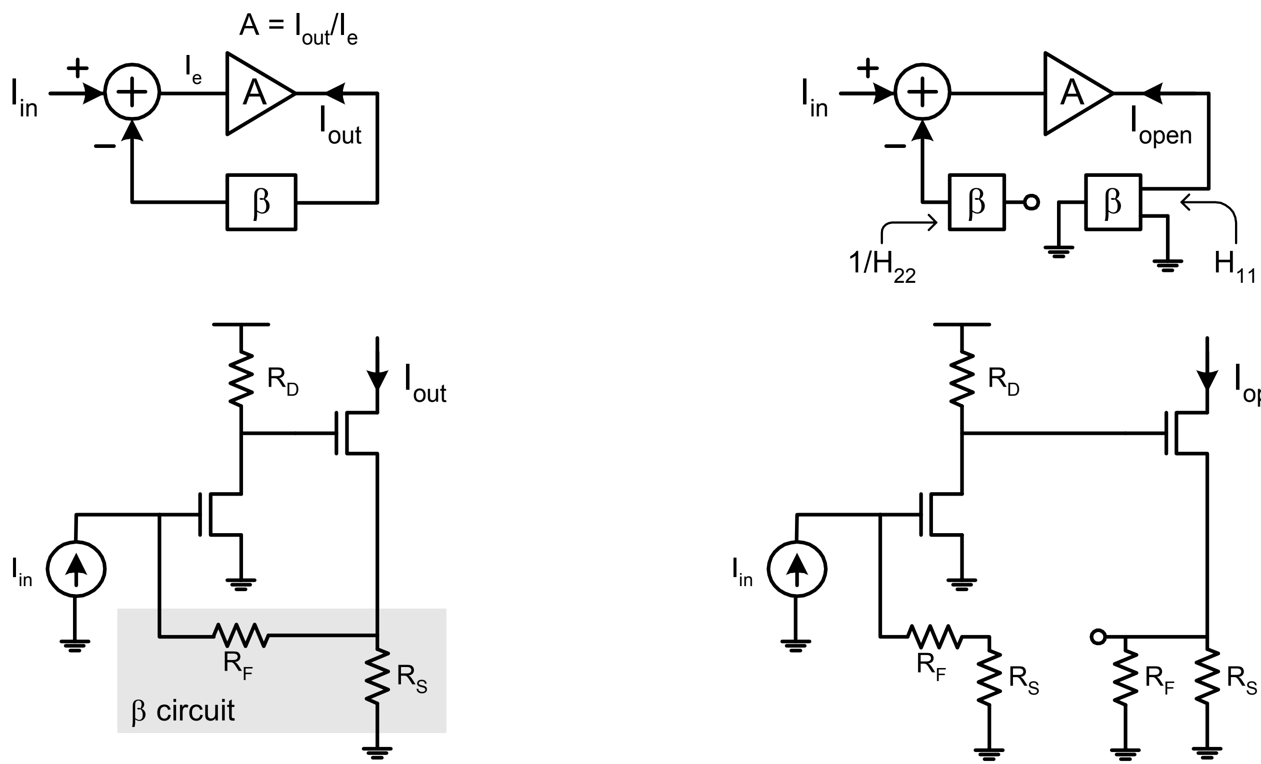




- The open-loop gain (transimpedance) can be solved to be:

$$A_{open-loop} = \frac{V_{open}}{I_{in}} = -g_m (R_S \parallel R_F)(R_D \parallel R_F)$$

Loading in Shunt-Series (Current-Current) FB Example



- The open-loop (current) gain be solved to be:

$$A_{open-loop} = \frac{I_{open}}{I_{in}} = -g_m (R_S + R_F) R_D \frac{1}{R_F \parallel R_S + 1/g_{m2}}$$

Next Time

- Reading
  - S&S Chapter 10.7~10.8

- What to look forward to…
  - We will shift gears a little bit and look at a few approaches for building CMOS Operational Amplifiers. We will begin with single-stage op amps and the proceed to two-stage op amps and some of their advantages and disadvantages.
  - The investigation of op amp designs with multiple poles forces us to then deal with stability issues. The final lecture will cover stability and compensation in amplifier design. Reading for the final lecture is S&S Chapter 8.8~8.11.