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Damped oscillations in the ratios of stock market indices

Ming-Chya Wu[(a)]

Research Center for Adaptive Data analysis, National Central University - Chungli 32001, Taiwan,
Institute of Physics, Academia Sinica - Nankang, Taipei 11529, Taiwan and
Department of Physics, National Central University - Chungli 32001, Taiwan

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Abstract – A stock market index is an average of a group of stock prices with weights. Different stock market indices derived from various combinations of stocks may share similar trends in certain periods, while it is not expected that there are fixed relations among them. Here we report our investigations on the daily index data of Dow Jones Industry Average (DJIA), NASDAQ, and S&P500 from 1971/02/05 to 2011/06/30. By analyzing the index ratios using the empirical mode decomposition, we find that the ratios NASDAQ/DJIA and S&500/DJIA, normalized to 1971/02/05, approached and then retained the values of 2 and 1, respectively. The temporal variations of the ratios consist of global trends and oscillatory components including a damped oscillation in 8-year cycle and damping factors of 7183 days (NASDAQ/DJIA) and 138471 days (S&P500/DJIA). Anomalies in the ratios, corresponding to significant increases and decreases of indices, only appear in the time scale less than an 8-year cycle. Detrended fluctuation analysis and multiscale entropy analysis of the components with cycles less than a half-year manifest a behavior of self-adjustment in the ratios, and the behavior in S&500/DJIA is more significant than in NASDAQ/DJIA.

Financial markets are a complex system, in which traders interact with one another and react to external information to determine the best prices for items. The emergence of econophysics as a new branch of statistical physics has advanced our understanding to financial activities in markets using the concepts and theories developed in physics [1–20]. Properties revealed from empirical data analysis of financial systems usually provide primary clues to understanding the underlying mechanisms and are essential for subsequent modelling [6–21]. These include financial stylized facts [2–6,22,23], such as fat tails in asset return distributions, absence of autocorrelations of asset returns, aggregational normality, asymmetry between rises and falls, volatility clustering [10], and phase clustering [18–20]. Successful empirical analysis and modelling of financial criticality have suggested possible physical pictures for financial crashes and stock market instabilities [11–17].

In this paper, we study the relations among the daily stock market indices of Dow Jones Industry Average (DJIA), NASDAQ, and S&P500, from 1971/02/05 to 2011/06/30. The data were downloaded from Yahoo Finance (http://finance.yahoo.com/). The original lengths of the data of the three indices are different. In the preprocessing of the data, the three indices are aligned by removing three data points in DJIA and S&P500 (1973/9/26, 1974/10/7, and 1975/10/16) which do not exist in NASDAQ. Finally there are 10197 data points involved in the study. Figure 1(a) shows the daily index data of the three stock markets. Though the indices of DJIA, NASDAQ, and S&P500 are distinct, there is a remarkable feature that by keeping DJIA as a reference and multiplying the NASDAQ index by a factor of 5.2 and S&P500 by 8.5, the curves of the rescaled indices coincide very well in several epoches, except large deviations in NASDAQ for the periods 1999–2001 and 2009–2011, as shown in fig. 1(b). In the year 2011, DJIA is the average price of 30 companies (http://www.djaverages.com/), NASDAQ consists of 1197 companies (http://www.nasdaq.com/), and the S&P500 index is an average result of 500 companies (http://www.standardandpoors.com). Some companies,

(a) E-mail: mcwu@ncu.edu.tw

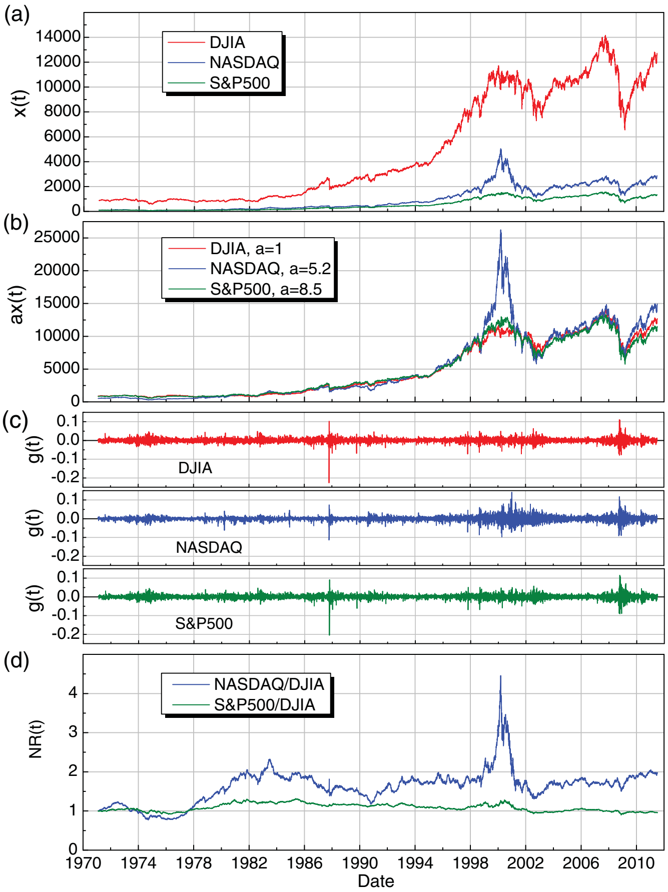

Fig. 1: (Colour on-line) (a) Daily indices of Dow Jones Industry Average (DJIA), NASDAQ, and S&P500. (b) Rescaled indices $ax(t)$. (c) The gains of the three indices. (d) Ratios of paired indices, normalized to the values on 1971/02/05.

such as Intel and Microsoft, are included in all the three markets, but most of their compositions are different. The relations among the indices are not crucially determined by the common companies. The coincidence of the three indices via scaling is thus nontrivial. It is interesting to investigate the properties of the relation among them revealed in some extent from the daily data.

Consider two indices x_i and x_j. The ratio between two indices at time t_n

$$R_{ij}(t_n) = \frac{x_i(t_n)}{x_j(t_n)}, \tag{1}$$

can be alternatively formulated as

$$R_{ij}(t_n) = R_{ij}(t_{n-1})\frac{1+g_i(t_{n-1})}{1+g_j(t_{n-1})}, \tag{2}$$

where

$$g_i(t_n) = \frac{x_i(t_{n+1}) - x_i(t_n)}{x_i(t_n)}, \tag{3}$$

is the gain of the index x_i. The gain time series of the three indices are shown in fig. 1(c). Since the initial time t_{n-1} in eq. (2) can be chosen arbitrarily, we "normalize" the ratio to the initial value of R_{ij} at t_0, *i.e.*,

$$NR_{ij}(t_n) \equiv \frac{R_{ij}(t_n)}{R_{ij}(t_0)} = \prod_{m=1}^{n} \frac{1+g_i(t_{m-1})}{1+g_j(t_{m-1})}. \tag{4}$$

Using 1971/02/05 as the initial time for the three indices, $NR_{ij}(t_n)$ (hereafter abbreviated as $NR(t)$ for simplicity) of NASDAQ/DJIA and S&P500/DJIA are shown in fig. 1(d). The normalized ratio of NASDAQ/DJIA increased before 1982 from 1 to 2, and then saturated with fluctuations. While there was a sharp change in the period from 1998 to 2002 (more precisely a peak in 2000), it returned to 1.5 in 2003 and then grew up to 2 gradually. On the other hand, the normalized ratio S&P500/DJIA varied around 1 with variation magnitudes within ± 0.3. Consequently, a general feature of the normalized ratio is that it approached and then retained the values of 2 and 1 for NASDAQ/DJIA and S&P500/DJIA, respectively. For the cases in which different dates are used to normalize the ratios, there is an overall factor $R_{ij}(t_0)/R_{ij}(t_0')$ to $NR(t)$ in eq. (4) and the behaviors remain unchanged. The scenario is similar to a mechanical system with a "restoring force" acting on it: when the ratio becomes too large or small, it inclines to retain an equilibrium state.

To explore the evolution of the ratios, we analyze the variations of $NR(t)$ in different time scales using the empirical mode decomposition (EMD) [24], developed for nonlinear and nonstationary time series analysis. The EMD method has been developed on the assumption that any time series consists of simple intrinsic modes of oscillations [24]. The adaptive decomposition scheme explicitly utilizes the actual time series for the construction of the decomposition base rather than decomposing it into a prescribed set of base functions. The decomposition is achieved by iterative "sifting" processes for extracting modes by identification of local extremes and subtraction of local means [24]. The iterations are terminated by a criterion of convergence. For details of the algorithms, reference is made to refs. [24,25]. Under the procedures of EMD [24,25], the ratio time series $NR(t)$ is decomposed into n intrinsic mode functions (IMFs) c_k's and a residue r_n,

$$NR(t) = \sum_{k=1}^{n} c_k(t) + r_n(t). \tag{5}$$

The IMFs are symmetric with respect to the local zero mean and have the same numbers of zero crossings and extremes, or a difference of 1. All the IMFs are orthogonal to each other [24]. Thus, the decomposition via the EMD scheme satisfies the requirement of completeness and orthogonality. According to the algorithm of EMD, the first component c_1 has the highest frequency, the secondary component c_2 has a frequency about half of c_1, and so on. Ideally, the frequency content of each component is not overlapped with others such that the characteristic frequencies of all components are distinct.

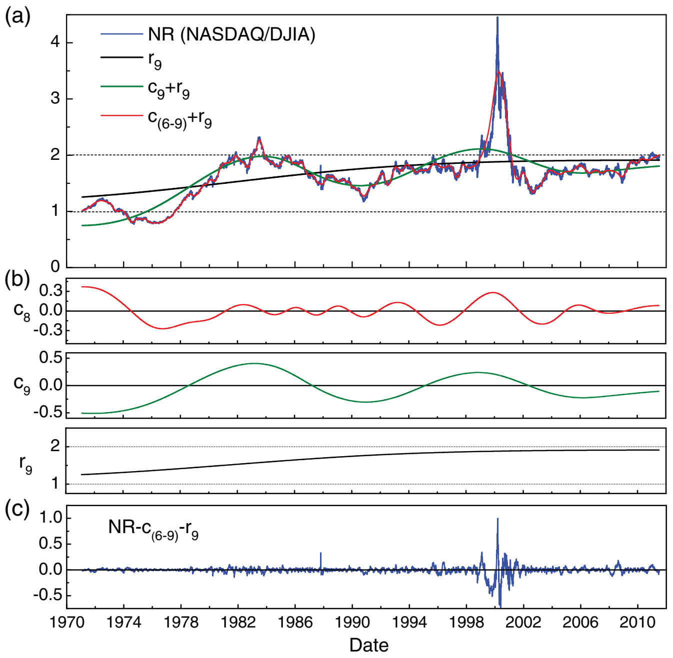

Fig. 2: (Colour on-line) Empirical mode decomposition (EMD) of the ratio NR for NASDAQ/DJIA. (a) Comparisons of NR, IMFs and IMF combinations. (b) IMFs c_8, c_9, and residue r_9. (c) The data of $NR - c_6 - c_7 - c_8 - c_9 - r_9$.

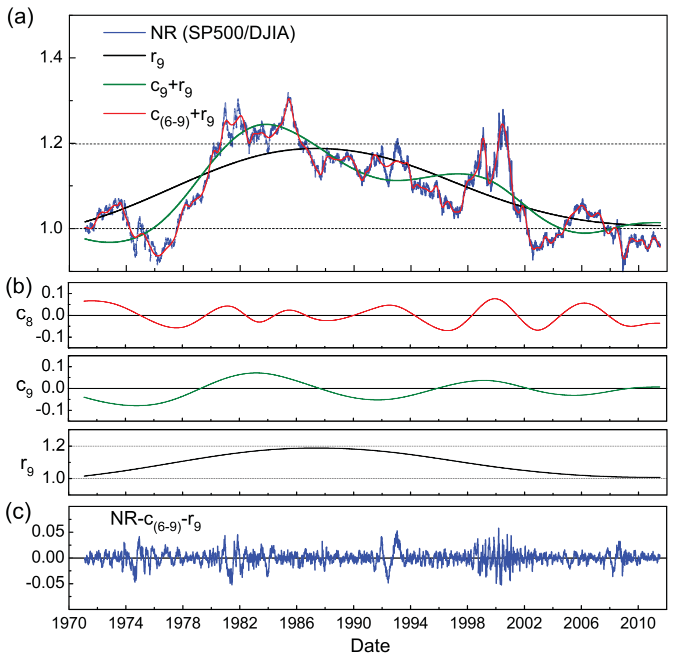

Fig. 3: (Colour on-line) Empirical mode decomposition (EMD) of the ratio NR for S&P500/DJIA. (a) Comparisons of NR, IMFs and IMF combinations. (b) IMFs c_8, c_9, and residue r_9. (c) The data of $NR - c_6 - c_7 - c_8 - c_9 - r_9$.

In time domain, one component can then be characterized by its own range of periods. Here, both the NR of NASDAQ/DJIA and S&P500/DJIA are decomposed into 10 components. Using the property that each component has a distinct period, we sum over different components to assess the behaviors of the ratios in different time scales. Figures 2(a) and 3(a) show the comparisons of NR, residue r_9 and combinations of the residue and IMFs, $c_9 + r_9$ and $c_{(6-9)} + r_9$ (here $c_6 + c_7 + c_8 + c_9$ has been abbreviated as $c_{(6-9)}$ for simplicity). For the decomposition, we are more interested in the residue r_9 and IMFs c_8 and c_9, shown in fig. 2(b) and fig. 3(b). The residue r_9 is the trend of the ratio NASDAQ/DJIA which approaches 2 gradually from 1.2 (fig. 2(b)), while r_9 of S&P500/DJIA grows up from 1 to 1.2 and then decreases back to 1 (fig. 3(b)). The IMF c_9 reveals that the variations of the ratios in the scale of an 8-year cycle behave as a damped oscillation in the form of $\exp[-(t_n - t_0)/\gamma]$ with damping factors $\gamma \approx 7183$ days (NASDAQ/DJIA) and 138471 days (S&P500/DJIA) determined from the local minima of IMF c_9. Thus, the combination of c_9 and r_9 shows the converge of oscillations to values 2 and 1 for NASDAQ/DJIA and S&P500/DJIA, respectively. Meanwhile, the IMF c_8 corresponding to a (2–4)-year cycle is accompanied with frequency modulation in late of 1990s, implying the trigger of the anomaly in amplitude change and its recovery to regular situation lasts 1.5 oscillatory cycles, about 4–6 years. Remarkably, this anomaly does not appear in IMF c_9. It is a local event in time with time scale less than an 8-year cycle.

The components of the ratios in the cycle less than half-year (about 125 days) are derived by taking the difference between NR and $c_{(6-9)} + r_9$. The data of $NR - c_{(6-9)} - r_9$ are shown in fig. 2(c) and fig. 3(c) for NASDAQ/DJIA and S&P500, respectively. We analyze their statistical properties by the detrended fluctuation analysis (DFA) [26–28] and the multiscale entropy (MSE) [29] analysis, and the results are presented in fig. 4. The DFA analysis measures the fluctuation $F(n)$ of δNR (defined as $\delta NR = NR - c_{(6-9)} - r_9$) with respect to a linear fit of the data (δNR_n) in a time window n, and use an index α defined from

$$F(n) = \sqrt{\frac{1}{T}\sum_t [\delta NR(t) - \delta NR_n(t)]^2} \sim n^\alpha \qquad (6)$$

to describe the correlation property of the data [26–28]. The results of $\alpha = 1.4851$ for NASDAQ/DJIA and $\alpha = 1.3859$ for S&P500/DJIA in fig. 4(a) suggest that the property of $NR - c_{(6-9)} - r_9$ is similar to a Brownian motion with more negative correlation (<1.5) in the time scale less than half-year (125 days) in fig. 4(a), indicating the anti-persistent behaviors in the ratios. The relatively stronger anti-persistent behavior in S&P500/DJIA than in NASDAQ/DJIA is considered as a signature of more significant self-adjustment in the ratio of S&P500/DJIA. The change of slope at 125 days is due to the removal of high-order IMFs ($c_{(6-9)}$ and r_9). The slopes in this regime indicate that effective changes of the ratios in S&P500/DJIA ($\alpha = 0.4084$) is smaller than in NASDAQ/DJIA ($\alpha = 0.5643$). Note that the above

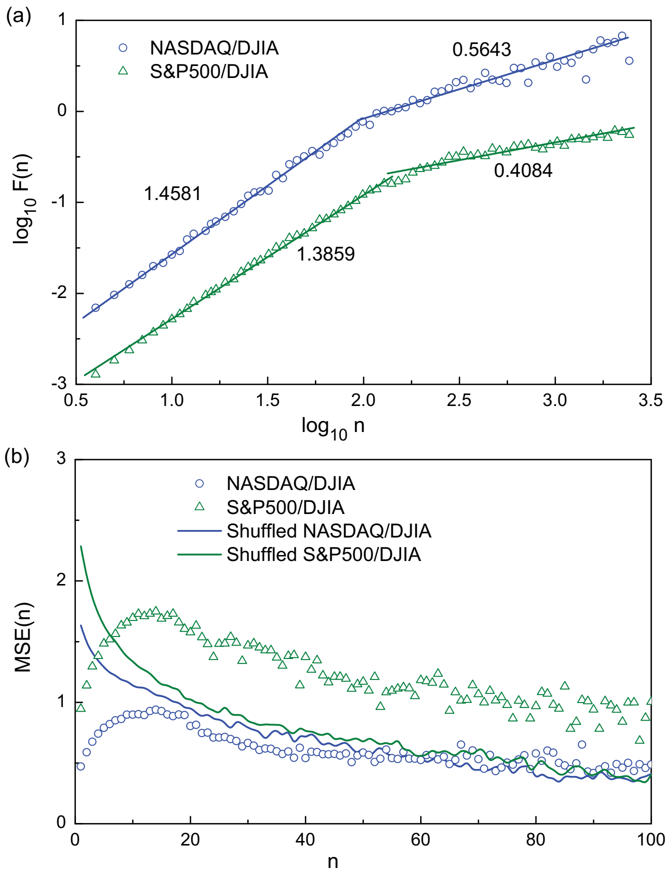

Fig. 4: (Colour on-line) Statistical properties of $NR - c_{(6-9)} - r_9$. (a) Detrended fluctuation analysis (DFA). The numbers indicate the α values of the linear seqments. (b) Multiscale entropy (MSE) analysis. The shuffled data are generated by randomizing $NR - c_{(6-9)} - r_9$ using normal distribution.

analysis is independent of the dates used to normalize the ratios, since the difference between the decompositions of NR and $NR \cdot R_{ij}(t_0)/R_{ij}(t'_0)$ is an overall factor to IMFs, which has no effect on their statistical properties.

Furthermore, the MSE analysis measures the scale dependence of the complexity in the data [29]. The analysis is implemented by calculating the entropies of a set of resampled data in different window sizes, and the relative complexity of the data is evaluated with respect to a reference defined from the corresponding shuffled data or some standard noises. The results in fig. 4(b) show that the information content of $NR - c_{(6-9)} - r_9$ of NASDAQ/DJIA is richer than that of S&P500/DJIA in all time scales. The existence of the detailed structures in the ratio of S&P500/DJIA (fig. 3(c)) is again a signature of more abundant self-adjustments in it than in NASDAQ/DJIA. Remarkably, their entropies reach maxima at about 14 days, implying that the reassessments on ratios are relatively more active in this time scale. The entropy of $NR - c_{(6-9)} - r_9$ for NASDAQ/DJIA is lower than the shuffled data, generated by randomizing the time series of $NR - c_{(6-9)} - r_9$ using normal distribution, in the scale less than 60 days, and that for S&P500/DJIA is less than the shuffled data in the scale less than 7 days. Interestingly, the information content in $NR - c_{(6-9)} - r_9$ for NASDAQ/DJIA is relatively lower than the corresponding shuffled data resembling to a white noise. There is a weaker correlation between NASDAQ and DJIA than between S&P500 and DJIA. As a result, larger deviations of the rescaled indices in fig. 1(b) for DJIA and NASDAQ than DJIA and S&P500 can be observed in the period from 1999 to 2002.

To gain knowledge for the dynamical properties the ratios, we calculate the dynamical cross-correlations for pairs of the stock market indices using logarithmic return defined by

$$lr_i(t_n) = \log \frac{x_i(t_{n+1})}{x_i(t_n)}. \quad (7)$$

The dynamical cross-correlation between returns of two indices is defined as

$$\rho_{ij}(t_n) = \frac{(lr_i - \langle lr_i \rangle)(lr_j - \langle lr_j \rangle)}{\sigma_i \sigma_j}, \quad (8)$$

with $\sigma_i^2 = \langle lr_i^2 - \langle lr_i \rangle^2 \rangle$ the variance of the index, and $\langle \cdots \rangle$ indicates an average over a time window T. Figures 5(a) and (b) show the dynamical cross-correlations between DJIA and NASDAQ with time windows $T = 10$ days and $T = 100$ days. The correlation between two stock indices are more positive for $T = 100$ days than $T = 10$ days. Despite the phase differences in the short time scale, the variations of the indices in the large time scale are generally positive correlated (more in phase). Figure 5(c) shows the window size dependence of the average correlation of the stock indices. The average correlation between S&P500 and DJIA is stronger than NASDAQ and DJIA for all window sizes, consistent with inference from the MSE analysis in fig. 4(b) that the information content in &P500/DJIA in the cycle less than a half-year is richer than NASDAQ/DJIA. Furthermore, DJIA and NASDAQ have the strongest correlation at $T = 60$ days, while the correlation strength between DJIA and S&P500 grows gradually with time and saturates at $T > 1000$ days.

Next, by expanding the product in eq. (4), the normalized ratio $NR_{ij}(t_n)$ can be rewritten as

$$NR_{ij}(t_n) = \frac{1 + \sum_{k=1}^{n} G_i^{(k)}}{1 + \sum_{k=1}^{n} G_j^{(k)}}, \quad (9)$$

with

$$G_i^{(k)} = \frac{1}{k!} \sum_{t_{m_1} \neq t_{m_2} \cdots} \prod_{l=1}^{k} g_i(t_{m_l}). \quad (10)$$

The term $G_i^{(1)}$ in eq. (10) is a sum of all the gains. Figure 6(a) shows the probability density function of the gain g for the three indices. The means of the grains are 0.00032095, 0.00040822, and 0.00031729 for DJIA, NASDAQ, and S&P500, respectively. The value of $G_i^{(1)}$

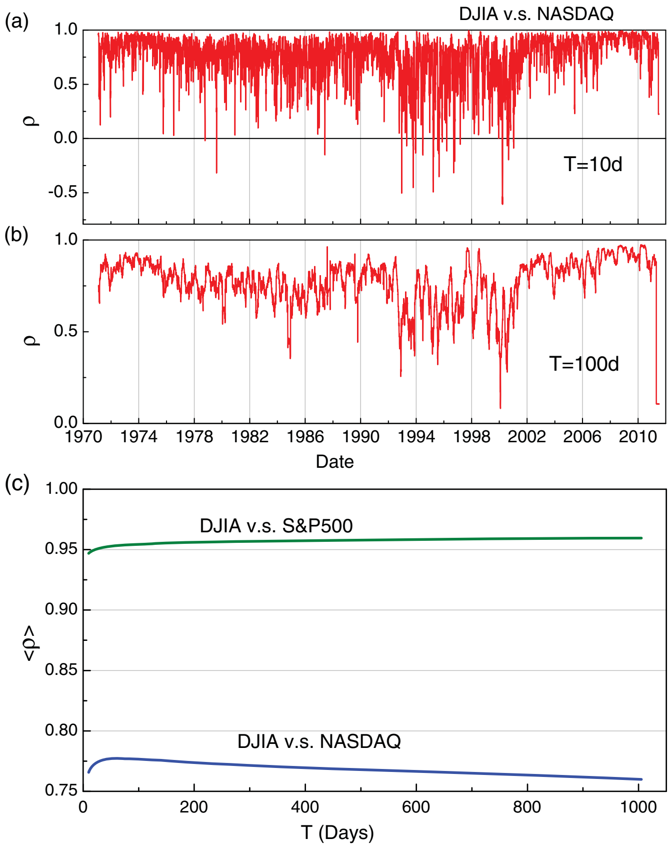

Fig. 5: (Colour on-line) Dynamic cross-correlation stock market indices with window size (a) $T = 10$ days, and (b) $T = 100$ days. (c) Window size dependence of the average correlation. DJIA and NASDAQ has the strongest correlation at $T = 60$ days, and the correlation between DJIA and S&P500 grows gradually with time and saturates at $T > 1000$ days.

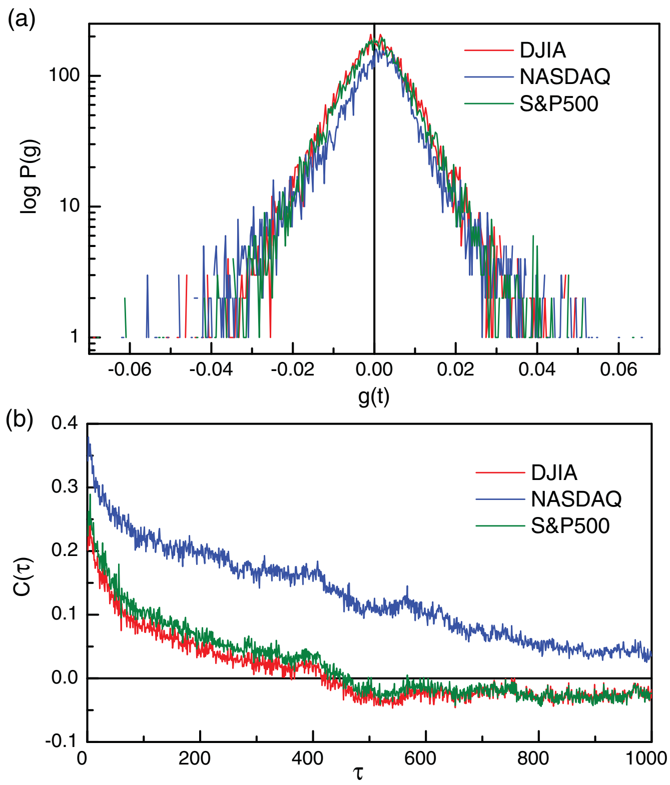

Fig. 6: (Colour on-line) Statistical properties of the stock market indices. (a) Probability distribution function of the gains. (b) Autocorrelation functions of the absolute gains. The correlation length is 194 days for DJIA, 766 days for NASDAQ, and 238 days for S&P500.

is in the order of 1. The term $G_i^{(2)}$ is proportional to the autocorrelation function of the gain, defined as $C(\tau) = \int_{t_0}^{t_N-\tau} \delta g(t)\delta g(t+\tau)\mathrm{d}t/\eta^2$, with variance $\eta^2 = \langle g^2 - \langle g\rangle^2\rangle$, and its value is also in the order of 1. The $G_i^{(k)}$'s with $k \geqslant 3$ are combinations of the sum of gains and autocorrelation functions. Further calculations of $G_i^{(k)}$ show that the values of all $G_i^{(k)}$'s of eq. (10) are in the order of 1. Consequently, all $G_i^{(k)}$'s substantially have equal contributions to the ratios. We then calculate the autocorrelation of the absolute gain and the results are shown in fig. 6(b). Using the exponential decay model to fit the autocorrelation function, the correlation length is determined to be 194 days for DJIA, 766 days for NASDAQ, and 238 days for S&P500, which are less than 4 years. Therefore, the damped oscillation in an 8-year cycle is not a consequence of cross-correlation and autocorrelation of the indices.

In conclusion, we have analyzed the ratios of the daily index data of DJIA, NASDAQ, and S&P500 from 1971/02/05 to 2011/06/30. The results show that though three indices are distinct from one another, using suitable scaling factor, the indices can be made coincidence very well in several epoches, except NASDAQ in the periods 1999–2001 and 2009–2011. This feature indicates the existence of definite ratios among them. Sophisticated time series analysis based on EMD method show that the ratios NASDAQ/DJIA and S&P500/DJIA, normalized to 1971/02/05, approached and then retained 2 and 1, respectively, from 1971 to 2011, through damped oscillatory components in the 8-year cycle and damping factors of about 29 years (7183 days for NASDAQ/DJIA) and 554 years (138471 days for S&P500/DJIA). The damped oscillation of the 8-year cycle is not associated with the characteristic time scales in the autocorrelation and cross-correlation of the indices. Furthermore, the peak of NASDAQ/DJIA in the period from 1998 to 2002, which is considered as an anomaly in the ratio, does not appear in the 8-year cycle. It is a local event in the time scale less then 8-year. Thus, the behavior of damped oscillation of the ratios converging to fixed values is independent of such anomalies. For the components with cycles less than half-year, behaviors of self-adjustments are observed in the ratios, and there is a relatively active re-assessment on the ratio in the time scale of 14 days revealed from the MSE analysis. The behavior of self-adjustment in the ratio for S&P500/DJIA is more significant than in

NASDAQ/DJIA. As the damped components set reasonable bounds to the variations of the indices, our findings may be informative for risk evaluation of the markets.

Finally, we would like to propose a further study of stock market modelling. In standard finance theory, it is assumed that arbitrage opportunities disappear quickly as arbitrage is performed by traders [30]. The strategies derived from algorithmic trading engines, which quote inter-product ratios and inter-market ratios and apply statistical arbitrage approaches to anticipate deviations, are on shorter time scales [4]. It is interesting to investigate whether such processes can lead to the damped oscillatory behaviors in longer time scales observed in this study.

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