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Phase correlation of foreign exchange time series

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Abstract

Correlation of foreign exchange rates in currency markets is investigated based on the empirical data of USD/DEM and USD/JPY exchange rates for a period from February 1 1986 to December 31 1996. The return of exchange time series is first decomposed into a number of intrinsic mode functions (IMFs) by the empirical mode decomposition method. The instantaneous phases of the resultant IMFs calculated by the Hilbert transform are then used to characterize the behaviors of pricing transmissions, and the correlation is probed by measuring the phase differences between two IMFs in the same order. From the distribution of phase differences, our results show explicitly that the correlations are stronger in daily time scale than in longer time scales. The demonstration for the correlations in periods of 1986–1989 and 1990–1993 indicates two exchange rates in the former period were more correlated than in the latter period. The result is consistent with the observations from the cross-correlation calculation.

Keywords: Foreign exchange rate; Time series; Phase correlation

1. Introduction

Financial markets are complex systems consisting of a large number of traders, institutions, and regulatory agents interact one another on the basis of market information to determine asset prices. Traditional studies of financial systems relies heavily on economic fundamentals such as dividend yield, long-short interest rate spreads, risk, book value, etc, and tend to address issues on drawing trading strategies for traders and investors. With the increase of knowledge on financial systems and developments of new algorithms for statistical analysis, some previous studies have provided rich information for such purposes [1].

However, previous studies have also suffered by a limit of scope from the statistics of return and its derivatives. As a result, cross disciplinary studies on financial systems have attracted much attention in recent decades [2–7]. With the aid of ideas and techniques from other fields, there have been significant advancements on the studies of economy science. One of great achievements has been the applications of statistical mechanics to economic systems, which has been later referred to *econophysics* [5]. Some correspondences between quantities in economic systems and physical systems were found, and knowledge in physics such as phase transitions in criticality [8], finite-size scaling theory [2,5,6], etc, were suggested to be fundamental concepts behind.

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There are also developments on the analysis of financial time series in methodology [3,4,9]. For example, the method of random matrix theory has been developed to study statistical structure of multivariate time series [3,4], and given remarkable agreement between theoretical prediction and empirical data [3,7]. Furthermore, the wavelet transform modulus maxima approach [10] has been applied to study non-stationary time series such as physiologic systems [11–14] and economic systems [9]. Quite recently, Wu et al. proposed a new approach to study stock time series [15]. The approach was based on the concept of instantaneous phase defined from the return time series can catch the characteristic structures of financial time series [15]. To implement the proposal, the Hilbert–Huang time signal analysis method [16] was used to define and evaluate instantaneous phase of return time series. Based on the investigations of phase distribution and phase correlation of the so-called intrinsic mode functions (IMFs), Wu et al. concluded that the return time series fall into a class which is different from other non-stationary time series, and the statistics of phase differences further provided useful observations on the trading activities in Dow-Jones and NASDAQ stock markets.

In this paper, we will follow the approach proposed in Ref. [15] to study correlation of foreign exchange time series. We use the empirical data of USD/DEM and USD/JPY exchange rates for the study. There are some significant differences between foreign exchange markets and stock markets. In contrast to stock markets which are highly regulated, foreign exchange markets function under a very loose, essentially self-policing environment. Furthermore, the foreign exchange market is the largest market in the world, with \$1.9 trillion in all currencies changing hands each day [17]. However, due to relatively moderate price variations of foreign exchange rates in currency markets, there are fewer reports on the investigations of foreign exchange rates in comparison with rich studies on stock markets. It is heuristic to demonstrate the application of the approach to foreign exchange time series.

The main purpose of this study is to provide an alternative and promising scheme for the demonstration of correlations of exchange rates in currency markets. To have quantitative descriptions on the correlative behaviors revealing from the return of foreign exchange time series, we first employ the empirical mode decomposition (EMD) method [16] to decompose a return time series into a set of IMFs and then apply the Hilbert transform to calculate instantaneous phases of these IMFs. We measure the correlation between two exchange time series by calculating the distribution of phase differences of the IMFs in the same order. Our results show explicitly the correlations are stronger in daily time scale than in longer time scales, and two exchange time series were more correlated in the period of 1986–1989 than in the period of 1990–1993.

The rest part of this paper is organized as follows. In next section, we briefly introduce the source of the empirical data under consideration. Return time series of USD/DEM and USD/JPY are shown in Section 3 for an exploration. The analyses of phase correlation are presented in Section 4. Finally, we conclude our results in Section 5.

2. Data

The empirical data used in this paper includes the transaction prices of USD/DEM and USD/JPY exchanges over the period from February 1 1986 to December 31 1996 [18]. The ownership of the data belongs to the Olsen & Associates and the authorized use in the current work was under the agreement between the author's institution (Academia Sinica) and Olsen & Associates. The original data of the USD/DEM and USD/JPY exchange time series were separatively recorded and the data lengths were not consistent. After deleting the dates without records, and then aligning the opening and closing prices by date for each contract, we finally got a set of time series with totally 3843 sampling points for the study. In general, the number of sampling point is too few for scaling analysis, but is enough for a whole-set statistics to give reliable estimation. Therefore, we will focus on the latter in this paper.

3. Time series of intraday returns

To examine the pricing transmission of the USD/DEM and USD/JPY exchange rates, we define the intraday return $R(t)$ as

$$R(t) = \frac{Y_c(t) - Y_o(t)}{Y_o(t)}, \tag{1}$$

where $Y(t)$ is the foreign exchange time series, and the subscript "o" denotes the opening price and "c" for the closing price. The general features of intraday return time series of USD/DEM and USD/JPY exchange rates are shown in Fig. 1. Except for inherent stochastic behaviors reveal in such time series [15], the correlative behaviors among them can be found but are implicit over the whole period. The technique used in this work is intended to provide statistical demonstrations for correlative behaviors.

We further define the probability distribution (or more precisely, probability density function) P as the normalized distribution of a measure ρ, which satisfies the normalization condition

$$\int_{-\infty}^{\infty} P(\rho)\,\mathrm{d}\rho = 1, \tag{2}$$

where the measure ρ can be return $R(t)$ or phase difference ϕ to be defined and used in the later discussions.

The probability distributions P of intraday returns for USD/DEM and USD/JPY exchange rates are shown in Fig. 2. Fig. 2(a) is the probability distributions $P(R)$ of intraday returns shown in linear scale, and Fig. 2(b) is the same but in semi-logarithmic scale. The curves of Fig. 2(b) show similar profiles with those of stock return time series considered in Refs. [2,5,15], while there is a significant peak at $R = 0$ due to the definition of R in Eq. (1) and properties of currency market. It was also shown in Ref. [15] that return time series of foreign exchange rate falls into the same class as stock returns. Therefore, essential properties of the returns for exchange time series resemble those of stock returns, such as heavy tail and Lévy stable distribution. However, due to the limitation that the size of our empirical data is too short for scaling analysis, we will not discuss this

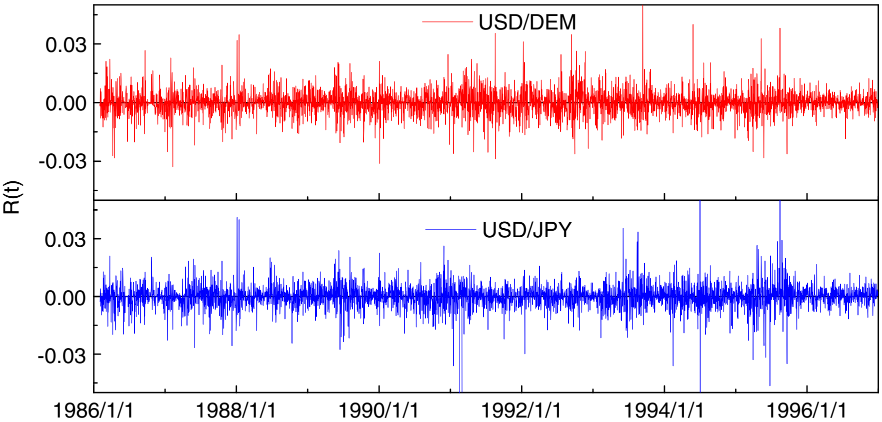

Fig. 1. Time series of intraday returns for the USD/DEM and USD/JPY exchange rates from February 1 1986 to December 31 1996.

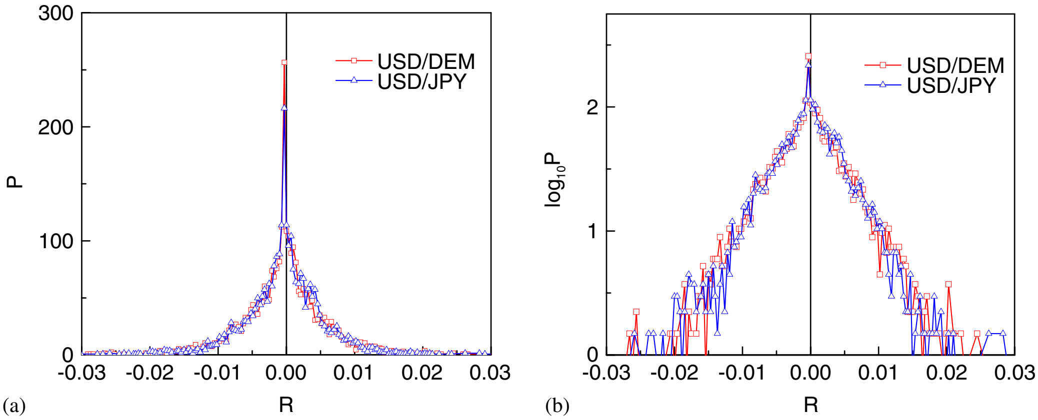

Fig. 2. Probability density function $P(R)$ of intraday returns in (a) in linear scale, and (b) in semi-logarithmic scale.

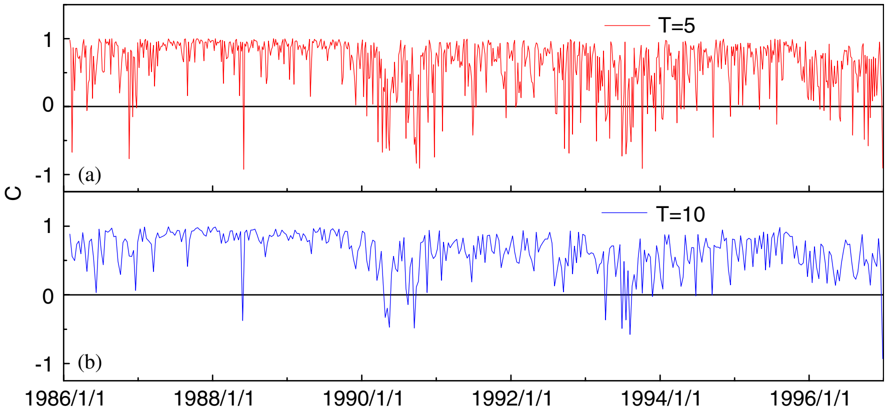

Fig. 3. Cross-correlation coefficient C with (a) $T = 5$ and (b) $T = 10$ for the USD/DEM and USD/JPY exchange time series.

issue in this paper. Here we note that the probability distributions P for two exchanges are in general the same in Fig. 2(a) and (b). This shows returns of exchange rates of different currencies share the same intrinsic properties.

Conventional approach for demonstrating the behaviors of correlation among a number of stocks and markets is by the so-called cross-correlation coefficient C_{ij} defined as the statistical overlap of the fluctuations $\delta R_i(t) = R_i(t) - \langle R_i(t) \rangle$ between two markets i and j, that is

$$C_{ij} = \frac{\langle \delta R_i(t) \delta R_j(t) \rangle}{\sigma_i(t) \sigma_j(t)}, \tag{3}$$

where $R_i(t)$ is the intraday return of market i, and $\sigma_i^2 = \langle [\delta R_i(t)]^2 \rangle$. The average $\langle \cdots \rangle$ is over a time period T or a fixed number of time sampling point in empirical data. By the definition, C_{ij} has a value ranging from -1 (the weakest correlation) to 1 (the strongest correlation). The cross-correlation coefficients between USD/DEM and USD/JPY exchange time series with $T = 5$ and 10 are shown in Fig. 3(a) and (b), respectively. These figures shows C_{ij} for USD/DEM and USD/JPY exchange time series is closer to 1 in a period from 1987 to 1989, and is relatively closer to 0 in 1990 and 1993.

The advantage of the evaluation of cross-correlation coefficient is the resultant quantity represents continuously the dynamical correlation for a pair of time series. This provides a window for the survey of evolution of correlative behaviors in the period of interest. However, a shortcoming of the same is the detailed structures of correlations may be smeared by the average over a time period in Eq. (3).

4. Phase correlation

In this section, we will employ the approach proposed in Ref. [15] to analyze the correlation among foreign exchange rates of USD/DEM and USD/JPY. The analyses will be achieved by performing statistics on the phase differences between two exchange time series. The approach is based on the concept of the instantaneous phase defined from the return time series can catch the characteristic features of financial time series, and the idea is originated from the fact that phases of a time series usually contain rich information about the structures of the time series [15]. In the approach, the Hilbert–Huang time signal analysis method [16], which is suitable for the analysis of non-stationary time series, is adapted to define and calculate instantaneous phase.

The Hilbert–Huang method consists of the EMD and the Hilbert spectral analysis [16]. The EMD method is developed from the assumption that any time series is consisted of simple intrinsic modes of oscillation, and the essence of the method is to identify the intrinsic oscillatory modes by their characteristic time scales in the data empirically, and then decompose the data accordingly [16]. This is achieved by an algorithm with a series of processes for sifting data to generate IMFs. The IMFs introduced by the EMD are a set of well-behaved intrinsic modes which are symmetric with respect to the local zero mean and have the same numbers of zero crossings and extremes. Therefore, the Hilbert spectral analyses can be directly worked on the resulting IMFs to calculate instantaneous phases.

Now we briefly review the EMD method. The sifting algorithm to create IMFs in EMD consists of two steps. First, the local extremes in the return time series data $R(t)$ are identified. Then, all the local maxima are connected by a cubic spline line $U(t)$ forming the upper envelope of the time series, and another cubic spline line $L(t)$ forming the lower envelope. Both envelopes will cover all the original time series, and the mean of upper envelope and lower envelope $m_1(t)$ given by

$$m_1(t) = \frac{U(t) + L(t)}{2} \tag{4}$$

is a running mean. The running mean $m_1(t)$ is then subtracted from the original time series $R(t)$ to yield the first component, $h_1(t)$,

$$R(t) - m_1(t) = h_1(t). \tag{5}$$

The resulting component $h_1(t)$ is an IMF if it satisfies the conditions: (i) $h_1(t)$ is free of riding waves. (ii) It displays symmetry of the upper and lower envelopes with respect to zero. (iii) The numbers of zero crossing and extremes are the same, or only differ by 1. If $h_1(t)$ is not an IMF, the sifting process has to be repeated as many times as it is required to reduce the extracted signal to an IMF. In the subsequent steps of sifting process, $h_1(t)$ is treated as the data,

$$h_1(t) - m_{11}(t) = h_{11}(t). \tag{6}$$

Again, if the function $h_{11}(t)$ does not yet satisfy requested conditions (i)–(iii), the first sifting process continues up to k times until some acceptable tolerance is reached,

$$h_{1(k-1)}(t) - m_{1k}(t) = h_{1k}(t). \tag{7}$$

If the resulting time series is the first IMF, then it is designated as $c_1 = h_{1k}(t)$. Subsequently, the first IMF is subtracted from the original data, and the difference r_1 given by

$$R(t) - c_1(t) = r_1(t) \tag{8}$$

is a residue. The residue $r_1(t)$ is taken as if it were the original data, and we apply to it again the sifting process. Following above procedures, the process of finding more intrinsic modes, c_i, continues until the last mode is found. The final residue will be a constant or a monotonic function which represents the general trend of the time series data. Finally, we get

$$R(t) = \sum_{i=1}^{n} c_i(t) + r_n(t), \tag{9}$$

$$r_{i-1}(t) - c_i(t) = r_i(t), \tag{10}$$

where r_n is a residue.

Notably, according to the algorithm for generating IMFs, the lowest order of IMF has the highest frequency. In terms of intermittency (i.e., variative period), this means the length of intermittency is proportional to the order of IMFs. The intermittency can be here considered as a window used to eliminate the end effects and to facilitate computation. However, a characteristic intermittency in trading time of a currency market is indefinite. For a truly non-stationary process, there is no time scale to guide the choice of the window size. This is the nature of the financial time series [15,19], and we thus do not impose definite intermittencies in the sifting process as an additional criterion.

Fig. 4 shows the IMFs obtained by the EMD method from the two return time series. Note that in the practical EMD procedures, the number of IMFs is case by case and depends on the properties of primary time series. Here, a return time series is decomposed into 11 IMFs in the sifting processes but only five IMFs are shown in Fig. 4. Among these IMFs, the c_1 IMF catches the finest structures, and c_2 is the next. It was reported that the c_1 component reveals the stochastic properties of financial time series [15], which can be demonstrated by existing stochastic volatility models [20]. Furthermore, it is also interesting that the correlative behaviors in the return time series of Fig. 1 are more apparent from these IMFs. Since the length of

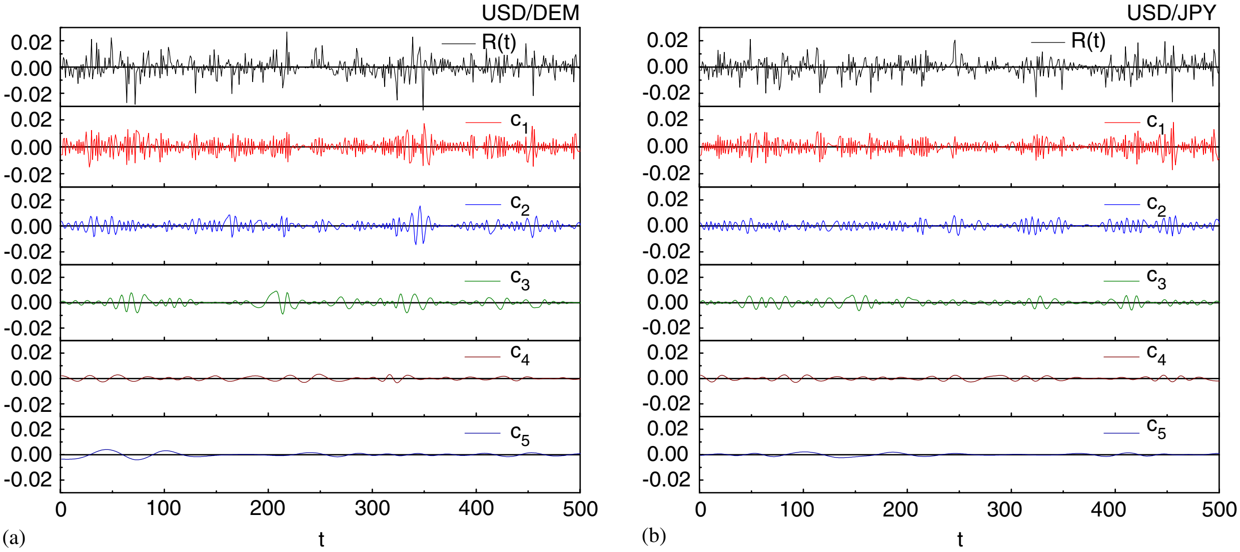

Fig. 4. Intraday returns and the first five IMFs obtained by the EMD method for (a) USD/DEM and (b) USD/JPY exchange time series.

intermittency is proportional to the order of IMFs, the correlative behaviors among time series can also be demonstrated in terms of various intermittency via the analyses of correlations among the same order of IMFs. This is beneficial to the analyses of relations between correlative strength and time scale. In particular, it is intuitively expected that the strengths of correlations of exchange rates in currency markets should vary with time scale.

After IMFs being obtained from the EMD, we can proceed to apply the Hilbert transform to each IMF component to calculate the instantaneous phases of IMFs. The Hilbert transform can be shortly summarized as firstly calculating of the conjugate pair of $c_r(t)$, i.e.,

$$y_r(t) = \frac{1}{\pi} P \int_{-\infty}^{\infty} \frac{c_r(t')}{t - t'} \, dt', \tag{11}$$

where "P" indicates the Cauchy principal value. Then, the two functions $c_r(t)$ and $y_r(t)$ forming a complex conjugate pair can be expressed as

$$c_r(t) + iy_r(t) = A_r(t)\, e^{i\phi_r(t)}, \tag{12}$$

with amplitude $A_r(t)$ and the phase $\phi_r(t)$ defined by

$$A_r(t) = [c_r^2(t) + y_r^2(t)]^{1/2}, \tag{13}$$

$$\phi_r(t) = \arctan\left(\frac{y_r(t)}{c_r(t)}\right). \tag{14}$$

Consequently, we can calculate instantaneous phase of the rth IMF according to Eq. (14).

Fig. 5 shows instantaneous phase variations of the first-IMFs (c_1 in Fig. 4) for the return time series of the USD/DEM and USD/JPY exchanges. Note that in some epoches the phases of these time series follow the same paths. This implies collective behaviors in the periods. The amplitudes of such time series have been shown to be in Boltzmann distribution [15], and we will not discuss here since our analyses do not concern with the amplitude of return time series. However, we shall mention that the properties we observed can further be used to demonstrate that the return time series can be modelled by time-varying amplitudes in Boltzmann distribution with time-varying phases distributed in particular patterns [19].

To study correlation between the exchange rates of USD/DEM and USD/JPY, we further define the phase differences between pairs of IMFs in the same order. We take USD/DEM as a reference and define the relative

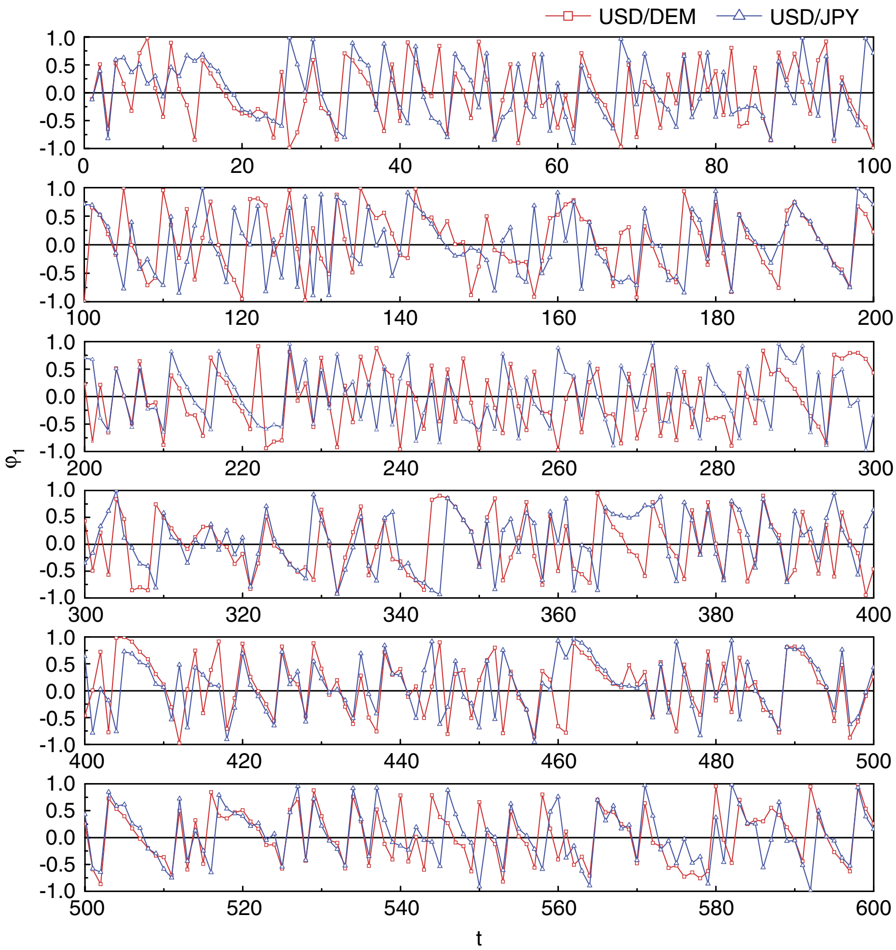

Fig. 5. Instantaneous phase variations of the first-IMFs (c_1 in Fig. 4) of the returns of USD/DEM and USD/JPY exchange time series.

phase difference $\Delta\phi_r$ as

$$\Delta\phi_r = \phi_r(\mathrm{USD/JPY}) - \phi_r(\mathrm{USD/DEM}), \tag{15}$$

and the results for $\Delta\phi_1$, $\Delta\phi_2$ and $\Delta\phi_3$ are shown in Fig. 6(a). The corresponding probability density functions are shown in Fig. 6(b), and a summary of skewness and kurtosis statistics of the phase differences are listed in Table 1.

From Fig. 6(b), we can make quantitatively descriptions on the correlations between two exchange rates. In Fig. 6(b), the distribution of phase difference $\Delta\phi_1$ is narrower and more peaked at $\Delta\phi_r = 0$ than those of $\Delta\phi_2$ and $\Delta\phi_3$. This observation is apparent from the statistics of kurtosis listed in Table 1, in which the kurtosis changes from $0.5826(\Delta\phi_1)$ to $-0.3373(\Delta\phi_2)$ and then to $-0.2488(\Delta\phi_3)$. This implies return variations contributed by the first-IMFs are more in phase than those of higher-order IMFs. In other words, two exchange rates are more correlative in daily time scale. Furthermore, no significant difference is found between distributions of $\Delta\phi_2$ and $\Delta\phi_3$. This further implies the correlative behaviors between USD/DEM and USD/JPY exchange rates were similar in time scales longer than daily. This scenario is completely different from single currency at different spots. The feature observed may be interpreted by the fact that USD has been a hard currency in currency markets. The exchange rates of USD/DEM and USD/JPY were correlated under the influence of USD. According to the decomposition scheme of the EMD, the first IMF catches the finest structures of fluctuations. For financial time series, such fluctuations are characteristics of non-predictable and stochastic features [15]. In this time scale, the fluctuations are essentially generated by local trading activities in

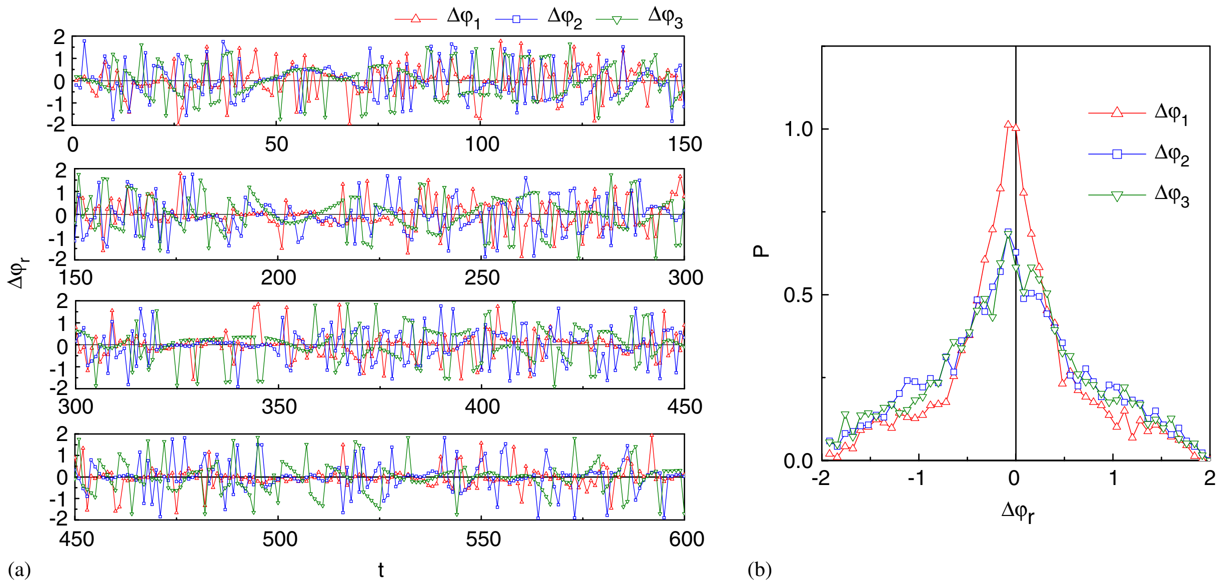

Fig. 6. Instantaneous phase difference of (a) the first three IMFs of intraday returns of the USD/DEM and USD/JPY exchange time series; (b) probability density function P of the phase difference $\Delta\phi_r$.

Table 1
Summary of skewness and kurtosis statistics for the phase differences $\Delta\phi_1$, $\Delta\phi_2$, and $\Delta\phi_3$ of USD/DEM and USD/JPY for the period of 1986–1996, and $\Delta\phi_1$ for 1986–1989 and 1990–1993

	$\Delta\phi_1$	$\Delta\phi_2$	$\Delta\phi_3$	$\Delta\phi_1$(1986–1989)	$\Delta\phi_1$(1990–1993)
Skewness	0.0458	0.0341	−0.0260	0.0340	0.0454
Kurtosis	0.5826	−0.3373	−0.2488	1.9112	0.1108

which effects contributed by individual transactions dominate. Stronger interactions leading to stronger correlations are possible in open markets. In contrast, high-order IMFs are derivatives from local means of the original time series, in which short-time scale fluctuations have been smeared out. The smearing is similar with local average of the original time series under a specified window size. In this case, the factors of governmental policy and other balance mechanisms are dominant factors determining the profiles of the time series. In principle, governmental policy and some balance mechanisms usually aim to direct the exchange rate of its currency in an independent and stable way. They work as a self-modulation on the basis of economic fundamentals to stabilize currency markets. A harmony of interdependence and correlation in foreign exchange markets reveals such that similar correlations represented by the distribution of $\Delta\phi_2$ and $\Delta\phi_3$ can be observed. As a result, the correlative behaviors in longer time scales are weaker than those in daily time scale.

To demonstrate the scheme of phase statistics to correlation between exchange rate time series, we further consider $\Delta\phi_1$ of different time periods. For comparison, we take two periods, one from 1986 to 1989 and the other from 1990 to 1993. In our statistics, the period 1986–1989 consists of 1367 data points and the period 1990–1993 consists of 1400 data points. We calculate the distribution of $\Delta\phi_1$ for these two periods and the results are shown in Fig. 7. The result confirms that the correlation between two exchange rate time series in the period of 1986–1989 was stronger than in the period of 1990–1993, which is consistent with the result of the cross-correlation calculation shown in Fig. 3. Note that in 1990–1993, foreign exchange rates of DEM and JPY in currency markets were affected by the unification of East Germany and West Germany and the era of bubble economy of Japan. The factors directing the pricing transmissions of USD/DEM and USD/JPY exchanges were not USD but economic environments and governmental policies of the unified Germany and

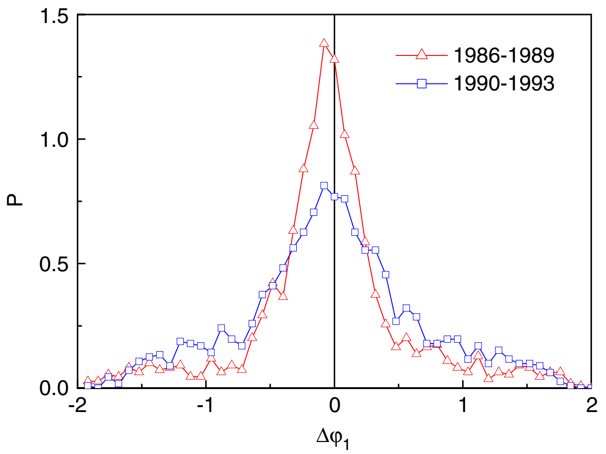

Fig. 7. Probability density function P of the phase difference $\Delta\phi_1$ for periods 1986–1989 and 1990–1993.

Japan. Therefore, two currencies were more independent, and the USD/DEM and USD/JPY exchange rates became weakly correlated in this period.

5. Conclusions

In conclusion, we have investigated the correlations of foreign exchange rates of USD/DEM and USD/JPY for the period from February 1 1986 to December 31 1996 from the aspect of phase correlation. Following the approach proposed in Ref. [15], we defined the instantaneous phase from the return time series of exchange rate. To achieve this, the Hilbert–Huang method was employed to decompose empirical time series into a number of IMFs and the Hilbert transform was then applied to calculate the instantaneous phase of the resultant IMFs. Then phase differences between two exchange rate time series were calculated and statistics on the phase differences were performed to probe their correlations. Our results showed explicitly that two exchange rates were more correlative in daily time scale than in longer time scales. This feature was explained from the trading environment with self-modulation mechanisms and time scale for free of interventions in currency markets.

We also calculated the distributions of the phase differences between the first-IMFs of two exchange time series in periods from 1986 to 1989 and from 1990 to 1993. The result indicates that two exchange rate time series were more correlated in the period of 1986–1989 than in the period of 1990–1993, which is consistent with direct observations from the cross-correlation calculation. The weaker correlation was explained from the economic environments and governmental policies after the unification of East Germany and West Germany and in the era of bubble economy of Japan in early 1990s.

Finally, it should be emphasized that even though existing analyses based on cross-correlation coefficient can also provide information on the correlative behaviors among markets, our approach based on phase statistics has provided an alternative and promising scheme for the demonstration of correlations among financial time series. It is also expected that the method can be applied to the studies of other time series, such as physiological time series [21], seismic time series, temperature variations, and other social models [19].

Acknowledgments

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References

[1] See e.g., B. Zhou, J. Business Econ. Stat. 14 (1996) 45.
[2] R.N. Mantegna, H.E. Stanley, Nature 376 (1995) 46.
[3] L. Laloux, P. Cizeau, J.-P. Bouchaud, M. Potters, Phys. Rev. Lett. 83 (1999) 1467.
[4] V. Plerou, P. Gopikrishnan, B. Rosenow, L.A.N. Amaral, H.E. Stanley, Phys. Rev. Lett. 83 (1999) 1471.
[5] R.N. Mantegna, H.E. Stanley, An Introduction to Econophysics, Correlations and Complexity in Finance, Cambridge University Press, Cambridge, 2000.
[6] J. Voit, The Statistical Mechanics of Financial Markets, second ed., Springer, New York, 2003.
[7] W.-J. Ma, C.-K. Hu, R.E. Amritkar, Phys. Rev. 70 (2004) 026101.
[8] V. Plerou, P. Gopikrishnan, H.E. Stanley, Nature 421 (2003) 130.
[9] K. Ohashi, L.A.H. Amaral, B.H. Natelson, Y. Yamamoto, Phys. Rev. E 68 (2003) 065204(R).
[10] J.F. Muzy, E. Bacry, A. Arneodo, Int. J. Bifurcation Chaos Appl. Sci. Eng. 4 (1994) 245.
[11] S. Thurner, M.C. Feurstein, M.C. Teich, Phys. Rev. Lett. 80 (1998) 1544.
[12] P.C. Ivanov, L.A.N. Amaral, A.L. Goldberger, S. Havlin, M.G. Rosenblum, Z.R. Struzik, H.E. Stanley, Nature (London) 399 (1999) 461.
[13] A. Marrone, A.D. Polosa, G. Scioscia, Phys. Rev. E 60 (1999) 1088.
[14] L.A.N. Amara, P.C. Ivanov, N. Aoyagi, I. Hidaka, S. Tomono, A.L. Goldberger, H.E. Stanley, Y. Yamamoto, Phys. Rev. Lett. 86 (2001) 6026.
[15] M.-C. Wu, M.-C. Huang, H.-C. Yu, T.C. Chiang, Phys. Rev. E 73 (2006) 016118.
[16] N.E. Huang, Z. Shen, S.R. Long, M.C. Wu, H.H. Shih, Q. Zheng, N.-C. Yen, C.-C. Tung, H.H. Liu, Proc. R. Soc. London Ser. A 454 (1998) 903.
[17] BIS (Bank for International Settlements), Triennial Central Bank Survey of Foreign Exchange and Derivatives Market Activity in April 2004—Preliminary global results, September, 2004.
[18] The empirical data is part of the Olsen & Associates data distribution: HFDF93, which is available from Olsen & Associates, Research Institute for Applied Economics, Seefeldstrasse 233, CH-8008 Zurich, Switzerland.
[19] M.-C. Wu, Phase distribution patterns of time series and their implications, in preparation.
[20] For example, there are the Ornstein-Uhlenbeck (OU), the Heston model, and the exponential OU (expOU) model. See, e.g., J. Perelló, J. Masoliver, Phys. Rev. E 67, 037102 (2003)
J. Perelló, J. Masoliver, N. Anento, Physica A 344, 134 (2004); and references therein.
[21] M.-C. Wu, C.-K. Hu, Phys. Rev. E 73 (2006) 051917.