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Phase Statistics Approach to Time Series Analysis

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In this paper, an approach we introduced recently to study physiological and financial time series [Phys. Rev. E **73**, 051917 (2006); Phys. Rev. E **73**, 016118 (2006)] is reviewed. The approach mainly consists of an application of the Hilbert-Huang method to decompose an empirical time series into a number of intrinsic mode functions (IMFs), calculation of the instantaneous phase of the resultant IMFs, and the statistics of the instantaneous phase for each IMF. To illustrate the approach, we consider cardiorespiratory synchronization and the phase distribution and phase correlation of financial time series as examples. The formulation of the approach is systematic and can be applied to the analysis of other time series.

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I. INTRODUCTION

Dynamical systems can usually be recorded by a successive recording processes, and the characteristic behaviors can be caught in corresponding time series. Some of these time series are distinguishable by the characteristic structures of their wave forms. For example, the bursting features in seismic time series can be easily distinguished from the essentially regular waveforms of respiratory time series. For the investigations of empirical time series, some studies focus on phenomenological interpretations. The data processing method plays a crucial role in the obtained results. An essential task for studies is to process such signals and pickup essential component(s) from experimental data. Even though proper filters can be used to filter out noises from real data, the capabilities and the effectiveness of the filtration are usually questionable. This is due to the fact that most of these approaches require the original time series to be stationary and/or linear. However, empirical signals are usually noisy, nonlinear, and non-stationary. Furthermore, there is also no strict criteria to judge what is the inherent dynamics and what is the contribution of the external factors and noise in experimental data. Improper approaches might then lead to misleading results.

In this paper, we will review the phase statistics approach introduced recently to study physiological [1] and financial time series [2]. The approach mainly consists of application of the Hilbert-Huang method [3] to decompose an empirical time series into a number of intrinsic mode functions (IMFs), the calculation of instantaneous phase of the resultant IMFs, and the statistics of the instantaneous phase for each IMF. The Hilbert-Huang method introduced by Huang *et al.* [3] consists of the empirical mode decomposition (EMD) and the Hilbert spectral analysis. It was designated for the analysis of nonlinear and non-stationary time series. Unlike conventional filters, the EMD method provides an effective way to extract physical rhythms from experimental data. This advantage can, thus, provide a more reliable interpretation from empirical time series. To illustrate the approach, here we consider cardiorespiratory synchronization and phase distribution and the phase correlation of financial time series as examples.

Cardiorespiratory synchronization is a phenomena originating from the coupling between the cardiac and the respiratory subsystems. The nature of the couplings has been extensively studied from measured data in recent years [4–13]. Recently, Schäfer *et al.* [14, 15] and Rosenblum *et al.* [16] found that there were sufficiently long periods of hidden synchronization and concluded that the cardiorespiratory synchronization and respiratory sinus arrhythmia (RSA) are two competing factors in cardiorespiratory interactions. Up to now, cardiorespiratory synchronization has been reported in young healthy athletes [14, 15], healthy adults [17–19], heart transplant patients [17,20], infants [21], and anesthetized rats [22]. Most of the studies support the existence of cardiorespiratory synchronization. The essential part of the investigation is the extraction of respiratory rhythms from noisy respiratory signals. A technical problem in the analysis of the respiratory signal then arises: insufficiently filtered signals may still have too many noises, and over-filtered signal may be too regular to lose the characteristics of respiratory rhythms. To overcome these difficulties, Wu and Hu [1] propose use-

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ing the Hilbert-Huang method for such studies and got significantly reasonable results. In the first part of this paper, we will review briefly the work of Wu and Hu [1].

As a second example, we will consider the application of the EMD to the study of financial time series. Traditional analysis of financial systems is usually based on fundamental statistics of return time series and tends to address issues on drawing trading strategies for traders and investors. With the power of new algorithms for statistical analysis, some previous studies have provided rich information for such purposes [23]. However, previous studies have also suffered from the limited scope provided by the statistics of conventional derivatives from returns. As a result, cross-disciplinary studies on financial systems have attracted much attention in recent decades [24–28]. When considering the issue as a generic time series analysis problem, there have been developments in methodology [24,25,29], such as the method of random matrix [24–26] and the wavelet transform modulus maxima approach [29–34]. The wavelet analysis has difficulty with its non-adaptive nature, so once the basic wavelet is selected, it is used to analyze all the data. However, some wavelets are Fourier based, so they suffer the shortcoming of Fourier spectral analysis for only giving a physical meaningful interpretation to linear phenomena [3]. Nevertheless, financial time series are nonlinear, so analyses by using these approaches may lose information on the nonlinear properties. In light of the above situation, Wu *et al.* developed a new approach to study financial time series [2,35]. In their approach, the EMD was used to define and evaluate the instantaneous phase of the time series. Based on this method, Wu *et al.* studied the returns of the Dow Jones Industrial Average 30 (DJIA) and the NASDAQ stock indices to extract the characteristic structures of the empirical data. The main issues they addressed were the phase distribution of financial time series and the phase correlation between two stocks [2]. In the second part of this paper, we will then briefly review the approach and discuss the new results they obtained based on the approach.

The rest part of this part is organized as follows: In Sec. II, we briefly introduce the Hilbert-Huang method. In Sec. III, we apply the Hilbert-Huang method to the study of cardiorespiratory synchronization. The application of the same method to financial time series is presented in Sec. IV. Finally, we summarize our results in Sec. V.

II. HILBERT-HUANG METHOD

As we mentioned above, the Hilbert-Huang method consists of the EMD and the Hilbert spectral analysis. The EMD method is developed from the assumption that any time series consists of simple intrinsic modes of oscillation, and the essence of the method is to identify the intrinsic oscillatory modes by their characteristic time scales in the data empirically and then decompose the data accordingly [3]. This is achieved by sifting data to generate IMFs. The IMFs introduced by the EMD are a set of well-behaved intrinsic modes, and these functions satisfy the conditions that they are symmetric with respect to the local zero mean and have the same numbers of zero crossings and extremes. Therefore, the Hilbert transform can be directly used to calculate the instantaneous phase after the decomposition processes. The algorithm to create IMFs in the EMD has two main steps [1,3]:

1. *Step-1*: First, the local extremes in the experimental respiratory time series data $\{x(t)\}$ are identified. Then, all the local maxima are connected by a cubic spline line $U(t)$ while the same procedure is applied for the local minima to produce the lower envelope $L(t)$. Both envelopes will cover all of the original time series. The mean of upper envelope and the lower envelope, $m_1(t)$, given by

$$m_1(t) = \frac{U(t) + L(t)}{2} \tag{1}$$

is a running mean. Subtracting the running mean $m_1(t)$ from the original time series $x(t)$, we get the first component $h_1(t)$,

$$x(t) - m_1(t) = h_1(t). \tag{2}$$

The resulting component $h_1(t)$ is an IMF if it satisfies the following conditions: (i) $h_1(t)$ is free of riding waves. (ii) It displays symmetry of the upper and the lower envelopes with respect to zero. (iii) The numbers of zero crossings and extremes are the same, or only differ by 1. Beside these, an additional condition based on the intermittence [1] can be imposed here to sift out waveforms with a certain range of intermittence for the purpose of physical consideration. If $h_1(t)$ is not an IMF, the sifting process has to be repeated as many times as is required to reduce the extracted signal to an IMF.

In the subsequent sifting process steps, $h_1(t)$ is treated as the datum to repeat steps mentioned above,

$$h_1(t) - m_{11}(t) = h_{11}(t). \tag{3}$$

Again, if the function $h_{11}(t)$ does not yet satisfy criteria (i)-(iii), the sifting process continues up to k times until some acceptable tolerance is reached:

$$h_{1(k-1)}(t) - m_{1k}(t) = h_{1k}(t). \tag{4}$$

2. *Step-2*: If the resulting time series is the first IMF, it is designated as $c_1 = h_{1k}(t)$. The first IMF is

then subtracted from the original data, and the difference r_1 given by

$$x(t) - c_1(t) = r_1(t), \tag{5}$$

is the first residue. The residue $r_1(t)$ is taken as if it were the original data and is applied again the sifting process of *Step-1*.

Following the above procedures of *Step-1* and *Step-2*, we continue the process to find more intrinsic modes c_i until the last one. The final residue will be a constant or a monotonic function that represents the general trend of the time series. Finally, we get

$$x(t) = \sum_{i=1}^{n} c_i(t) + r_n(t), \tag{6}$$

$$r_{i-1}(t) - c_i(t) = r_i(t). \tag{7}$$

Note that among these IMFs, the first IMF has the highest oscillatory frequency, and in our practical EMD process, there is a general relation of intermittence for different modes:

$$\tau_n = 2^{n-1} \cdot \tau_1, \tag{8}$$

where τ_n denotes the intermittence of the nth mode. In other words, if c_1 has an intermittence ranging from τ_1 to $2 \cdot \tau_1$, then the c_n mode has an intermittence ranging from $2^{n-1} \cdot \tau_1$ to $2^n \cdot \tau_1$.

The instantaneous phase of the resultant IMFs can then be calculated by using the Hilbert transform. For the kth mode, this can be done by first calculating the conjugate pair of $c_k(t)$, *i.e.*,

$$y_k(t) = \frac{1}{\pi} P \int_{-\infty}^{\infty} \frac{c_k(t')}{t - t'} dt', \tag{9}$$

where P indicates the Cauchy principal value. With this definition, the two functions $c_k(t)$ and $y_k(t)$ forming a complex conjugate pair define an analytic signal. Accordingly, we define

$$c_k(t) + i y_k(t) = A_k(t) e^{i\phi_k(t)}, \tag{10}$$

with the amplitude $A_k(t)$ and the phase $\phi_k(t)$ defined by

$$A_k(t) = \left[c_k^2(t) + y_k^2(t)\right]^{1/2}, \tag{11}$$

$$\phi_k(t) = \tan^{-1}\left(\frac{y_k(t)}{c_k(t)}\right). \tag{12}$$

Then, we can calculate the instantaneous phase according to Eqs. (9) and (12).

III. CARDIORESPIRATORY SYNCHRONIZATION

In this section, we present an application of the Hilbert-Huang method to the study of cardiorespiratory synchronization [1]. Cardiorespiratory synchronization is considered as a process of adjustment of rhythms due to physiological interactions between the cardiac and the respiratory subsystems. These interactions can lead to a perfect locking of their phases whereas their amplitudes remain chaotic and non-correlated [36].

For the study of cardiorespiratory synchronization, the EMD is useful for the extraction of respiratory rhythms from empirical data which usually has noise. This is achieved by taking into account the physiological situation as the sifting process in the EMD is performed. More specifically, the number of respiratory cycles per minute for human beings is such a condition. The number of respirations of human beings has a rather wide range; it is about 18 cycles per one minute for adults, and about 26 cycles per one minute for children. For different healthy states, the number of cycles may also vary case by case. To include most of these possibilities, one may take respiratory cycles ranging from 10 to 30 times per minute; each respiratory cycle then takes roughly $2 - 6$ seconds.

The empirical data consisting of 20 data sets were collected by the Harvard medical school in 1994 [37]. Each data set included electrocardiographic (ECG) data and respiration signals. The continuous ECG and respiration data were digitized at 250 Hz (respiratory signals were latter preprocessed to be at 5 Hz). Each heartbeat was annotated using an automated arrhythmia detection algorithm, and each beat annotation was verified by visual inspection. Among these, records f1y01, f1y02, $\cdots$ and f1y10 were obtained from ayoung cohort, and records f1o01, f1o02, $\cdots$ and f1o10 were obtained from an elderly cohort. Each group of subjects included equal numbers of men and women.

The procedures for the analysis are as follows [1]: (i) Apply the EMD to decompose the recorded data into several IMFs. We use the time scale of the respiratory cycle as the criteria in the sifting process. Since the respiratory signal was preprocessed to a sampling rate of 5 Hz, there are $(10 - 30)$ data points in one cycle. Then, for example, we can use $c_1 : (3 - 6)$, $c_2 : (6 - 12)$, $c_3 : (12 - 24)$, $\cdots$, *etc.* After the sifting processes of the EMD, the original respiratory data are decomposed into n empirical modes c_1, c_2, $\cdots$, c_n, and a residue r_n. (ii) Visually inspect the resulting IMFs decomposed by using the EMD. If the amplitude of a certain mode is dominant and the wave-form is well distributed, then the data are said to be well decomposed, and the decomposition is successfully completed. Otherwise, the decomposition may be inappropriate, and we have to repeat step (i) with different parameters. Fig. 1 shows a typical EMD for a cardiorespiratory time series.

The physical meanings of IMFs can be understood from their intermittencies. Here, we should note that the variability of respiratory signals are substantially preserved in a certain IMF by using the property of an adap-

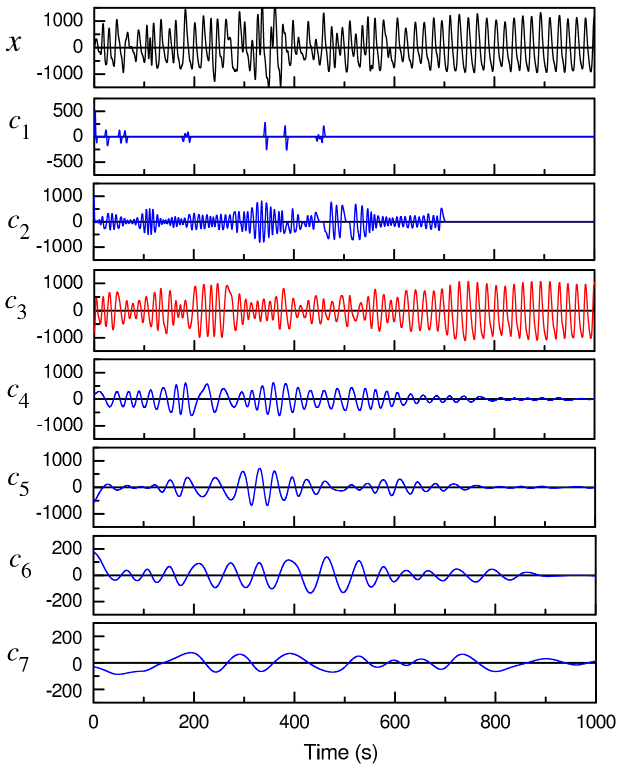

Fig. 1. Example of EMD for a typical respiratory time series data (code f1o01). The criteria for intermittence in the sifting process is $(3-6)$ data points per cycle for c_1. Signal $x(t)$ is decomposed into 14 components including 13 IMFs and 1 residue; here, only the first 7 components are shown. After Ref. [1].

tive basis instead of the a-priori basis used in other methods, such as Fourier-based analysis and Wavelet methods [3]. In other words, the frequency in the EMD method is not global, but is local in time (*i.e.*, time-dependent). Here, we should emphasize that in the study, only one IMF should be taken for the respiratory rhythm and that any sum of a few of IMFs can not be used [1]. Therefore, one should properly choose the intermittence such that the respiratory time signal can be correctly gathered into a single IMF.

Next, we construct the cardiorespiratory synchrogram (CRS), which is a visual tool for inspecting synchronization. Let us denote the phase of the respiratory signal calculated by using Eq. (12) as ϕ_r and the heartbeat as ϕ_c. If the two phases couple in a fashion that a cardiovascular system completes m heartbeats in n respiratory cycles, then a roughly fixed relation can be proposed. In general, there will be a phase-locking condition [14,15,36]

$$|n\phi_r - m\phi_c| = \text{const.}, \tag{13}$$

with m and n being integer, or a weaker type of synchronization named frequency locking [15,36],

$$|n\phi_r - m\phi_c| < \text{const.} \tag{14}$$

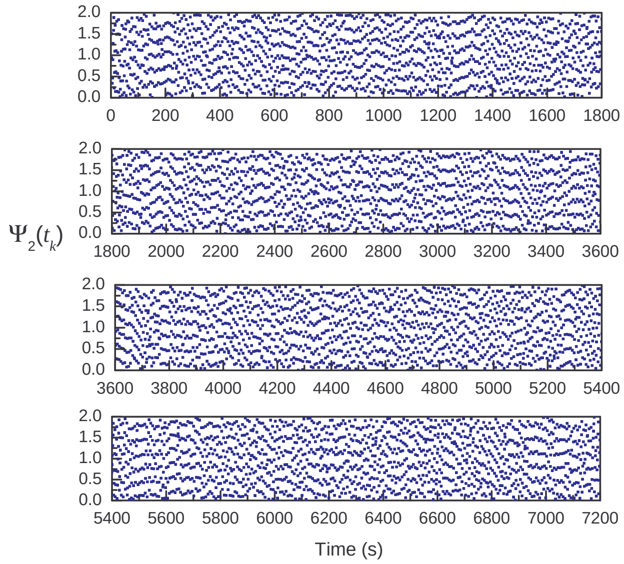

Fig. 2. Cardiorespiratory synchrogram for a typical subject (code f1o06). Empirical data are preprocessed by using the EMD method. There is about an 800 sec synchronization at $2800-3600$ sec, as well as several spells of $50-300$ sec at other time intervals. After Ref. [1].

More precisely, frequency locking should be regarded as modulation, not synchronization [15].

According to Eq. (13), when the ECG completes m cycles while the respiration completes n cycles, synchronization of m cardiac cycles with n respiratory cycles occurs. If the heartbeat event time t_k is the time frame in which the length of the time intervals are not fixed, but vary with the time, then Eq. (13) implies the relation

$$\phi_r(t_{k+m}) - \phi_r(t_k) = 2\pi n. \tag{15}$$

Furthermore, by defining

$$\Psi_m(t_k) = \frac{1}{2\pi}\left[\phi_r(t_k) \bmod 2\pi n\right]. \tag{16}$$

and plotting $\Psi_m(t_k)$ versus t_k, synchronization will result in n horizontal lines in case of $n:m$ synchronization. By choosing n adequately, a CRS can be developed for detecting synchronization between the heartbeat and respiration [14,15].

The example of $3:1$ synchronization with $n=6$ and $m=2$ is shown in Fig. 2, where phase locking appears in several epochs, *e.g.*, at $2800 \sim 3600$ sec. There is also frequency locking, *e.g.*, at 400 sec, near which there are n parallel lines with the same positive slopes. Fig. 3 shows a histogram of the phases for the phase-locking period from 2800 to 3600 sec in Fig. 2. Significantly higher distribution can be found at $\Psi_2 \approx 0.25, 0.6, 0.9, 1.25, 1.6$ and 1.9 in the unit of 2π, indicating that heartbeat events occur roughly at these respiratory phase during this period.

The same procedures and analysis can be applied to all subjects. Due to the limited space, we will not discuss

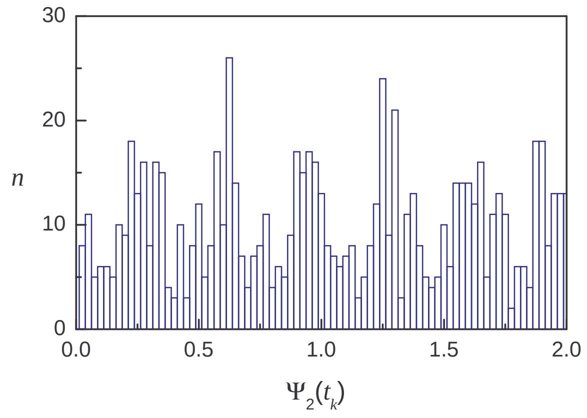

Fig. 3. Histogram of the phase for the phase locking period from 2800 sec to 3600 sec for a typical subject (code f1o06) shown in Fig. 2. After Ref. [1].

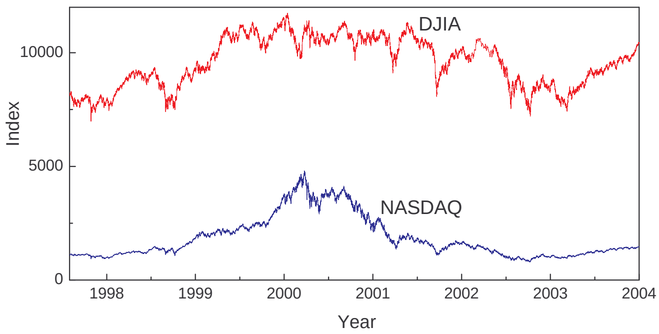

Fig. 4. DJIA and NASDAQ index data from 1 August 1997 to 31 December 2003.

this further, but refer the reader to Ref. [1] for details and other demonstrations.

IV. PHASE STATISTICS OF FINANCIAL TIME SERIES

In this section, we present the application of the Hilbert-Huang method to the study of financial time series [2,35]. The empirical analyses are based on the DJIA and the NASDAQ from the Trade and Quotation (TAQ) database and the Yahoo database [38]. The intraday 10-minute scale values for both the DJIA and the NASDAQ spanning from August 1, 1997, through December 31, 2003, cover the whole six-and-half hours trading from 9 : 30 to 15 : 50 EST. Fig. 4 shows the DJIA and the NASDAQ index data from 1 August 1997 to 31 December 2003.

In the study of financial time series, one usually uses the logarithmic return. For the index values denoted by a time series $Y(t)$, the time series of logarithmic returns of an asset priced at $Y(t)$ over a time scale τ is defined as

$$R_\tau(t) = \ln Y(t) - \ln Y(t-\tau), \tag{17}$$

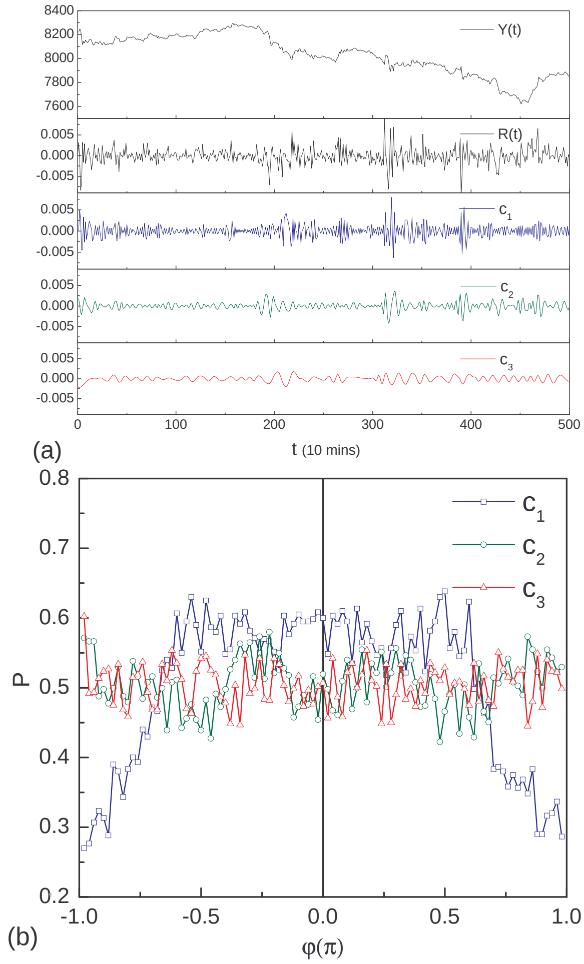

Fig. 5. (a) Intraday DJIA index and the corresponding return sampled by 10 minutes and the first 3 IMFs; (b) probability distribution of phases. After Ref. [2].

where τ is a multiple of the primary time sampling unit Δt. Since the time scale τ (in unit of Δt) is a parameter used to sample the time series of returns, we can take different τ for $R_\tau(t)$ to explore behaviors of the returns with intraday and interday frequencies. Furthermore, we define the probability distribution (or more precisely, probability density function) P as the normalized distribution of a measure ρ, which satisfies

$$\int_{-\infty}^{\infty} P(\rho)\, d\rho = 1, \tag{18}$$

where the measure ρ can be R_τ, the phase or phase difference defined in the later discussions.

The application of the phase statistics approach to investigate the return time series of the DJIA and the NASDAQ is based on the concept that the instantaneous phase can catch the characteristic features of a financial time series [2, 35]. The idea originated from the fact that the phases of a time series usually contain rich information about the structures of the time series. To faithfully extract information, the approach further

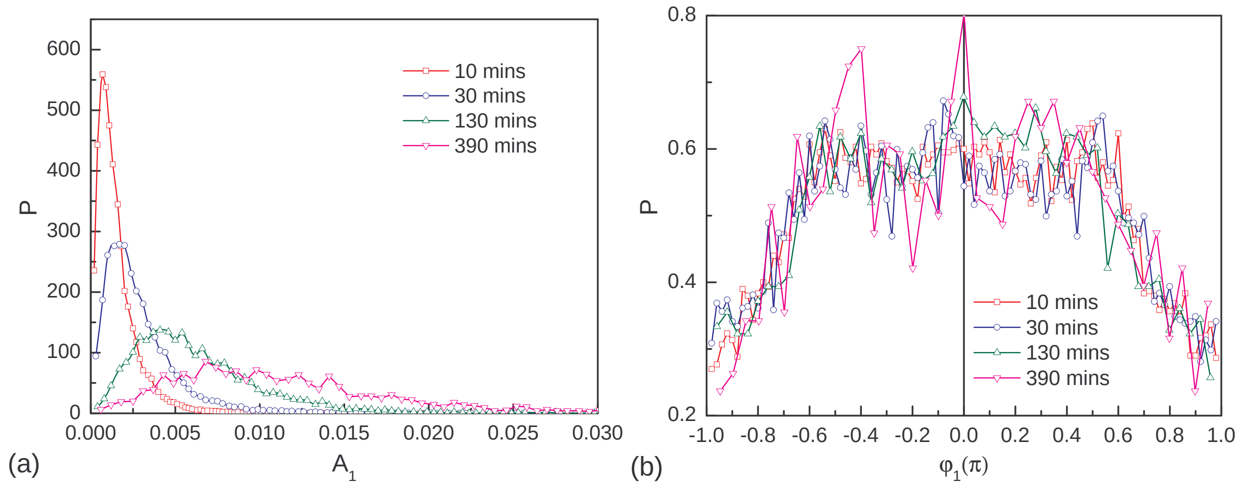

Fig. 6. Probability distributions of (a) amplitudes and (b) phases for the first-IMFs of the returns of the DJIA index sampled by 10, 30, 130, and 390 minutes. After Ref. [2].

applies the Hilbert-Huang method [3] to define and calculate the instantaneous phase. Since a characteristic intermittency in the trading time of a stock market is indefinite, one cannot impose definite intermittencies in the EMD. Hence, the structures of the time series with primary time sampling intervals are closely preserved in the first IMF.

We first take the intraday returns $R_\tau(t)$ with a time sampling interval of 10 minutes as the primary time series and then perform the EMD to decompose $R_\tau(t)$ into 14 IMFs. The results are shown in Fig. 5(a). It is obvious that c_1 catches the main structures of $R_\tau(t)$ because the time series of $R_\tau(t)$ is mainly characterized by its highest frequency component. Similarly, we can perform the EMD on the time series with time sampling intervals of 30, 130, and 390 minutes. We find that except for the first-IMFs of these time series, the phases of the other IMFs are randomly distributed and have equal probabilities for all possible phases, *i.e.*, $-\pi \leq \phi \leq \pi$, as shown in Fig. 5(b). Figs. 6(a) and 6(b) show the amplitude and the phase distributions of the first-IMFs of these time series, respectively.

The probability density functions of the amplitudes for the first-IMFs are general Boltzmann distributions. Among these, the phase distribution is quite interesting. Most phases of the IMFs locate at $-0.5\pi \leq \phi \leq 0.5\pi$. The clustered distribution of phase originates from abrupt changes in the behaviors of the index time series, which is a nature of a time series with intermittency close to the sample time scale τ. We find these behaviors exist for all sample time scales (time sampling intervals of multiples of 10 minutes) of the intraday data. The same analysis can be applied to the NASDAQ index time series. It is remarkable that the distributions of phases are the same in spite of the compositions of their stocks. This implies that it is a characteristic behavior of such kinds of time series. For comparison, we

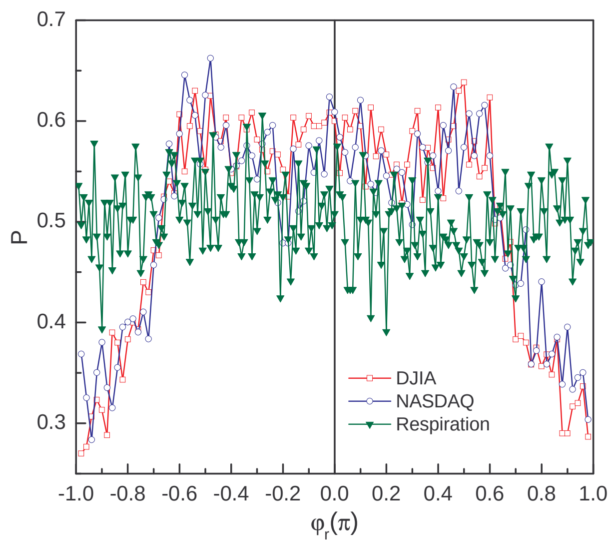

Fig. 7. Probability distributions of phases for the first-IMFs of the returns for the DJIA and the NASDAQ indices sampled by 10 minutes, and the third IMF of a typical respiratory time series (f1o06). After Ref. [2].

calculate the phase distribution of a typical respiratory time series (code: f1o06) and compare the probability distributions of phases for returns of the DJIA and the NASDAQ indices and for the respiratory time series in Fig. 7. From the results, that the return time series and the respiratory time series belong to different classes is quite apparent.

In Fig. 4, the DJIA and the NASDAQ indices show explicit correlations in several epoches. For example, two indices decline in March of 2001 and abruptly decline in September of 2001 due to 9/11 attack. These big changes are in-phase. There are also out-of-phase changes, such as those in the period from February to March of 2000. In other periods, there are also similar behaviors on shorter

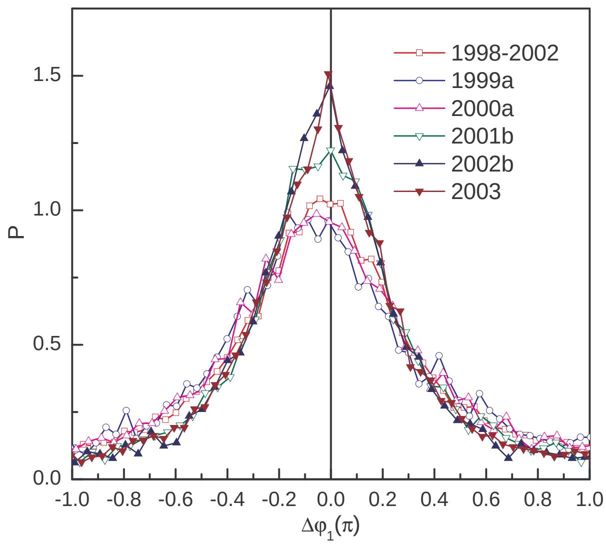

Fig. 8. Probability distributions of phase differences between the first-IMFs of returns of the DJIA and the NASDAQ indices for 1998-2002 and for certain periods and events. After Ref. [2].

time scales. Therefore, in spite of the substantial differences between two markets in the definitions of the indices and in compositions, there are correlative behaviors between the two markets. More specifically, the DJIA is a price-equally-weighted index while the NASDAQ is a value-weighted index. Further, the stocks in the DJIA are more stable and mature than those in the NASDAQ, and the stocks in the NASDAQ are usually more active than those in the DJIA. These differences generally manifest themselves in the transaction volume and in reactions to events. Since the index simply represents an average of the trading activities in a market, an investigation of the correlation between two indices is not a comparison of the former which is more suitable by the scaling analysis (see Ref. [1] for detail), but is associated with the latter. We, thus, have to exclude the effect from transaction volume. This can be achieved by using the Hilbert transform. The phase variation of a time series calculated by the Hilbert transform is equivalent to the variations of the time series under amplitude normalization.

To investigate the correlative behaviors between trading activities in the DJIA and the NASDAQ, we calculate the instantaneous phases of several epoches of the return time series of the DJIA and the NASDAQ indices. To be statistical meaningful, each epoch will have more than $3,000$ sampling points. Here, we define the phase differences of the first-IMFs for different indices. Taking the DJIA as a reference, we define the phase difference $\Delta\phi_1$ as

$$\Delta\phi_1 = \phi_1(\text{NASDAQ}) - \phi_1(\text{DJIA}) \tag{19}$$

and then calculate the probability distributions for certain epoches and events. Fig. 8 shows the probability distributions of phase differences between the first-IMFs of returns of the two indices for 1998 – 2002, the first half year of 1999 (indicated by 1999a), the first half year of 2000 (indicated by 2000a), the last half years of 2001 and 2002 (indicated by 2001b and 2002b, respectively), and the whole year of 2003. We find that there is a remarkable change in the behavior of the trading activities both in the DJIA and the NASDAQ since 9/11 attack. More specifically, Fig. 8 shows that there were more correlative activities after 9/11 so that the distribution functions of 2001b and 2002b were quite different from those before 9/11. Note that there was a similar spectrum in the year 2003, which implies the scenario persisted in later trading activities.

V. SUMMARY

We have briefly explored the scheme of the phase statistics approach by introducing the Hilbert-Huang method and its application to the study cardiorespiratory synchronization and the analysis of financial time series. The remarkable advantage of the EMD method is that it can catch primary structures of intrinsic rhythms from empirical data based on its adaptive feature [39]. This property is especially suitable for performing phase statistics on empirical time series. By imposing intermittency criteria based on physiological conditions revealed by empirical time series, this feature also allows us to effectively keep the signal structures and avoid the introduction of the artificial signals that easily appear in the Fourier-based filters with a-priori bases that cannot process properly variative intermittencies in a nonlinear time series. Furthermore, the introduction of IMFs in the EMD provides a reasonable definition of the instantaneous phase. This advantage is considered to be helpful for drawing reliable conclusions from studies of empirical data. From our results [1], we also found the existence of cardiorespiratory synchronization with several locking ratios occurring in several subjects as in Refs. [14,15].

Furthermore, we have also reviewed investigations of phase distribution and the phase correlation of the DJIA and the NASDAQ indices based on the paper of Ref. [2]. The EMD method was used to decompose the return time series into several IMFs, and the Hilbert transform was used to calculate the instantaneous phase of the first three IMFs. We find that except for the first-IMFs of these time series, which have phases mainly distributed within the range of $-0.5\pi \leq \phi \leq 0.5\pi$, the phases of the other IMFs are randomly distributed and have equal probabilities for all possible phases. This behavior exists in all sample time scales (time sampling intervals of multiples of 10 minutes) of the intraday data. The phase distributions corresponding to abruptly change behaviors indicate non-predictable and stochastic features of the index. Furthermore, our results show that explicitly the phase spectra of the return time series fall into a class different from other signals, such as a time series of human respiration.

The investigations on the correlations between the DJIA and the NASDAQ indices by using the phase difference for various epochs show a remarkable picture on trading activities. The phase distribution between two indices became closer after the event of 9/11. This implies an explicit change in the behavior of trading activities of the DJIA and the NASDAQ after September 2001 [2]. A similar spectrum in the last half of 2002 and the whole year of 2003 (Fig. 8) further implies the scenario persisted in later trading activities. This was seen as faster information transmission and stronger event dependence in the stock markets after 9/11 [2]. According to the impressive achievements of the application of the phase statistics approach to time series analysis with the aid of the Hilbert-Huang method, we expect the approach presented herein to also be useful for statistical analysis of other time series, such as time series of temperature variation, seismic time series, and other social models.

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