Lecture 5:

Operational Amplifiers and Op Amp Circuits

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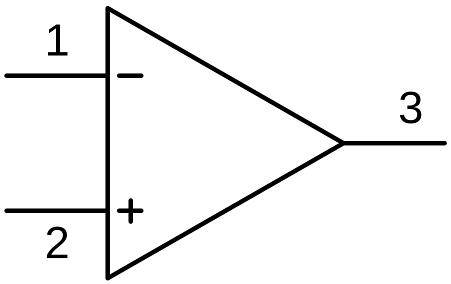
Overview

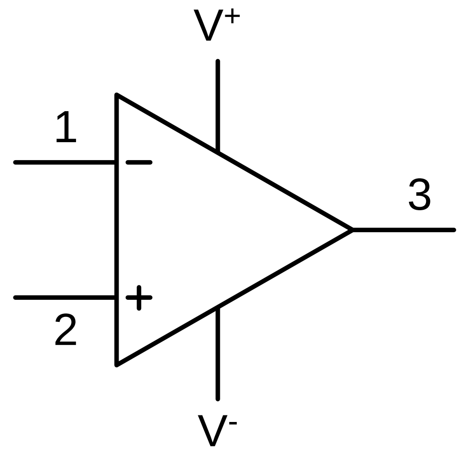
- **Reading**
  - S&S: Chapter 2
- **Supplemental Reading**

- **Background**
  - Armed with our circuit analysis tools, we begin the real meat of the material for this course by looking at op amps. Op amps were initially constructed out of vacuum tubes, then discrete transistor components. With the advent of the integrated circuit, op amp ICs came out in the 60's. They are extremely useful because they are versatile and one can do almost anything with op amps. To start, we will look at an ideal version of the op amp and see how they are useful. Then, we will investigate various non-idealities of real amplifier design and how they affect op amp circuits.

Op-Amp Terminals



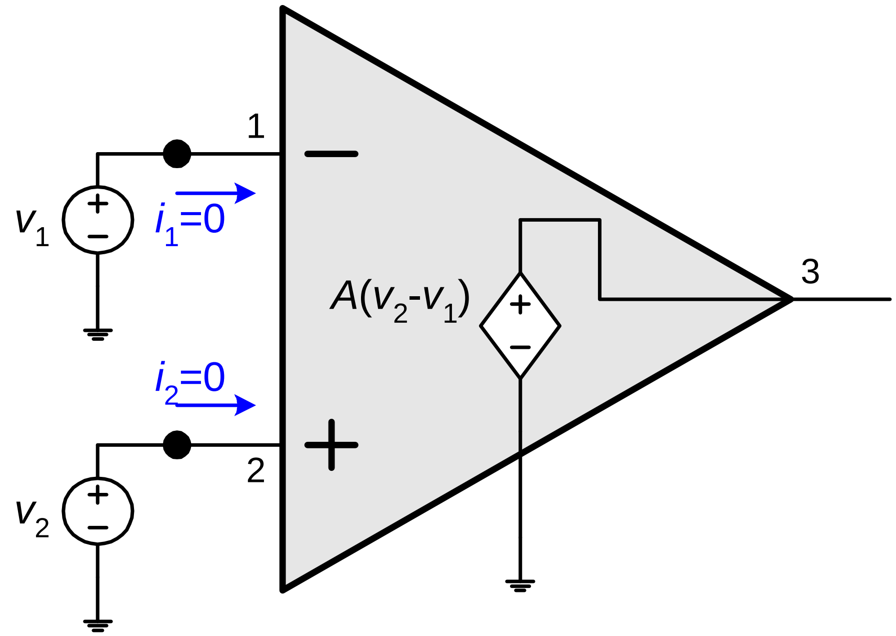
op amp symbol
(we will use most often)



op amp symbol
with power supply connections

- At a minimum, op amps have 3 terminals: 2 input and 1 output.
- An op amp also requires dc power to operate. Often, the op amp requires both positive and negative voltage supplies.

Ideal Op Amp

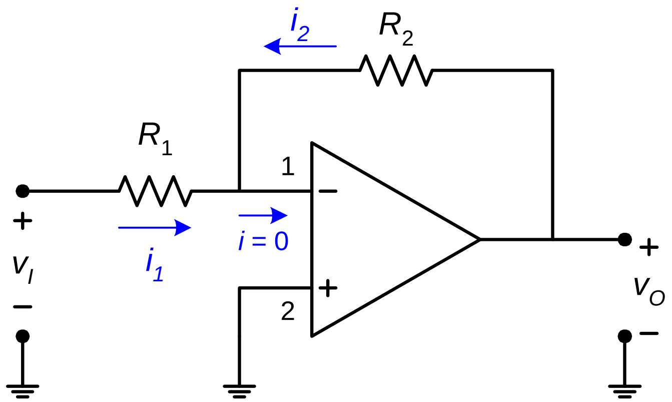


- The op amp is designed to sense the difference between the voltage signals applied to the two input terminals and then multiply it by some gain factor *A* such that the voltage at the output terminal is $A(v_2-v_1)$.
  - One of the input terminals (1) is called an inverting input terminal denoted by '-'
  - The other input terminal (2) is called a non-inverting input terminal denoted by '+'
  - The gain *A* is often referred to as the differential gain or open-loop gain
  - Notice that we can model the amplifier as a voltage-controlled voltage source

Ideal Op Amps Characteristics

- Ideal op amp characteristics:
  - Does not draw input current so that the input impedance is infinite (i_1=0 and i_2=0)
  - The output terminal can supply an arbitrary amount of current and the output impedance is zero
  - The op amp only responds to the voltage difference between the signals at the two input terminals and ignores any voltages common to both inputs. In other words, an ideal op amp has infinite **common-mode rejection**.
  - The frequency response of an ideal op amp is flat for all frequency. In other words, it amplifies signals of any and all frequencies by the same amount *A*.
  - Lastly, *A* is or can be treated as being infinite. Useful b/c we can easily specify a closed-loop gain as will see later.
- We will see later that real op amps do not have the characteristics above, but we strive to make them behave as close to an ideal op amp as possible.

Op Amps in the Inverting Configuration



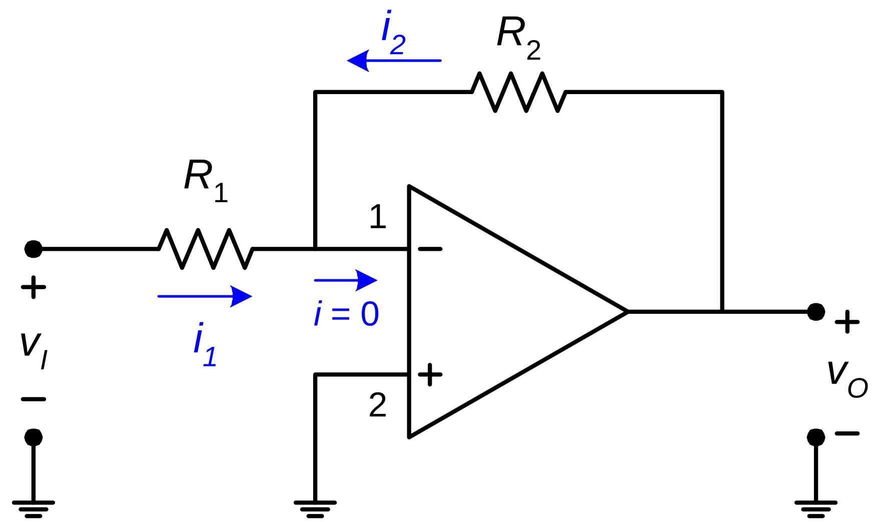
- Let's look at an op amp in an inverting closed-loop configuration.
  - There are two resistors R_1 and R_2
  - R_2 is called the (negative) **feedback** resistor and also "closes" the loop. A resistor between terminals 1 and 3 would be a positive feedback resistor.
- Closed-Loop Gain G
  - Defined,

$$G \equiv \frac{v_O}{v_I}$$

  - Assume A is infinite and the amp is trying to produce a voltage on terminal 3. Then, the voltage difference between terminals 1 and 2 should be very small, v_2-v_1→0 and A→inf. By definition…

$$v_2 - v_1 = \frac{v_O}{A} \cong 0$$

  - So, we say there is a virtual short between the two terminals and that terminal 1 is a virtual ground since terminal 2 is grounded.



- Use KCL to solve for the close-loop gain.

$$i_1 = \frac{v_I - v_1}{R_1} \cong \frac{v_1}{R_1} \qquad i_2 \cong \frac{v_O - 0}{R_2} = \frac{v_O}{R_2}$$

$$i_1 + i_2 = \frac{v_1}{R_1} + \frac{v_O}{R_2} = 0 \Rightarrow \frac{v_1}{R_1} = -\frac{v_O}{R_2} \Rightarrow G = \frac{v_O}{v_1} = -\frac{R_2}{R_1}$$

- We can adjust the closed-loop gain by changing the ratio of R2 and R1.
- If the input is a sine wave, then the output is a sign wave phase-shifted by 180 degrees.
- The closed-loop gain is (ideally) independent of op amp open-loop gain *A* and we can make it arbitrarily large or small and of desired accuracy depending on the accuracy of the resistors.
- This is a classic example of what negative feedback does. It takes an amplifier with very large gain and through negative feedback, obtain a gain that is smaller, stable, and predictable. In effect, we have traded gain for accuracy. This kind of trade off is common in electronic circuit design… as we will see.

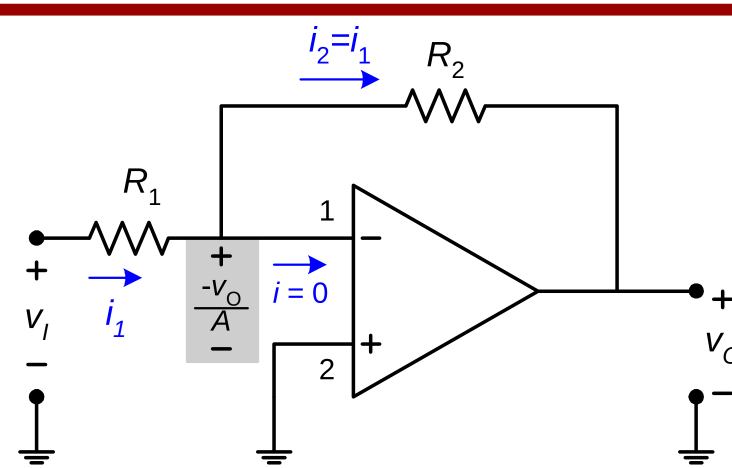
Finite Open-Loop Gain

- Since infinite *A* is not physically possible, what happens when *A* is finite?
  - Instead of a virtual ground, assume input terminal 1 has potential $-v_O/A$

$$G \equiv \frac{v_O}{v_1} = \frac{-R_2/R_1}{1+(1+R_2/R_1)/A}$$

  - As A → infinity, G → $-R_2/R_1$
    and the voltage at terminal 1 goes to 0… the virtual ground assumption we made earlier
  - To minimize the effects of open-loop gain on G, we want…

$$1+\frac{R_2}{R_1} << A$$



$$i_1 = \frac{v_1-(-v_O/A)}{R_1} = \frac{v_1+v_O/A}{R_1}$$

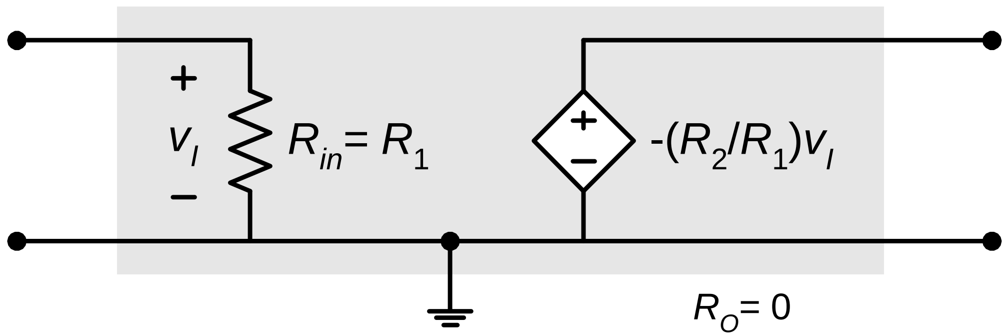
$$v_O = -\frac{v_O}{A} - i_1 R_2 = -\frac{v_O}{A} - \left(\frac{v_1+v_O/A}{R_1}\right)R_2$$

Input and Output Resistance

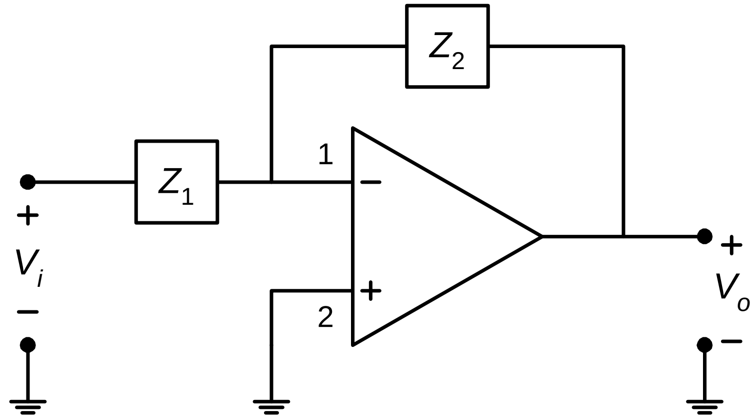
- Assuming an ideal op amp with infinite open-loop gain A, in the closed-loop inverting configuration, the input resistance is R_1.

$$R_{in} \equiv \frac{v_I}{i_1} = \frac{v_I}{v_I / R_1} = R_1$$

  - To make R_{in} high, need to make R_1 high which is not practical
    - See Example 2.2 in S&S for a possible solution to this problem
  - The output resistance is 0 since it is the output of a voltage source
- We can model the closed-loop inverting amplifier with the following equivalent circuit using a voltage-controlled voltage source…



Inverting Configuration with General Impedances

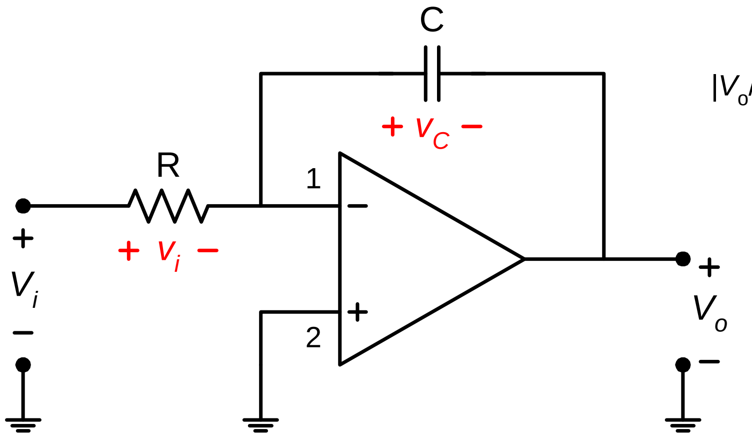


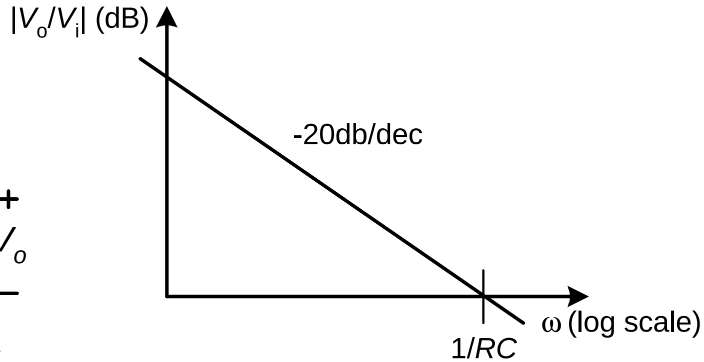
- Let's replace R1 and R2 in the inverting configuration with impedances $Z_1(s)$ and $Z_2(s)$.
- We can write the closed-loop transfer function as…

$$\frac{V_o(s)}{V_i(s)} = -\frac{Z_2(s)}{Z_1(s)}$$

- By placing different circuit elements into Z_1 and Z_2, we can get interesting operations. Some examples…
  - Integrator
  - Differentiator
  - Summer
  - Unity–Gain Buffer

Inverting Integrator





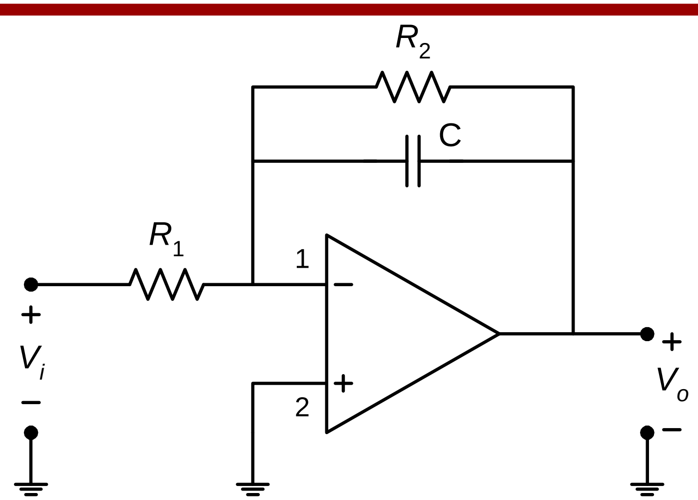
- We replace Z_2 (the negative feedback impedance) with a capacitor and Z_1 is a resistor.

$$\frac{V_o(s)}{V_i(s)} = -\frac{1/sC}{R} = -\frac{1}{sRC} \Rightarrow \left|\frac{V_o(jw)}{V_i(jw)}\right| = \left|-\frac{1}{jwRC}\right| = \frac{1}{wRC} \Rightarrow 1 = \frac{1}{wRC} \rightarrow w = \frac{1}{RC}$$

- How about in the time domain?

$$v_R = v_i \text{ and } v_C = v_C(0) + \frac{1}{C}\int_0^t i_C(t)dt$$

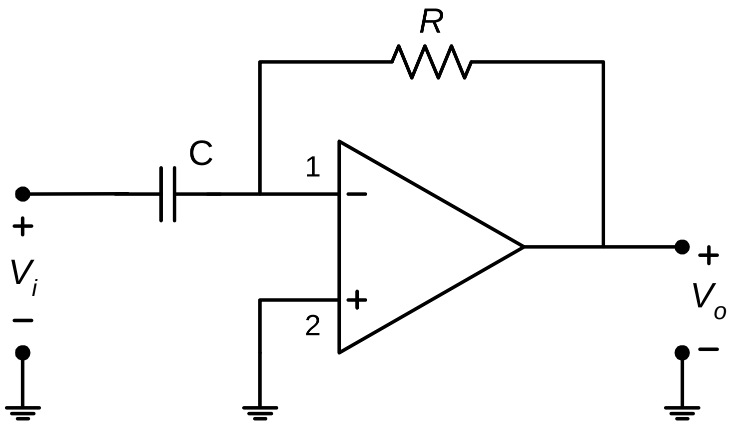
$$v_O(t) = -\frac{1}{RC}\int_0^t v_I(t)dt - v_C(0)$$

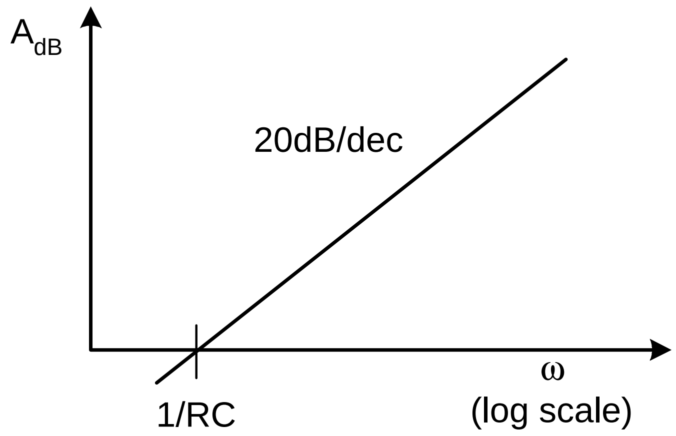


- While the DC gain in the previous integrator circuit is infinite, the amplifier itself will saturate. To limit the low-frequency gain to a known and reliable value, add a parallel resistor to the capacitor.

$$\frac{V_o(s)}{V_i(s)} = -\frac{Z_2(s)}{Z_1(s)} = -\frac{R_2 \parallel C}{R_1} = \frac{-R_2/R_1}{1+sR_2C}$$

Differentiator

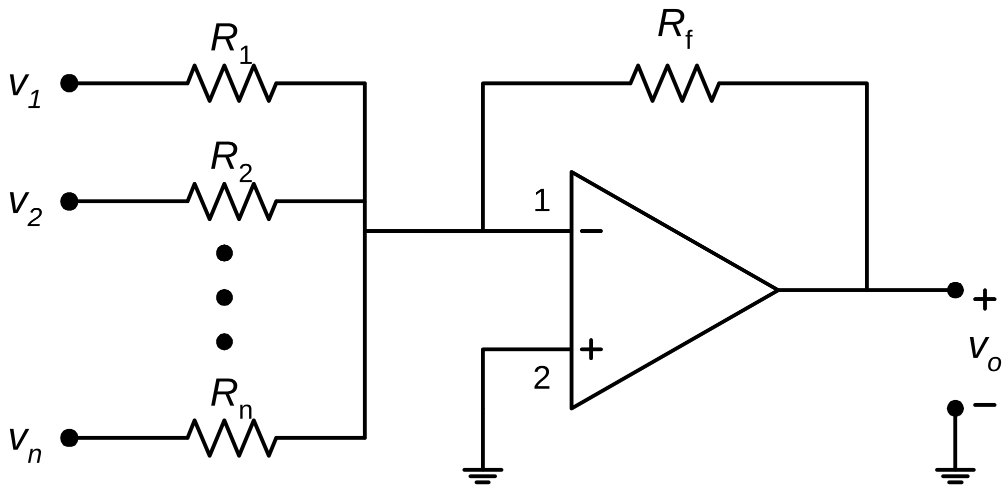




$$\frac{V_o(s)}{V_i(s)} = -\frac{Z_2(s)}{Z_1(s)} = -\frac{R}{1/sC} = -sRC$$

$$\left|\frac{V_o(jw)}{V_i(jw)}\right| = \left|-jwRC\right| = wRC \rightarrow w = \frac{1}{RC} \text{ when } A_{dB} = 0$$

Weighted Summer



- You can also building a summer.

$$i_1 = \frac{v_1}{R_1}, i_2 = \frac{v_2}{R_2}, ..., i_n = \frac{v_n}{R_n}$$

$$i = i_1 + i_2 + \cdots + i_3$$

$$v_O = 0 - iR_f$$

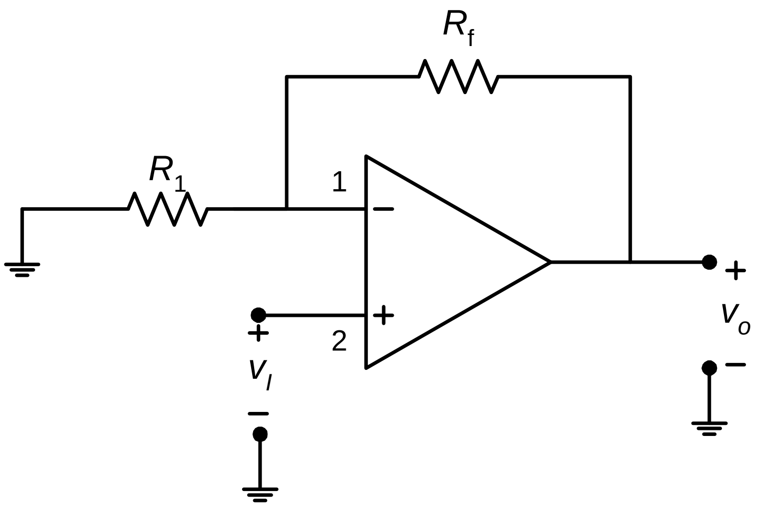
$$v_o = -\left(\frac{R_f}{R_1} v_1 + \frac{R_f}{R_2} v_2 + \cdots + \frac{R_f}{R_n} v_n \right)$$

Non-Inverting Configuration

- To avoid the inversion, shown is a non-inverting configuration

$$\frac{v_O}{A} = v_2 - v_1 \rightarrow 0 \text{ as } A \rightarrow \infty$$

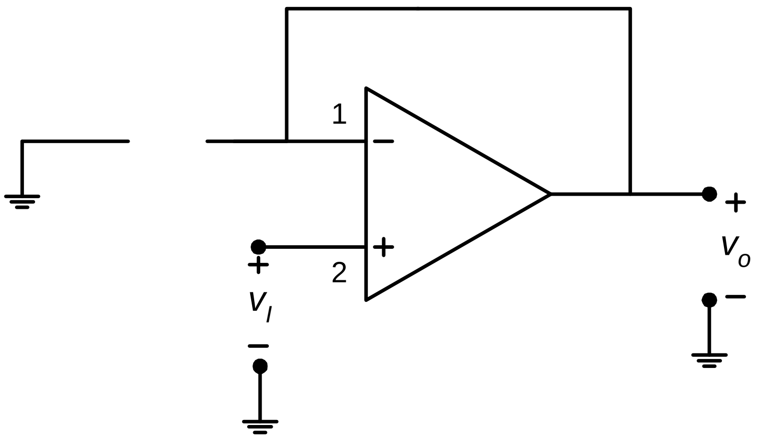
$$\frac{v_I}{R_1} = \frac{v_O - v_I}{R_f} \rightarrow \frac{v_O}{v_I} = 1 + \frac{R_f}{R_1}$$



- Now what happens as $R_1 \rightarrow$ infinity and $R_f \rightarrow 0$

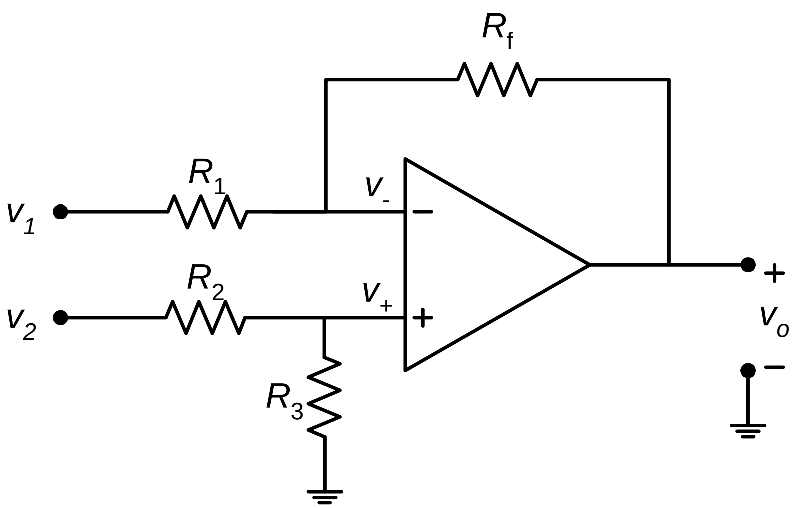
$$\frac{v_O}{v_I} = 1 + \frac{1}{\infty} \rightarrow 1$$

  – Unity-Gain Amplifier



Difference Amplifier

Now, we can combine the non-inverting amplifier and inverting amplifier configurations to be able to take a difference between two inputs. You can use superposition or brute force it…



$$\frac{v_O}{A} = v_+ - v_- \to 0 \text{ as } A \to \infty \Rightarrow v_+ = v_-$$

$$v_+ = \frac{R_3}{R_2 + R_3} v_2$$

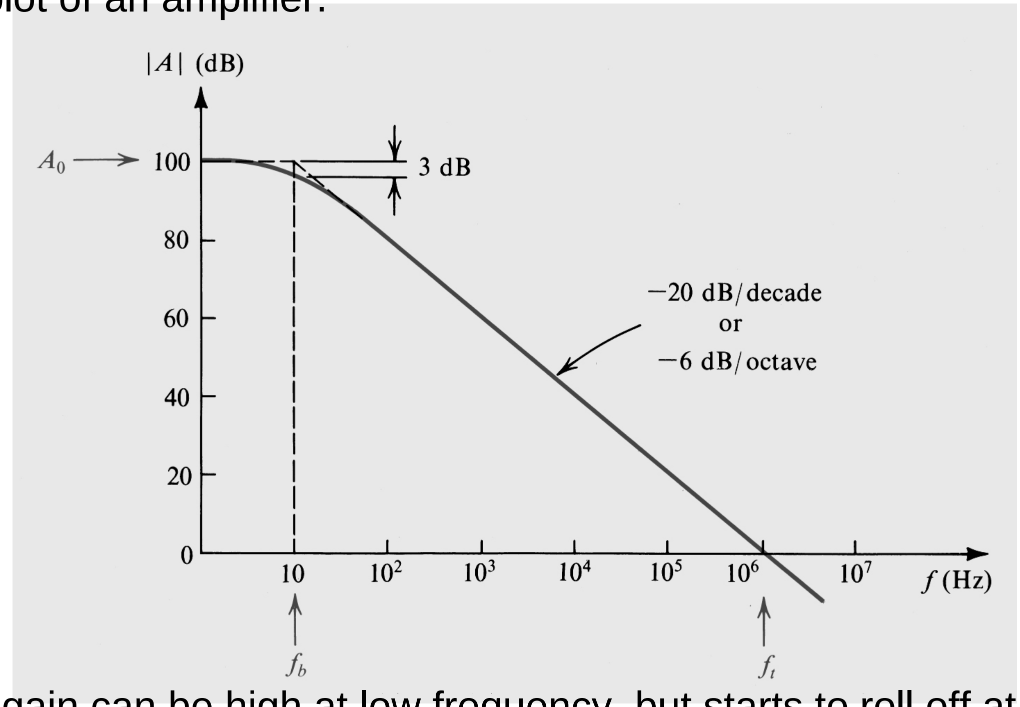
$$\frac{v_O - v_-}{R_f} = \frac{v_- - v_1}{R_1} \to \frac{v_O}{R_f} + \frac{v_1}{R_1} = \left(\frac{1}{R_f} + \frac{1}{R_1}\right) v_- \to$$

$$\frac{v_O}{R_f} + \frac{v_1}{R_1} = \left(\frac{R_f + R_1}{R_f R_1}\right)\left(\frac{R_3}{R_2 + R_3}\right) v_2 \to v_O = R_f \left(\frac{R_f + R_1}{R_f R_1}\right)\left(\frac{R_3}{R_2 + R_3}\right) v_2 - \frac{R_f}{R_1} v_1$$

$$v_O = \frac{1 + R_f / R_1}{1 + R_2 / R_3} v_2 - \frac{R_f}{R_1} v_1$$

Finite Open-Loop Gain and BW

- So far, we have assumed infinite gain and infinite bandwidth (BW) for the amplifier, but that is not reality. Amplifiers have finite gain and BW. Here's an example of the open-loop gain vs. frequency plot of an amplifier.



- Notice that the gain can be high at low frequency, but starts to roll off at a low frequency also. They are also "frequency compensated" to roll off at -20dB/dec (or a single pole) to guarantee that op amp circuits will be stable (more on this later in the semester when we talk about the guts of building amplifiers and feedback).

- We can represent frequency response characteristics of this amplifier as we did for a single-time constant low-pass filter.

$$A(s) = \frac{A_0}{1 + s/w_b}$$

$$A(jw) = \frac{A_0}{1 + jw/w_b}$$

- For frequencies much greater than w_b ($w >> w_b$) we can approximate the gain as…

$$A(jw) = \frac{A_0 w_b}{jw}$$

$$|A(j0)| = \frac{A_0 w_b}{w} = 1 \rightarrow w_t = A_0 w_b$$

w_t is called the **unity-gain BW**. So the gain can be represented as

$$A(s) = \frac{w_t}{s}$$

- So given this equation, we can find the gain at any frequency.

Frequency Response of Closed-Loop Amplifiers

- Let's look at the closed-loop gain equation we derived earlier for for an amplifier with finite op-amp open-loop gain *A*.

$$\frac{v_O}{v_i} = \frac{-R_2/R_1}{1+(1+R_2/R_1)/A} \quad \text{and} \quad A(s) = \frac{A_0}{1+s/w_b}$$

$$\frac{v_O(s)}{v_i(s)} = \frac{-R_2/R_1}{1+\frac{1}{A_0}\left(1+\frac{R_2}{R_1}\right)+\frac{s(1+R_2/R_1)}{w_t}}$$

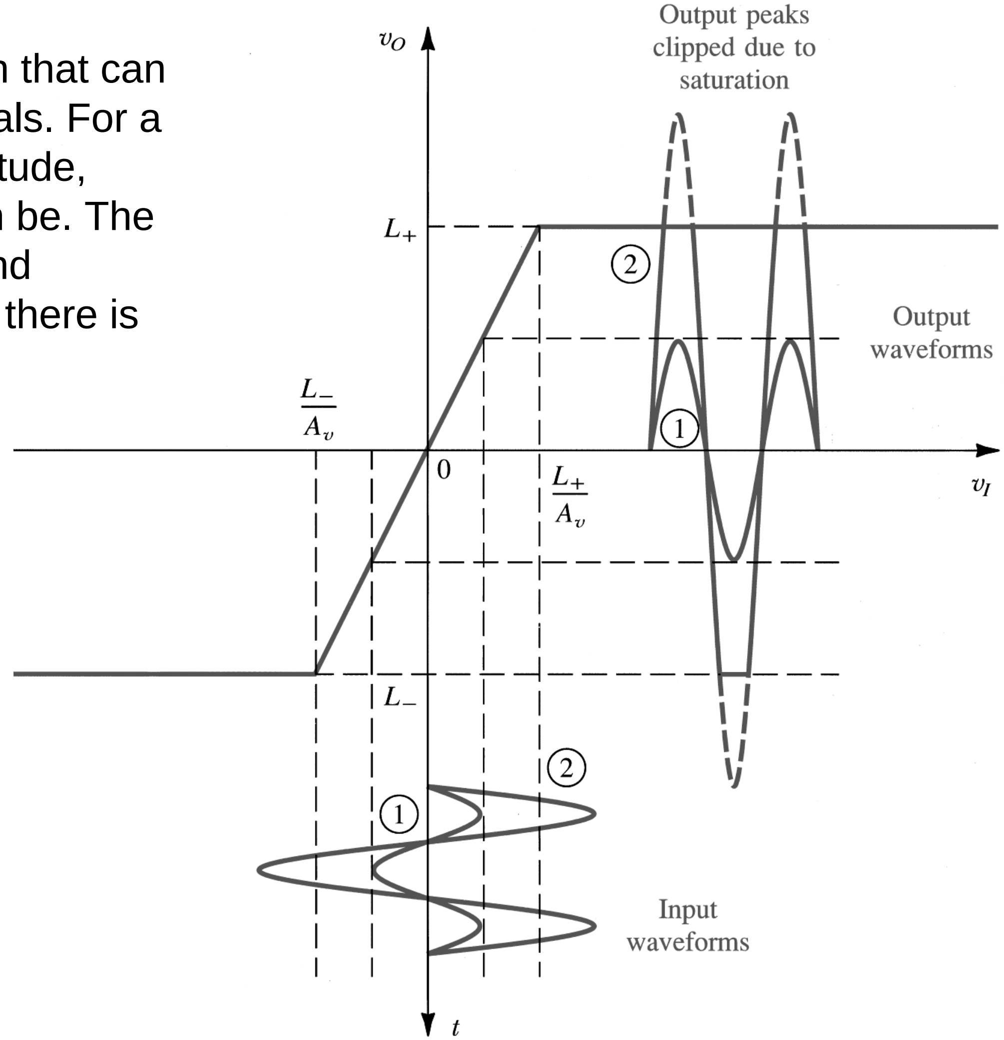
- if $A_0 >> 1+R_2/R_1$, then we can approximate the equation as…

$$\frac{v_O(s)}{v_i(s)} = \frac{-R_2/R_1}{1+\frac{s(1+R_2/R_1)}{w_t}} = \frac{-R_2/R_1}{1+\frac{s}{w_{-3dB}}} \quad \text{where } w_{-3dB} = \frac{w_t}{(1+R_2/R_1)}$$

- Therefore, the closed-loop gain has a response that rolls off at –20dB/dec at a frequency, w_{-3dB}, that is a function of the gain set by the input and feedback resistors.

Output Saturation

- So far, we have been looking at the amplification that can be achieved for relatively small (amplitude) signals. For a fixed gain, as we increase the input signal amplitude, there is a limit to how large the output signal can be. The output saturates as it approaches the positive and negative power supply voltages. In other words, there is limited range across which the gain is linear.



Slew Rate (BW limited)

- Another source of nonlinear distortion comes from the limited slew rate of the amplifier. Remember, we modeled the amplifier as a single time constant circuit. Thus, an input signal sees attenuation beyond the BW of the op amp.
- Let's look at the time domain response of the circuit by taking the inverse Laplace transform of the amplifier's transfer function multiplied by a step with magnitude V_{in}.

$$V_O(s) = \frac{K}{1 + s/w_{-3dB}} \cdot \frac{V_{in}}{s}$$

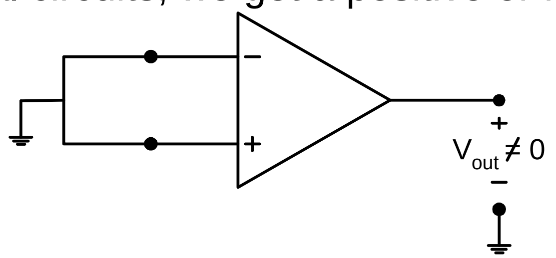
$$v_O(t) = L^{-1}\{V_O(s)\} = V_{in}K\left(1 - e^{-w_{-3dB}t}\right)$$

- The output does not change instantaneously. Rather, we see an exponential response that slews the output up. The maximum output slew rate is defined as the derivative of the output voltage at t=0.

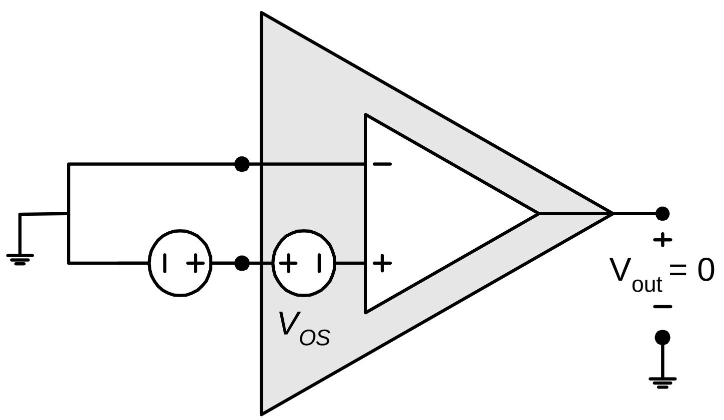
$$\text{SR} = \left.\frac{dv_O(t)}{dt}\right|_{\max} = w_{-3dB}KV_{in}$$

Voltage Offsets

- The circuit implementation of amplifiers is subject to a variety of imperfections during its fabrication. This imperfection can be due to physical imbalances that occurs even at DC (or zero frequency).
- To understand this problem, assume the two inputs to the amplifier are connected together. Instead of a zero output, in real circuits, we get a positive or negative voltage at the output.

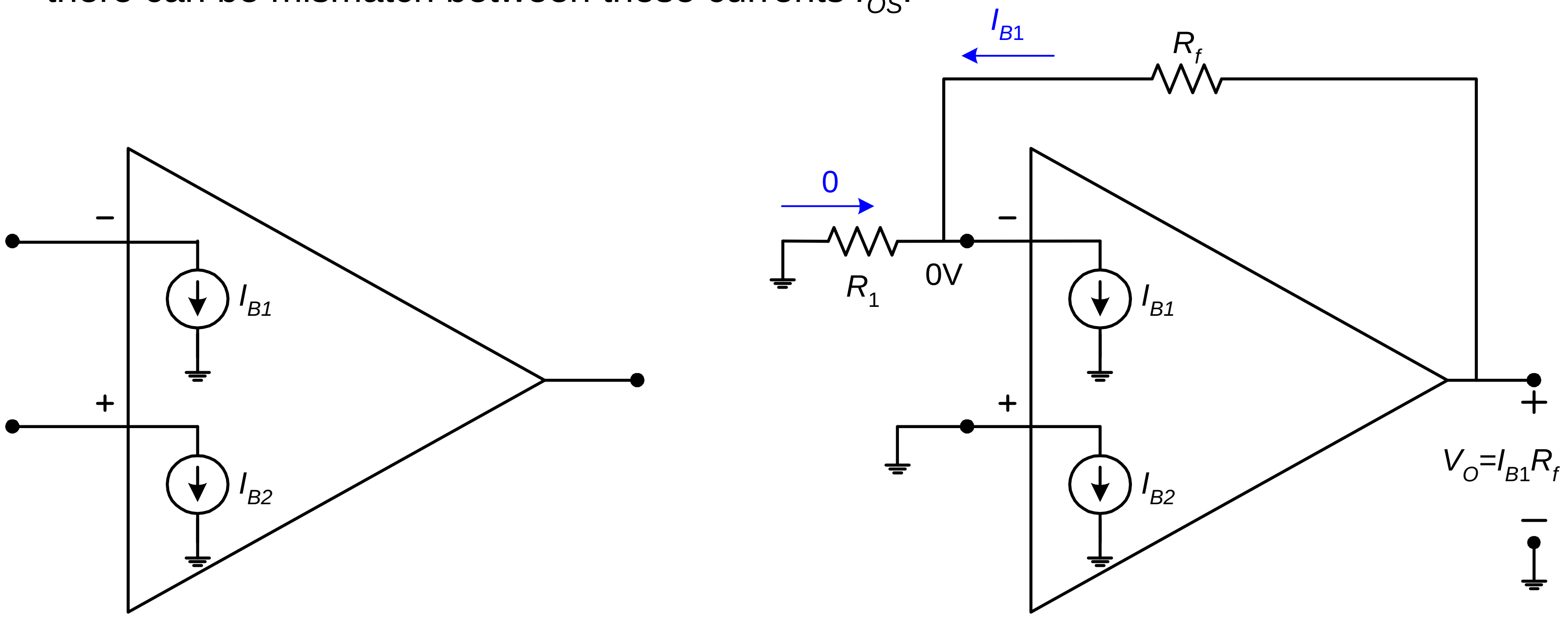


- One can fix the imbalance by adding a DC voltage offset between the two inputs until the output goes to zero. This is an **input offset voltage** (V_{OS}) in the amplifier which can be compensated for with a voltage of equal magnitude and opposite polarity.



Input Bias Circuits

- In real amplifiers, the two input terminals have to be supplied with dc currents called **input bias currents**. They can be represented by two current sources I_{B1} and I_{B2}. Furthermore, there can be mismatch between these currents I_{OS}.



- We can reduce the output voltage effects from the input bias current by adding a resistor into the positive terminal. However, mismatches between I_{B1} and I_{B2} ($I_{OS} = I_{B1} - I_{B2}$) results in an offset voltage $V_{OS}=I_{OS}R_f$.