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# Statistical properties, dynamic conditional correlation and scaling analysis: Evidence from Dow Jones and Nasdaq high-frequency data☆

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ABSTRACT

This paper investigates statistical properties of high-frequency intraday stock returns across various frequencies. Both time series and panel data are utilized to explore the properties of probability distribution, dynamic conditional correlations, and scaling analysis in Dow Jones Industrial Average (DJIA) and Nasdaq intraday returns across 10-min, 30-min, 60-min, 120-min, and 390-min frequencies. The evidence shows that both returns and volatility (standard deviation) increase with the increasing scaling from 10-min to 390-min intervals. By fitting an AR(1)-GARCH(1,1) model to intraday data, we find that AR(1) coefficients are negative for DJIA returns and positive for Nasdaq, exhibiting a positive and negative feedback strategy in DJIA and Nasdaq, respectively. The evidence also shows that these coefficients are statistically significant for either including or excluding opening returns for the 10-min and 30-min frequencies. By examining the dynamic conditional correlation between the DJIA and the Nasdaq across different frequencies, a positive correlation ranging from 0.6 to 0.8 was found. In addition, the variance of the dynamic correlation coefficients is decreasing and appears to be stable for the 2001–2003 period. Finally, both returns on DJIA and Nasdaq satisfy the stable Lévy distributions, implying that both markets can converge to equilibrium by self-governing mechanism after shocks. Results of this work provide relevant implications for investors and policy makers.



## 1. Introduction

Traditional analysis of stock returns relies heavily on economic fundamentals such as dividend yield, interest rate spreads, risk, and market–book ratio etc. (see, ex. Refs. [1–4]). The advantage of fundamental analysis is that the underlying economic rationale can be verified and the findings can then be used for either guiding investment decisions or monitoring market behavior by regulators. The fundamental analysis is, of course, crucial if the issue to be investigated is a longer run phenomenon based on quarterly and monthly data. Using longer horizon factors may not be feasible for analyzing high-frequency data because the variations of the return series are unlikely to be explained by the economic fundamentals. For this reason, analysts need to explore alternative data and techniques to provide more complete information of market behaviors.

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In this study, we apply modern time series techniques to detect the empirical regularities of high-frequency data for both the Dow Jones Industrial Average 30 (DJIA) and the Nasdaq stock indices. The reason for choosing these two indices stems from the fact that the former represents well-established and well-known firms in the US market, while the latter consists of high-tech and growth firms. These two indices thus represent not only the core of the US economy but also facilitate the menu for investors' choice in making the mean–variance investment decision. A successful empirical investigation emerging from this study is bound to provide insight into understanding the behavioral relation of high-frequency data and its validity across different scales.

This study is motivated by the conventional approach usually focusing on a particular point of time series data to derive the empirical regularities. This resulting statistical analysis may be misleading without considering a broader selection of data points. For instance, in investigating daily observations, researchers often use closing price (return) or average price (return) without carefully constructing an appropriate measure, not even taking into consideration the impact of the opening observations. Taking different points in time from a particular trading day to represent a daily observation may inherently introduce some sort of anomaly into data construction. As a result, it may produce a biased estimate and statistical inference. Second, the conventional analysis of the empirical issue is usually based on a particular scale of measure without concerning for its scaling variants. For instance, daily data are frequently used to examine the AR(1) process. This approach pays no attention to frequency variations, such as the validity of 10-min, 30-min, 60-min, 120-min, or 390-min horizons. Apparently, the derived empirical regularity is conditional on a particular time scale and lacks general implications. Third, although high-frequency data have been analyzed in a number of research papers [5–10], these papers focus mainly on a single market, especially the foreign exchange market. The exceptions are Wood et al. [11] and Abhyankar et al. [12], who analyze the equity markets. However, the dynamic relationship of intraday returns between two markets is mostly ignored in the market microstructure literature.

This paper differs from the extant literature in the following ways. First, in addition to exploring the time series properties involving high-frequency data, this study extends the conventional analysis to include the scaling dimension, since time series analysis can capture only limited information in terms of a particular time horizon. With the addition of the frequency-varying dimension, we will have more complete knowledge of the test, ranging from short-span to long-horizon data. Second, most of the empirical literature employs only time series data to investigate autocorrelation, without screening out the significance of opening intervals. By reshaping the time series data into panel data, we are able to compare the panel autocorrelations across different time frequencies. Our study shows that although both the DJIA and the Nasdaq indices exhibit the highest return in the opening interval at 9:30 A.M., the autocorrelation of return coefficients in a GARCH(1,1) specification are statistically significant for the 10-min and 30-min frequencies, either including or excluding the opening returns. Third, although a dynamic conditional correlation (DCC) technique was utilized to investigate the leads and lags across different markets, very few attempts have been geared to the analysis of intraday returns with different frequencies. Our evidence shows that using different scaling would lead to cross-correlation variations, suggesting that the validity of dynamic correlations between two time series is conditional on a particular time scale.

Finally, we analyze scaling behaviors of the time series on returns to probe the stability of time series distributions. Following the scheme proposed by Mantegna and Stanley [13,14], we perform scaling analyses on DJIA and Nasdaq changes with various time intervals. Both exhibit well-behaved scaling and belong to a stable distribution based on the criterion of Lévy's $\alpha$ stable distribution condition [15].

This paper is organized as follows. Section 2 briefly outlines the data and the construction of series frequency with various intervals for the intraday data of the DJIA and Nasdaq indices. Section 3 presents some summary statistics of returns and volatility for both indices. Section 4 investigates the time series and panel autocorrelations between the DJIA and the Nasdaq. Section 5 discusses the dynamic conditional correlation between the DJIA and the Nasdaq based on different time frequencies. Section 6 presents the probability distribution and scaling analysis. Section 7 contains concluding remarks.

## 2. Data and stock returns across different frequencies

The data employed here consists of Dow Jones Industrial Average index's (DJIA) and Nasdaq-100 index's (Nasdaq) 10-min intraday returns provided by the Bloomberg real-time data service. The DJIA stocks are the most actively traded securities, and the capital size of the firms in the DJIA also helps to ensure a high degree of liquidity. Alternatively, the stocks listed on the Nasdaq are characterized by growing high-tech firms, which are associated with higher price volatility.

The 10-min intraday scale values for both the DJIA and the Nasdaq span the period from August 1, 1997, through December 31, 2003, including 1543 trading days with 60,177 intraday observations starting from 9:30 to 15:50 EST (Eastern Time Zone). The overnight (or over-weekend) period constitutes an unusual time period, since it involves an interval much longer than 10-min. Therefore, the value of a stock index at opening prices is expected to present an anomaly when compared with other data points. Following the analysis in Ref. [9], we constructed 10-min returns with the daily transaction record extending from 9:30 to 15:50, a total of 39 10-min returns for each day.[1] The 10-min horizon is short enough that the realized returns and volatility can be measured well and yet it is also long enough that the confounding influences from market microstructure behavior such as the "bid-ask bounce" first noted in Ref. [18] can be largely mitigated.

[1] Alternatively, Engle and Russell [16] have developed the autoregressive conditional duration (ACD) model to investigate high-frequency stock market data. In the ACD model the expected duration between trades depends on past durations. Here we follow Andersen and Bollerslev's approach to investigate

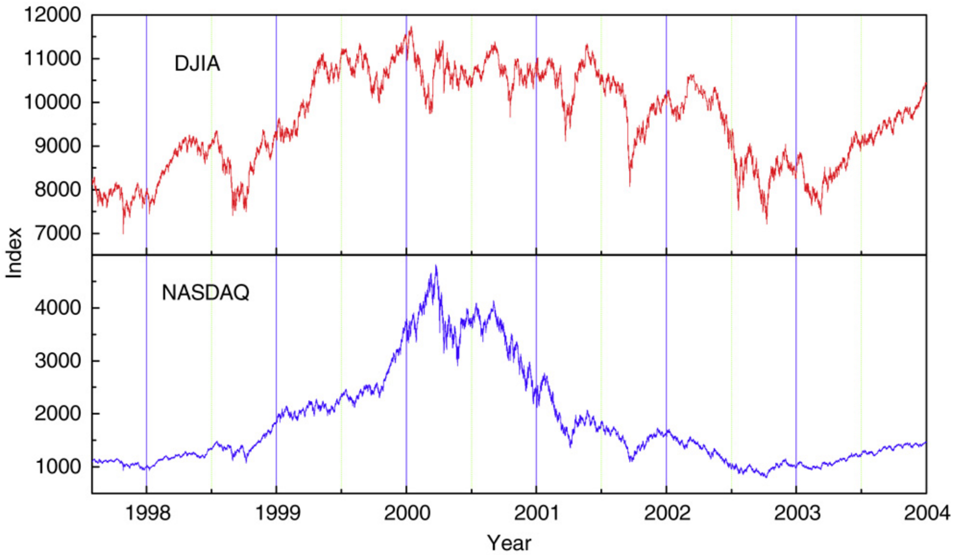


**Fig. 1.** Time series plots of 10-min frequencies of DJIA and Nasdaq indices (Aug. 1, 1997 – Dec. 31, 2003).

Fig. 1 shows time series plots of the DJIA and Nasdaq indices sampled at 10-min frequencies. As mentioned above, the trading time is defined in a continuous pattern, starting from the opening of the day to the closing, and this process repeats again on the next trading days.

As shown in Fig. 1, both the DJIA and Nasdaq seem to follow a similar pattern and trend over time, perhaps because both are influenced by common economic fundamentals. However, their short-run variations are somehow different, exhibiting different extents of correlations and speeds of change. In particular, returns on the DJIA display a relatively stable movement, whereas the Nasdaq experiences a dramatic change, showing a speedy upward trend around mid-2000, followed by a sharp downward trend thereafter.[2] This difference may be rooted in the nature of the corporations being weighted in their indices: the DJIA comprises well-established companies, while the Nasdaq is made up of growing high-tech firms. The latter is viewed to have a higher return that compensates for higher risk. To further explore the underlying characteristics of these two series, it is convenient to start with the investigation of the basic statistics for the returns of both indices.

## 3. Summary statistics of DJIA and Nasdaq

### 3.1. Basic statistics

Presented in Table 1 are the summary statistics reporting intraday returns from 10 min to 390 min and interday returns from one day to five days and one week to five weeks. As reported in Table 1, the average return on the DJIA at a 10-min interval is 0.000006 with a standard deviation of 0.0019. The distribution is slightly right-skewed (with a skewness of 0.17) and has a high narrow peak (with a kurtosis of 23.37), suggesting that positive returns occur more often than negative returns in a 10-min interval series. With respect to the Nasdaq, both average returns (0.000013) and the standard deviation (0.0039) are higher than those of DJIA. The return series is more right-skewed (0.25), associated with a higher kurtosis (29.15). The feature of higher returns accompanied by higher risk is more pronounced when compared to the 10-min returns between the DJIA and the Nasdaq.

By checking the average returns across different time scaling, it is apparent that the returns of larger scales are almost equal to returns of 10-min multiplied by a 10-min scale. For example, the 390-min return almost equals $0.000006 \times 39 = 0.000234$. The standard deviation, however, was growing at a rate almost proportional to the square root of the sampling frequency. This result is consistent with that of Ref. [9] suggesting the similar ideas in foreign exchange (FX) market intraday returns. This finding also implies that high-frequency returns reveal some common features between stock and FX markets. In general, both returns and volatility (standard deviation) increase with increasing scaling from 10-min to 390-min intervals. However, with interday data scaling from one day to five days or one week to five weeks, higher average returns and volatility are seen to increase with increasing scaling, but the standard deviation does not grow at a rate proportional to

the time series properties of two high-frequency stock returns. However, unlike Andersen and Bollerslev [9] and Mian and Adam [17], we do not omit the closing-to-opening returns. Rather, we keep them in the data to conduct sensitivity analyses. After eliminating the omitted days for which all of the 10-min values of the index were not available, we obtained a total of 1543 trading days with 60,177 observations of 10-min index values.

2 Examination of their dynamic correlations between two series can be found in Section 5.

**Table 1**
Summary statistics across different frequencies of return and volatility series. This table summarizes the return ($R_{\tau,t}$) statistics across different frequencies of intraday, interday, and inter-week for both the DJIA and the Nasdaq. The sample period covers Aug. 1, 1997, to Dec. 31, 2003.

| $\tau$ | DJIA | | | | | | | | | Nasdaq | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Mean | Med. | Max. | Min. | Std. Dev. | Skew. | Kurt. | J.-B. | Prob. | Mean | Med. | Max. | Min. | Std. Dev. | Skew. | Kurt. | J.-B. | Prob. | Observ. |
| 10 min | 0.000006 | 0 | 0.034 | −0.0326 | 0.001974 | 0.167669 | 23.36597 | 1052495 | 0 | 0.0000125 | 0.000015 | 0.07377 | −0.0683 | 0.003978 | 0.24868 | 29.14976 | 1735339 | 0 | 60 884 |
| 20 min | 0.000012 | 0 | 0.03689 | −0.04292 | 0.002786 | 0.059889 | 18.74973 | 314653.8 | 0 | 0.0000248 | 0.000050 | 0.09912 | −0.06611 | 0.005607 | 0.515363 | 19.97172 | 366701.2 | 0 | 30 442 |
| 30 min | 0.000018 | 0 | 0.04339 | −0.0447 | 0.003411 | 0.264554 | 15.26449 | 127427.4 | 0 | 0.0000382 | 0.000050 | 0.14961 | −0.0788 | 0.00699 | 0.762882 | 23.8069 | 368044.5 | 0 | 20 294 |
| 40 min | 0.000024 | 0 | 0.03685 | −0.04647 | 0.003934 | −0.126545 | 12.73828 | 60185.15 | 0 | 0.0000496 | 0.000080 | 0.13195 | −0.06955 | 0.007932 | 0.471845 | 16.59085 | 117710.2 | 0 | 15 221 |
| 50 min | 0.000030 | 0.000075 | 0.04731 | −0.04894 | 0.00443 | −0.032442 | 13.01785 | 50916.79 | 0 | 0.0000626 | 0.000050 | 0.1129 | −0.07885 | 0.008902 | 0.47037 | 13.41195 | 55448.36 | 0 | 12 176 |
| 60 min | 0.000036 | 0.000020 | 0.04568 | −0.0434 | 0.004839 | 0.11569 | 10.18474 | 21847.34 | 0 | 0.0000767 | 0.000100 | 0.11919 | −0.07476 | 0.009908 | 0.45797 | 12.1145 | 35477.76 | 0 | 10 147 |
| 70 min | 0.000041 | 0.000030 | 0.04035 | −0.0586 | 0.0052 | −0.099977 | 10.91561 | 22719.79 | 0 | 0.0000883 | 0.000130 | 0.13467 | −0.07106 | 0.010627 | 0.463904 | 12.00188 | 29676.59 | 0 | 8 697 |
| 80 min | 0.000047 | 0 | 0.04772 | −0.06083 | 0.005556 | 0.00203 | 10.54224 | 18037.4 | 0 | 0.0001010 | 0.000140 | 0.14252 | −0.07561 | 0.011323 | 0.572785 | 12.1004 | 26676.1 | 0 | 7 610 |
| 90 min | 0.000053 | 0 | 0.05044 | −0.05621 | 0.005922 | 0.094176 | 9.375432 | 11465.43 | 0 | 0.0001130 | 0.000160 | 0.12283 | −0.07159 | 0.012055 | 0.34779 | 9.290563 | 11288.84 | 0 | 6 764 |
| 100 min | 0.000059 | 0 | 0.0482 | −0.05177 | 0.006263 | −0.011898 | 9.286831 | 10026.13 | 0 | 0.0001270 | 0.000130 | 0.16023 | −0.09494 | 0.012754 | 0.541165 | 12.86781 | 24997.6 | 0 | 6 088 |
| 110 min | 0.000064 | 0 | 0.0413 | −0.04717 | 0.006532 | 0.085298 | 8.365525 | 6644.942 | 0 | 0.0001380 | 0.000190 | 0.12811 | −0.07976 | 0.013252 | 0.397428 | 8.811231 | 7932.576 | 0 | 5 534 |
| 120 min | 0.000070 | 0.000030 | 0.04717 | −0.04681 | 0.006844 | 0.004339 | 8.271554 | 5873.976 | 0 | 0.0001540 | 0.000160 | 0.17218 | −0.07951 | 0.014148 | 0.609759 | 12.45802 | 19222.74 | 0 | 5 073 |
| 130 min | 0.000076 | 0.000060 | 0.04089 | −0.05899 | 0.00702 | 0.005924 | 7.941249 | 4764.188 | 0 | 0.0001680 | 0.000160 | 0.11997 | −0.13896 | 0.014726 | 0.176103 | 8.862001 | 6729.295 | 0 | 4 683 |
| 390 min | 0.000233 | 0.000480 | 0.05857 | −0.06803 | 0.012658 | −0.016781 | 5.541697 | 420.2569 | 0 | 0.0005380 | 0.001390 | 0.17424 | −0.11252 | 0.02686 | 0.168805 | 5.40815 | 384.6023 | 0 | 1 561 |
| 1 day | 0.000234 | 0.000300 | 0.06348 | −0.07184 | 0.01285 | −0.09432 | 5.87337 | 557.2809 | 0 | 0.0003760 | 0.001560 | 0.14173 | −0.09669 | 0.021685 | 0.197128 | 5.56106 | 451.2681 | 0 | 1 613 |
| 2 day | 0.000472 | 0.001480 | 0.07741 | −0.08749 | 0.01858 | −0.293446 | 4.972641 | 142.2508 | 0 | 0.0007680 | 0.002850 | 0.1422 | −0.11893 | 0.030997 | −0.037244 | 4.621624 | 88.49917 | 0 | 806 |
| 3 day | 0.000682 | 0.001780 | 0.0958 | −0.10872 | 0.02137 | −0.376276 | 5.52345 | 155.1512 | 0 | 0.0011020 | 0.003930 | 0.14274 | −0.13514 | 0.036568 | −0.350185 | 4.151999 | 40.66928 | 0 | 537 |
| 4 day | 0.000961 | 0.002010 | 0.13305 | −0.12363 | 0.026925 | −0.295379 | 6.277294 | 186.2137 | 0 | 0.0015360 | 0.003550 | 0.15109 | −0.16624 | 0.043674 | −0.272676 | 3.919767 | 19.19925 | 0.000068 | 403 |
| 5 day | 0.001099 | 0.002525 | 0.09837 | −0.1426 | 0.027533 | −0.357011 | 5.447198 | 87.18964 | 0 | 0.0017600 | 0.006675 | 0.18969 | −0.17358 | 0.046693 | −0.24838 | 4.26787 | 24.87807 | 0.000004 | 322 |
| 1 week | 0.001096 | 0.002950 | 0.08426 | −0.14263 | 0.026995 | −0.508843 | 5.282978 | 86.94677 | 0 | 0.0017240 | 0.002895 | 0.18978 | −0.25305 | 0.045226 | −0.492549 | 6.38399 | 172.8701 | 0 | 334 |
| 2 week | 0.002115 | 0.005920 | 0.11924 | −0.14502 | 0.036082 | −0.528108 | 4.70597 | 28.01374 | 0 | 0.0033530 | 0.005140 | 0.25753 | −0.1805 | 0.063106 | 0.017988 | 4.063278 | 7.875821 | 0.019489 | 167 |
| 3 week | 0.003071 | 0.007070 | 0.10407 | −0.13242 | 0.044471 | −0.274524 | 3.074423 | 1.419836 | 0.491685 | 0.0055250 | 0.017370 | 0.20654 | −0.33079 | 0.083506 | −0.577294 | 4.536858 | 17.0894 | 0.000195 | 111 |
| 4 week | 0.004148 | 0.006060 | 0.11911 | −0.15116 | 0.05314 | −0.394225 | 3.262172 | 2.387595 | 0.303068 | 0.0062160 | 0.009640 | 0.27388 | −0.21807 | 0.087565 | 0.109033 | 3.383367 | 0.672725 | 0.714364 | 83 |
| 5 week | 0.004884 | 0.010855 | 0.15898 | −0.20933 | 0.064307 | −0.358524 | 3.81427 | 3.237287 | 0.198167 | 0.0086670 | 0.009990 | 0.26439 | −0.27257 | 0.107436 | 0.035241 | 3.116665 | 0.05109 | 0.974778 | 66 |

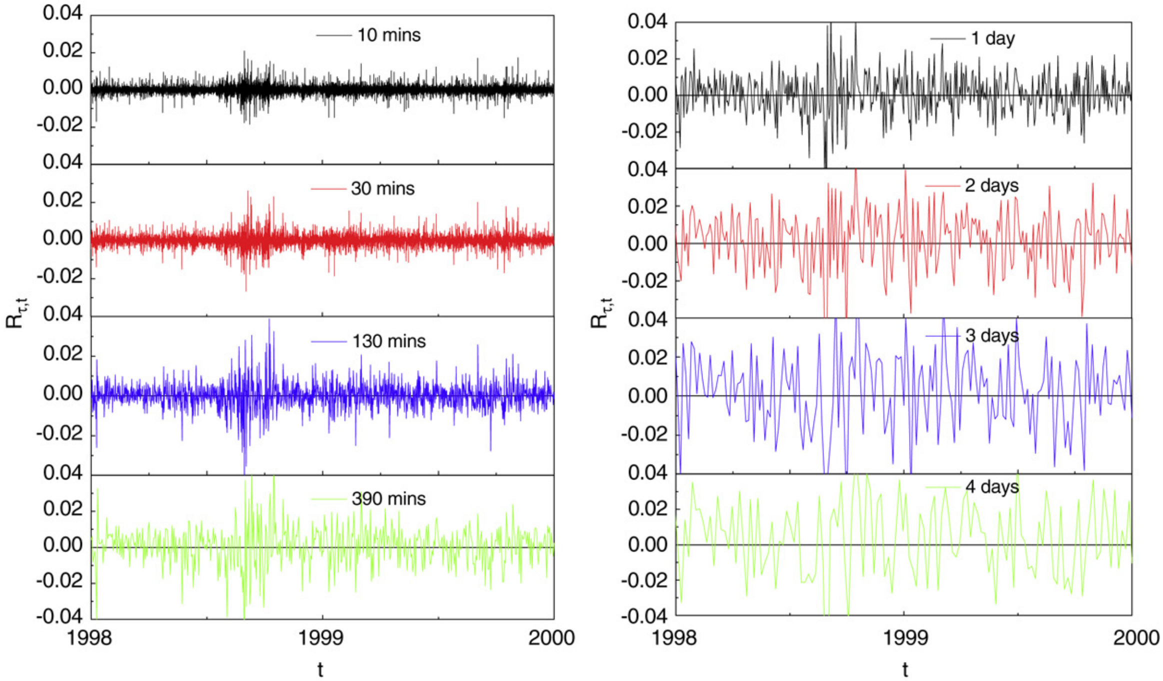


**Fig. 2.** Time series plots of stock returns for intraday and interday DJIA data.

the square root of the sampling frequency as intraday frequency does. These observations are different from those reported by Silva et al. [19]. Silva et al. [19] studied the distribution of stock returns of four individual large American companies at mesoscopic time lags for the period 1993 to 1999. There were no major market disturbances during this period, and Silva et al. found a linear relation between the square of variance and the time lag from five minutes to a month, after introducing an effective overnight time lag. However, we do not observe this feature for interday data even after the introduction of overnight time lags. More specifically, we did not find a single overnight time lag that is global for different time intervals from interday to weekly data. Since our empirical data are based on market indices from 1997 to 2003, they cover several macroeconomic events, such as the collapse of the dot-com companies and the 9/11 attacks. Whether market disturbances violate market efficiency [20] and, hence, the time-lag-variance proportionality may deserve further investigation.

By comparing the skewness, we find that most of the intraday returns have a positive sign, whereas the interday returns display a negative sign. This indicates that most of the daily or weekly interday returns are negatively skewed, meaning negative returns occur more often than positive returns, since the distribution has a longer left tail. This phenomenon occurs in both DJIA and Nasdaq markets.[3]

With respect to the kurtosis, all of the intraday returns show narrower peaks than normal, since the kurtosis is larger than 3. However, the kurtoses are declining (from more than 20 to almost 3) with increasing time intervals (from 10 min to five weeks) in both indices. Particularly, the kurtosis of the 10-min interval returns reaches the highest peak among all intraday interval measures, and it is decreasing with increasing scales to 130-min, daily, and weekly statistics.

Several regularities can be drawn from in this section: First, low-frequency returns in multiples of higher-frequency returns happen only in intraday returns; these results are not found significantly in interday returns. Second, the standard deviation of intraday returns is shown growing at a rate almost proportional to the square root of the sampling frequency; however, interday returns do not show a similar pattern. Third, most daily or weekly interday returns are negatively skewed; however, most intraday returns are positively skewed. Fourth, all scales of kurtosis of the intraday returns are greater than 3; however, the kurtosis declines as scale increases. Fifth, the intraday return series does not necessarily exhibit the best fit for normal distribution. Instead, daily returns (one-day series) show a better fit for normal distribution than those of other frequencies based on both skewness and kurtosis estimates, although they are still not perfect.[4]

Fig. 2 provides the time series plots of stock returns for the intraday DJIA data sampled by 10, 30, 130, and 390 min as well as interday data.[5]

[3] It may be seen that most of the interday intervals show left skewness, especially the DJIA. As noted by Andersen and Bollerslev [9], the negative skewness may be interpreted as evidence of the "leverage" and/or volatility feedback effects discussed by Black [21], Campbell and Hentschel [22] and Bekaert and Wu [23].

[4] The statistics show that daily returns have a skewness of −0.09 and kurtosis of 5.87 for the DJIA, and a skewness of 0.197 and kurtosis of 5.56 for the Nasdaq.

[5] We do not plot the Nasdaq to save space.

The volatility in Fig. 2 gets larger from 1998 to 1999 across all various frequencies, including both intraday and interday data. Moreover, these volatilities increase as scale increases. After checking the events during our sample period, we find that the consequence of Asian financial crisis in late 1998 might be the most significant impact factor. This also implies that the volatility spillover effect occurs across different continents' stock markets. Moreover, this spillover effect occurs not from big markets to small ones but from small to big ones.

### 3.2. Panel intraday returns and volatility

As we reported earlier, the intraday stock returns from 10 min to 390 min and the interday returns from one day to five days and one week to five weeks are on average higher for the Nasdaq compared to those of the DJIA. The higher returns are matched by higher volatilities measured by the standard deviations. As shown in Fig. 3, the differences in returns in Fig. 3(a) corresponding to the higher volatilities in Fig. 3(b) are seen to increase as the calculation of the intervals increases.

Next, it would be of interest to focus on the relation between return and volatility for a particular interval. Thus, we investigate the intraday behaviors of the return series among all 10-min intervals from 9:30 to 15:50. To this end, we reshape the time series data into panel data with 39 10-min intervals on every trading day across all 1543 days. Both 10-min intraday returns and volatilities from 9:30 a.m. to 15:50 p.m. for the DJIA and the Nasdaq are presented in Fig. 4(a) and (b), respectively. Both indices exhibit the highest return in the opening interval of 9:30 a.m. and follow a similar pattern across 10-min intervals. An especially high return in the opening interval reflects pronounced adjustments

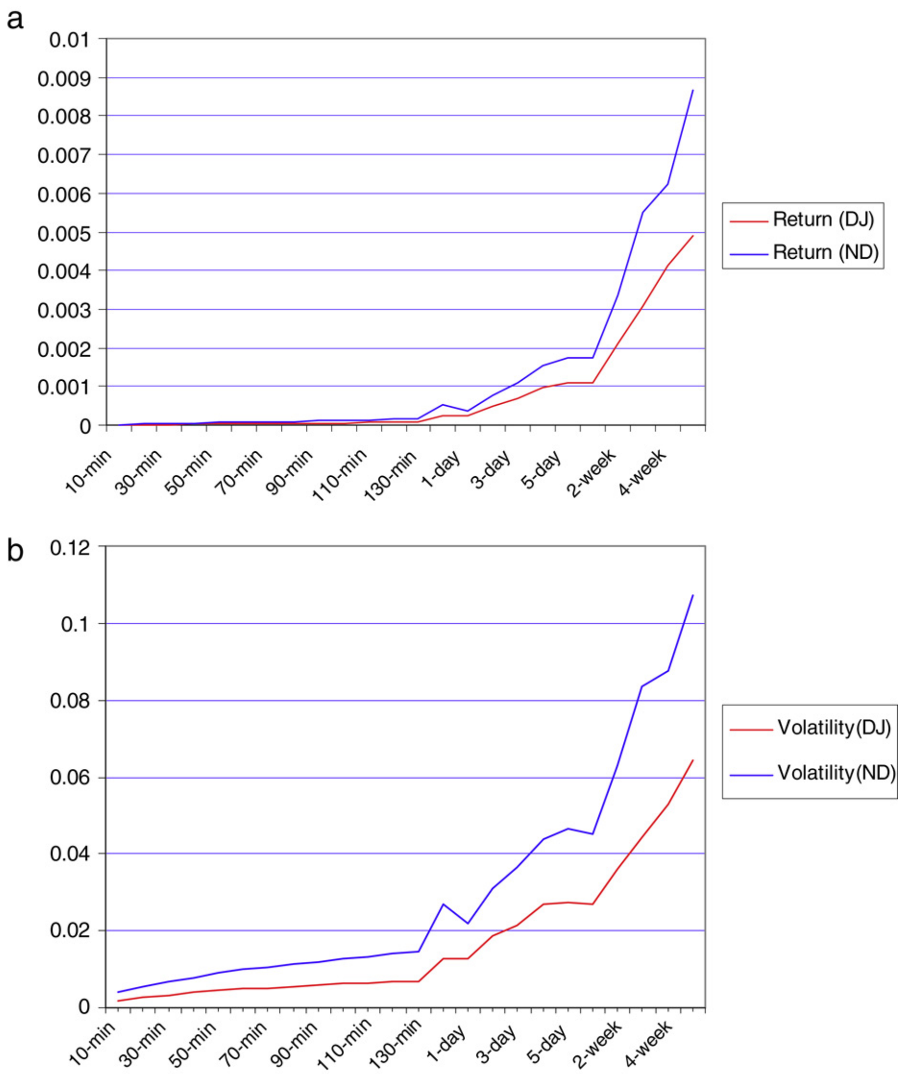


**Fig. 3.** (a) Plots of returns (vertical axis) of DJIA and Nasdaq from 10-min to 5 week intervals (horizontal axis), (b) Plots of volatilities (standard deviations along the vertical axis) of DJIA and Nasdaq from 10-min to 5-week intervals (horizontal axis).

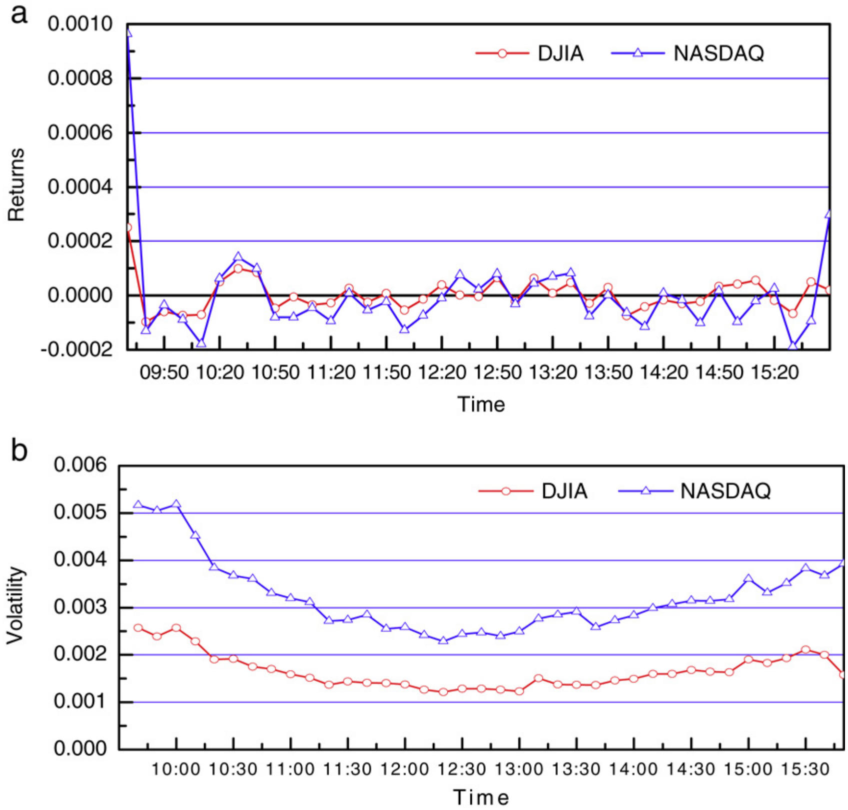


**Fig. 4.** (a) Panel data of 10-min intraday *returns* of DJIA and Nasdaq with opening interval, and (b) Panel data of 10-min intraday *volatility* of DJIA and Nasdaq with opening interval.

to the information accumulated overnight. This opening interval reveals a much higher return and volatility than any other 10-min interval. A parallel pattern, however, with a relatively moderate magnitude is displayed in the market closures.

The Nasdaq on average has a higher return than the DJIA, and this becomes more apparent in both the opening and closing intervals. The higher returns are associated with higher volatilities over the entire trading intervals. As shown in Fig. 4(b), the volatility against the 10-min trading interval displays a U-shaped curve. This U-shaped curve is consistent with the shape presented in equity [11] and derivative markets [24], implying a common feature associated with the volatility among various high frequencies of financial assets.[6]

Two points are worth noting. First, the opening interval always shows the highest volatility; lunch intervals display the lowest volatility. Second, it appears that the Nasdaq exhibits a greater curvature than the DJIA across all scales over the entire intervals. Evidently, larger scale not only creates higher volatility but also accompanied by a higher speed of change in volatility, so that a deeper curvature of the U-shape may result.

The finding of a U-shaped curve is consistent with social behavior during daily operations.[7] On the morning of each trading day, investors, in reacting to institutional arrangements for trading hours, tend to rack up voluminous transactions based on the information accumulated overnight, creating excessive volatility in the opening interval. Trading activity then slows down as investors collect news and process information over the course of the day. It reaches bottom around the lunch hour.[8] In projecting the closing hour, the accumulated trading activity rises and then accelerates before the market closes. To provide a rationale, recent studies [27,25,26] argue that this observed intraday U-shaped pattern in intraday stock market volatility is mainly attributable to the strategic interaction of traders around market openings and closings.

6 By increasing the time scale from 10-min to 30-min, we continue to find the U-shaped curve for both the DJIA and the Nasdaq. The plots are available upon request.

7 Evidence of the U-shaped pattern of intraday volatility can be found in Refs. [11,9,10], among others. Theoretical models of the U-shaped pattern appear in Refs. [25,26].

8 The U-shaped curve of volatility reflects the fact that the highest point of return volatility occurs around the opening 9:30 interval (0.006216804), followed by the 10:00, 9:40, and 9:50 intervals, respectively, and hits the lowest point around the 12:20 interval (0.001240272), followed by the second lowest at the 13:00 interval and third lowest at the 12:10 interval, respectively.

**Table 2**
Time series estimates of AR(1) for the DJIA and the Nasdaq across different time frequencies.

| $\tau$ | | DJIA | | Nasdaq | |
|---|---|---|---|---|---|
| | | Coefficient | *P*-value | Coefficient | *P*-value |
| 10 min | $\delta_\tau$ | 0.00000604 | 0.4498 | 0.0000126 | 0.4399 |
| | $\phi_\tau$ | −0.001695 | 0.6758 | −0.000938 | 0.8169 |
| | $R^2$ | 0.00028 | | 0.000002 | |
| 30 min | $\delta_\tau$ | 0.0000183 | 0.4716 | 0.0000385 | 0.4383 |
| | $\phi_\tau$ | 0.003285 | 0.64 | 0.005783 | 0.4102 |
| | $R^2$ | 0.000841 | | 0.000242 | |
| 60 min | $\delta_\tau$ | 0.0000361 | 0.465 | 0.0000768 | 0.4643 |
| | $\phi_\tau$ | −0.00124 | 0.9007 | 0.007409 | 0.4556 |
| | $R^2$ | 0.001899 | | 0.001201 | |
| 120 min | $\delta_\tau$ | 0.0000709 | 0.4423 | 0.000152 | 0.4431 |
| | $\phi_\tau$ | 0.027851 | 0.0476 | 0.014206 | 0.3118 |
| | $R^2$ | 0.006283 | | 0.000112 | |
| 390 min | $\delta_\tau$ | 0.000261 | 0.3782 | 0.000592 | 0.3819 |
| | $\phi_\tau$ | −0.022378 | 0.3797 | −0.098186 | 0.0001 |
| | $R^2$ | 0.006283 | | 0.00964 | |

Total sample included 60,879 observations for the 10-min series after adjustment. The estimated equation is: $R_{\tau,t} = \delta_\tau + \phi_\tau R_{\tau,t-1} + \varepsilon_{\tau,t}$, where $R_{\tau,t}$ is stock return applied to the DJIA and the Nasdaq series; $\delta_\tau$ is a constant term; $\phi_\tau$ is a constant coefficient; the subscript $\tau$ is a scale index; $\varepsilon_{\tau,t}$ is a vector of random error terms.

## 4. Autocorrelations of time series

### 4.1. Time series estimates of AR(1)

Since autocorrelation plays a central role in evaluating market efficiency, the recent literature has used AR(1) to detect feedback trading behavior [28,29]. Thus, it is of interest for us to investigate the sign of autocorrelation in order to understand more about investors' trading behaviors in both markets. In this section, we first consider time series autocorrelation models in our estimations. In expression, we write:

$$R_{\tau,t} = \delta_\tau + \phi_\tau R_{\tau,t-1} + \varepsilon_{\tau,t}, \tag{1}$$

where $R_{\tau,t}$ is stock returns applied to the DJIA and the Nasdaq series; $\delta_\tau$ is a constant term; $\phi_\tau$ is the coefficient for the AR(1) term; the subscript $\tau$ is a scale index, ranging from 10–390-min; and $\varepsilon_{\tau,t}$ is the random error term. The AR(1) term included in Eq. (1) accounts for autocorrelation, possibly arising from non-synchronous trading, price limitations, slow price adjustments, market frictions, or feedback trading (see Refs. [30–32,28,33–37]).

Estimations are made on the return series by setting $\tau = 10$ min, 20-min, ..., 390-min frequencies. In this time series estimation, the observations are arranged in the time sequence, including the lengthy opening interval. The estimates of the AR(1) coefficients for each $\tau$ frequency are reported in Table 2.

The evidence in Table 2 shows that the AR(1) coefficients on both the DJIA and the Nasdaq present mixed signs and lack of statistical significance. The exceptions are the coefficients for the 120-min intervals for the DJIA and the 390-min intervals for the Nasdaq. These significant statistics do not seem to have a consistent pattern. It appears to us that the unsatisfactory results may be attributable to the inclusion of the opening data point or, simply, to the misspecification of the model, or both.

### 4.2. AR(1)-GARCH(1,1) model

As documented by Laux and Ng [7] and Andersen and Bollerslev [9], since the high-frequency return volatility, such as that of the exchange rate and S&P 500 futures, displays a changing intraday pattern, we are led to consider the point that estimations based on Eq. (1) could be misspecified. Following the conventional approach, the conditional variance for high-frequency returns is assumed to follow a GARCH(1, 1) process as given by:

$$\sigma^2_{\tau,t} = \omega_\tau + \alpha_\tau \varepsilon^2_{\tau,t-1} + \beta_\tau \sigma^2_{\tau,t-1} \tag{2}$$

where $\sigma^2_{\tau,t}$ is the conditional variance for frequency $\tau$. Since volatility is likely to be time-varying and to present a clustering phenomenon, the unconditional returns distributions generated by a normal GARCH model will have fat tails. This is especially true for the high-frequency data. From this perspective, a student $t$-distribution [38] or a generalized error distribution (GED) [39] is usually assumed for the error process in the conditional mean equation. In this paper, we follow Nelson [39] by using the GED.[9] The estimates based on the log-maximum likelihood method are reported in Table 3.

[9] The parameters will be estimated by the log-maximum likelihood method. The density function in Nelson [39] is given by

$$f(\mu_{\tau,t}, \sigma_{\tau,t}, \nu) = \frac{\nu[\Gamma(3/\nu)]^{1/2}}{2[\Gamma(1/\nu)]^{-3/2}\sigma_{\tau,t}} \exp\left[-\left|\frac{\varepsilon_{\tau,t}}{\sigma_{\tau,t}}\right|^{\nu}\left[\frac{\Gamma(3/\nu)}{\Gamma(1/\nu)}\right]^{\nu/2}\right]$$

**Table 3**
Time series estimates of AR(1)-GARCH(1, 1) of DJIA and Nasdaq high-frequency returns.

| $\tau$ | Obs. | $\phi_\tau$ | P-value | $\alpha_\tau$ | P-value | $\beta_\tau$ | P-value | $\alpha_\tau + \beta_\tau$ |
|---|---|---|---|---|---|---|---|---|
| Panel A: DJIA including opening returns | | | | | | | | |
| 10 min | 60 528 | −0.0196 | 0.0000 | 0.2061 | 0.0000 | 0.7482 | 0.0000 | 0.9543 |
| 30 min | 20 176 | −0.0123 | 0.0742 | 0.2959 | 0.0000 | 0.4744 | 0.0000 | 0.7703 |
| 60 min | 10 088 | −0.0020 | 0.0749 | 0.0302 | 0.0000 | 0.9648 | 0.0000 | 0.9950 |
| 120 min | 5 045 | 0.0096 | 0.4578 | 0.0415 | 0.0000 | 0.9497 | 0.0000 | 0.9912 |
| 390 min | 1 613 | −0.0348 | 0.1808 | 0.0921 | 0.0000 | 0.8836 | 0.0000 | 0.9757 |
| 1 day | 1 613 | −0.0305 | 0.2581 | 0.0882 | 0.0000 | 0.8838 | 0.0000 | 0.9721 |
| Panel B: Nasdaq including opening returns | | | | | | | | |
| 10 min | 60 528 | 0.0271 | 0.0000 | 0.3371 | 0.0000 | 0.6952 | 0.0000 | 1.0323 |
| 30 min | 20 176 | 0.0134 | 0.0389 | 0.3586 | 0.0000 | 0.5233 | 0.0000 | 0.8819 |
| 60 min | 10 088 | 0.0025 | 0.3042 | 0.0284 | 0.0000 | 0.9695 | 0.0000 | 0.9979 |
| 120 min | 5 045 | 0.0283 | 0.0244 | 0.0358 | 0.0000 | 0.9612 | 0.0000 | 0.9970 |
| 390 min | 1 613 | −0.1273 | 0.0000 | 0.0936 | 0.0000 | 0.8949 | 0.0000 | 0.9885 |
| 1 day | 1 613 | 0.0014 | 0.9600 | 0.1007 | 0.0000 | 0.8888 | 0.0000 | 0.9895 |
| Panel C: DJIA excluding opening returns | | | | | | | | |
| 10 min | 58 976 | −0.0080 | 0.0583 | 0.1388 | 0.0000 | 0.8361 | 0.0000 | 0.9749 |
| 30 min | 18 624 | −0.0139 | 0.0384 | 0.0470 | 0.0000 | 0.9460 | 0.0000 | 0.9929 |
| 60 min | 9 312 | −0.0051 | 0.5705 | 0.0307 | 0.0000 | 0.9649 | 0.0000 | 0.9955 |
| 120 min | 4 656 | 0.0074 | 0.5791 | 0.0400 | 0.0000 | 0.9523 | 0.0000 | 0.9923 |
| Panel D: Nasdaq excluding opening returns | | | | | | | | |
| 10 min | 58 976 | 0.0528 | 0.0000 | 0.2024 | 0.0000 | 0.7970 | 0.0000 | 0.9995 |
| 30 min | 18 624 | 0.0298 | 0.0000 | 0.0505 | 0.0000 | 0.9474 | 0.0000 | 0.9979 |
| 60 min | 9 312 | 0.0064 | 0.4712 | 0.0320 | 0.0000 | 0.9660 | 0.0000 | 0.9981 |
| 120 min | 4 656 | 0.0312 | 0.0185 | 0.0355 | 0.0000 | 0.9619 | 0.0000 | 0.9974 |

There is only one observation in 390-min. interval per day; hence, there is no observation after excluding the opening interval in the 390-min. and daily frequency series. The AR(1)-GARCH(1, 1) model is $R_{\tau,t} = \delta_\tau + \phi_\tau R_{\tau,t-1} + \varepsilon_{\tau,t}$, $\sigma^2_{\tau,t} = \omega_\tau + \alpha_\tau \varepsilon^2_{\tau,t-1} + \beta_\tau \sigma^2_{\tau,t-1}$ where $\tau$ represents different frequencies. To maintain consistency in estimation, we set the scale of parameter at 1.5.

The evidence presented in Table 3 is quite consistent with respect to the sign and other statistical results. Specifically, AR(1) coefficients are negative for the DJIA returns and positive for the Nasdaq returns. The *p*-values suggest that these coefficients are statistically significant for the 10-min and 30-min frequencies, whether or not the data on opening returns are included in the estimations. The diverse signs of AR(1) coefficients reflect two distinct trading behaviors associated with investors involved in the DJIA and Nasdaq markets. Theory [28,29] suggests that the presence of positive feedback trading leads to negatively autocorrelated stock returns, while negative feedback trading tends to produce positively autocorrelated stock returns. Our evidence suggests that investors in the DJIA market have been dominated by the group of positive feedback traders, buying (selling) stocks after prices rise (fall), while investors in the Nasdaq market are mainly governed by a negative feedback group, buying (selling) after prices decline (rise).[10]

Another point that emerges from the empirical evidence in Table 3 is that the coefficients of the GARCH components are all highly significant, justifying the fact that stock return volatilities are characterized by a heteroskedastic process. Note that with the exception of the 30-min interval in the DJIA, $\hat{\alpha}_\tau + \hat{\beta}_\tau$ is very close to unity, indicating a high degree of persistence of volatility.

## 5. Time-varying correlation between the DJIA and Nasdaq

It is generally recognized that financial markets are highly integrated and efficient; price movements in one market are likely to spill over to another market instantaneously. Empirical evidence about stock return correlations abounds, ranging

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where $\Gamma(\cdot)$ is the gamma function and $\nu$ is a scale parameter or degree of freedom to be estimated. For $\nu = 2$, the GED yields the normal distribution, while for $\nu = 1$ it yields the Laplace or double-exponential distribution. Given initial values of $\varepsilon_{\tau,t}$ and $\sigma^2_{\tau,t}$, the parameter vector $\Theta \equiv (\delta_\tau, \phi_\tau, \omega_\tau, \alpha_\tau, \beta_\tau, \nu)$ can be estimated by the log-maximum likelihood method (log-MLE) over the sample period. The log-maximum likelihood function can be expressed as

$$L(\Theta) = \sum_{t=1}^{T} \log f(\mu_{\tau,t}, \sigma_{\tau,t}, \nu)$$

where $\mu_{\tau,t}$ is the conditional mean and $\sigma_{\tau,t}$ is the conditional standard deviation. Since the log-likelihood function is non-linear, the numerical procedure is used to derive estimates of the parameter vector.

10 As argued by Sentana and Wadhwani [28] and expounded by Antoniou et al. [29], positive feedback traders buy stocks after prices rise and sell stocks after prices fall. Shiller [20] found that a main reason that prompted investors to sell their stocks in October 1987 was that stock prices had fallen, thus inducing a fear of contagion in other investors. In contrast, the negative feedback traders sell stocks after prices increase and buy stocks after prices decline. Shiller argued that feedback models suggest that price is determined in part by its own lagged values, increases in price tending at times to foster further increases. However, as argued by Shiller, there is little, even a negative, serial correlation between price changes ([20], p. 375).

from individual stocks and mutual funds to stock indices for national markets.[11] For this reason the analysis of stock returns should not be restricted to a single market. Rather, in a general equilibrium environment, the interrelation between asset returns often carries some useful information content. One simple way to explore the relation of two asset returns is to calculate the correlation coefficient. However, a textbook type of correlation coefficient is usually assumed to be constant throughout a given window width. This approach is easy to calculate. However, it fails to capture the dynamics of financial markets, which are continuously subjected to ongoing shocks due to endogenous changes or innovations [42]. For this reason, we specify a multivariate model, which is capable of computing dynamic conditional correlation (DCC) coefficients.

Engle [44] and Cappiello et al. [45] provide a convenient dynamic system that specifies the conditional variance as:

$$H_{\tau,t} = D_{\tau,t} V_{\tau,t} D_{\tau,t}, \tag{3}$$

where $V_{\tau,t}$ is a symmetric conditional correlation matrix of $\varepsilon_t$, $D_{\tau,t} = \mathrm{diag}[\sqrt{\sigma^2_{\tau,ii,t}}]_{(2,2)}$. Eq. (3) suggests that the dynamic properties of the covariance matrix $H_{\tau,t}$ are determined by $D_{\tau,t}$ and $V_{\tau,t}$ for a given $\tau$. To remove the heteroskedasticity problem that may cause higher correlation [41], we standardize stock-return residuals via $\eta_{\tau,i,t} = \varepsilon_{\tau,i,t}/\sqrt{\sigma^2_{\tau,ii,t}}$, where $\eta_{\tau,i,t}$ is then used to estimate the parameters of the conditional correlation. The variance and covariance are assumed to be governed by Eqs. (4) and (5), respectively:

$$\sigma^2_{\tau,ii,t} = c_{\tau,i} + \alpha_{\tau,i}\varepsilon^2_{\tau,i,t-1} + \beta_{\tau,i}\sigma^2_{\tau,ii,t-1}, \quad i = 1, 2 \tag{4}$$

$$Q_{\tau,t} = (1 - \alpha_{\tau,i} - \beta_{\tau,i})\bar{Q}_\tau + \alpha_{\tau,i}\eta_{\tau,i,t-1}\eta'_{\tau,i,t-1} + \beta_{\tau,i}Q_{\tau,t-1}, \tag{5}$$

where $Q_{\tau,t} = (q_{\tau,ij,t})$ is the $2 \times 2$ time-varying covariance matrix of $\eta_{\tau,i,t}$, $\bar{Q}_\tau = E[\eta_{\tau,i,t}\eta'_{\tau,i,t}]$ is the $2 \times 2$ unconditional variance matrix of $\eta_{\tau,i,t}$, and $\alpha_{\tau,i}$ and $\beta_{\tau,t}$ are non-negative scalar parameters satisfying $(\alpha_{\tau,i} + \beta_{\tau,i}) < 1$. Since $Q_t$ does not generally have ones on the diagonal, we scale it to obtain a proper correlation matrix $V_{\tau,t}$. Thus,

$$V_{\tau,t} = (\mathrm{diag}(Q_{\tau,t}))^{-1/2} Q_{\tau,t} (\mathrm{diag}(Q_{\tau,t}))^{-1/2}, \tag{6}$$

where $(\mathrm{diag}(Q_{\tau,t}))^{-1/2} = \mathrm{diag}(1/\sqrt{q_{\tau,11,t}}, 1/\sqrt{q_{\tau,22,t}})$.

It can be shown that $V_{\tau,t}$ in Eq. (6) is a correlation matrix with ones on the diagonal and off-diagonal elements less than one in absolute value. A typical element of $V_{\tau,t}$ is in the form of:

$$\rho_{\tau,12,t} = q_{\tau,12,t}/\sqrt{q_{\tau,11,t}q_{\tau,22,t}}. \tag{7}$$

The dynamic correlation coefficient, $\rho_{\tau,12,t}$, can be obtained by using the element of $Q_{\tau,t}$ in Eq. (5).

The estimates of dynamic correlation coefficients,[12] $\rho_{\tau,12,t}$, between DJIA and Nasdaq index returns for one day, 10-min, and 30-min are shown in Fig. 5(a)–(c), respectively.[13]

Several observations are immediately apparent from these figures. First, although the correlation coefficients lie mainly in the range of 0.6 to 0.8 for most of the time, the estimated coefficients are time varying, reflecting some sort of portfolio shifting of the indices. Second, from a historical perspective, the variations of the correlations are seen to be declining, and the series appears to be more stable and displays less variance after the end of 2001. This suggests that both return series more or less conform to common factors in the post-2001 period, such as systematic risk, macroeconomic announcements, or Fed policy. This implies that the benefit of diversification by holding a combination of DJIA and Nasdaq stocks has declined in recent years. Third, correlation variations occur more frequently during downturns than upturns. This may be attributable to sector rotation between the new economy and the old economy in early 2000 or to diverse beliefs and expectations triggered by outbreaks of news. Fourth, the correlation coefficients increase their variability with frequent scales. It becomes more apparent in highly volatile periods. For example, if we look at the data between April 4, 2000, and April 12, 2000, the correlation coefficients for the daily data even display some negative values. To gain more insight into the dynamic nature

[11] A variety of papers have documented the fact that correlations across major stock markets change over time. King et al. [40] find that the covariances of stock returns change over time. Some evidence shows that correlations tend to increase during unstable periods [41]. Longin and Solnik [42] find that correlations between the major stock markets rise in periods of high volatility. Karolyi and Stulz [43] report that covariances are high, while returns on the national indices are high when "markets move a lot". All these papers are based mainly on daily data. Very few attempts have been devoted to analyzing dynamic conditional correlations in high-frequency data. Moreover, we are interested in exploring the results from varying different scales of data in the context.

[12] An alternative definition of the correlation coefficient (more precisely, cross-correlation coefficient, see Refs. [46,47]) denoted by $C_{\tau,ij,t}$ is defined as the statistical overlap of the fluctuations $\delta R_{\tau,i,t} = R_{\tau,i,t} - E\left(R_{\tau,i,t}\right)$ between the two stocks $i$ and $j$, that is, $C_{\tau,ij,t} = \frac{E\left(\delta R_{\tau,i,t}\delta R_{\tau,j,t}\right)}{\sigma_{\tau,i,t}\sigma_{\tau,j,t}}$, where $R_{\tau,t}$ is the logarithmic return, and $\sigma^2_{\tau,i,t} = E\left(\left[\delta R_{\tau,i,t}\right]^2\right)$. The average $E(\cdot)$ is over time period $T$. We are interested in exploring whether different scales of data types would cause different results in dynamic cross correlations. Based on this equation, we can perform two analyses: one with $T$ fixed to one day, and the other with $T$ fixed to a certain number of events. Using two ways (with and without deleting opening intervals) to investigate the DCC, we found that there is no difference between the two, and we do not report it here. Additional methods for measuring correlation can be found in Refs. [48,49].

[13] In our case, we fixed $T = 38$ after removing the 09:30 data point. The discontinuation of the correlation coefficients in the figures is due to missing data for the sample period from February 9, 2002, to May 9, 2002.

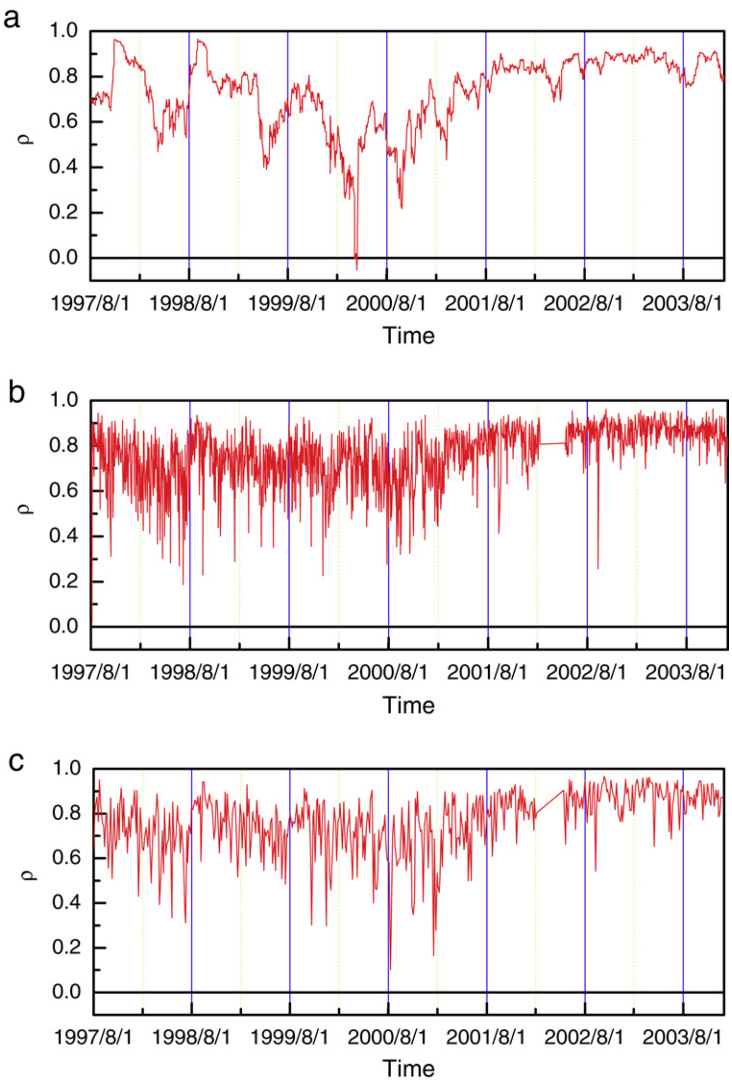


**Fig. 5.** Dynamic conditional correlation between the DJIA and the Nasdaq: (a) Daily dynamic conditional correlation between the DJIA and Nasdaq, (b) 10-min dynamic conditional correlation between the DJIA and the Nasdaq, (c) 30-min dynamic conditional correlation between the DJIA and the Nasdaq.

of these DCCs, the correlation coefficients have been fitted into a time series model, which allows the variances to evolve over time. Since plots of $\rho_{\tau,ij,t}s$ (from Fig. 5(a)–(c)) show non-stationarity, a first difference is required. Further, statistics (not reported) from autocorrelation and partial autocorrelation functions for the 10-min and 30-min series indicate that the MA(1) model appears to be a parsimonious representation. Thus, we write mean and variance equations as:

$$\Delta\rho_{\tau,ij,t} = \mu - \theta_1 \upsilon_{\tau,t-1} + \upsilon_{\tau,t}, \tag{8}$$

$$h_{\tau,\rho,t} = \omega_{\tau,0} + \omega_1 \upsilon^2_{\tau,t-1} + \omega_2 h_{\tau,\rho,t-1} \tag{9}$$

where $\mu$, $\theta_1$, and $\omega$ are parameters, and $\upsilon_{\tau,t}$ is the shock term. The variances expressed in (9) are assumed to evolve with a GARCH(1, 1) process, as popularized by Bollerslev et al. [50].

Since investment strategy, environment, and investor sentiment and psychology have displayed a distinct change since September 11, 2001, we use this date as a breakpoint based on the study by Enders and Sandler [51] to examine the DCC changes for 10-min, 30-min, and daily correlation series.[14] The results of the MA(1)-GARCH(1, 1) model are reported in Table 4 (see footnote 13). As shown in statistics of means and standard deviations, the mean values are consistently greater and have lower variances across all of the scales for the post-crisis period. This implies that both the DJIA and the Nasdaq

[14] Enders and Sandler [51] employ a Bai-Perron procedure to examine the structural changes. They find little has changed since 9/11. We choose this date to divide the data also based on its social costs and economic consequences in global markets [52]. See http://www.nato-pa.int.

**Table 4**
Time series analysis of dynamic conditional correlation coefficients at various time scales.

| Coefficient | $\Delta\rho_{\tau,t}$ (1 day) | | $\Delta\rho_{\tau,t}$ (30 min) | | $\Delta\rho_{\tau,t}$ (10 min) | |
|---|---|---|---|---|---|---|
| | Before crisis | After crisis | Before crisis | After crisis | Before crisis | After crisis |
| Panel A: Mean and standard deviation of $\rho_{\tau,t}$ | | | | | | |
| $\mu_\rho$ | 0.664 | 0.852 | 0.726 | 0.865 | 0.725 | 0.857 |
| $\sigma_\rho$ | 0.158 | 0.094 | 0.129 | 0.060 | 0.127 | 0.061 |
| Panel B: Mean equation | | | | | | |
| $C$ | −0.0005 | −8.72E−06 | −0.0002 | 0.0003 | −3.23E−05 | −2.17E−05 |
| | (0.628) | (0.018) | (0.229) | (1.869)** | (0.068) | (0.099) |
| $\theta_1$ | – | – | 0.894 | 0.975 | 0.887 | 0.929 |
| | | | (31.99)*** | (35.90)*** | (60.00)*** | (54.28)*** |
| Panel C: Variance equation | | | | | | |
| $\omega_0$ | 1.115E−05 | 2.4E−06 | 0.001 | 0.003 | 0.002 | 0.0002 |
| | (6.55)*** | (3.77)*** | (0.90) | (3.039)*** | (2.28)*** | (1.64) |
| $\omega_1$ | 0.054 | 0.115 | 0.027 | 0.363 | 0.048 | 0.024 |
| | (9.97)*** | (4.03)*** | (0.98) | (3.17)*** | (2.22)*** | (2.87)*** |
| $\omega_2$ | 0.936 | 0.738 | 0.896 | 0.076 | 0.771 | 0.909 |
| | (174.84)*** | (12.35)*** | (8.40)*** | (0.28) | (8.78)*** | (20.23)*** |
| $\bar{R}$ | 0.000 | 0.000 | 0.422 | 0.510 | 0.400 | 0.441 |
| *SSE* | 0.029 | 0.013 | 5.170 | 0.604 | 14.836 | 1.811 |
| *LB*(10) | 7.436 | 3.41 | 14.809 | 4.094 | 21.86*** | 10.203 |

a. The estimated equations are: $\Delta\rho_{\tau,t} = \mu - \theta_1 \upsilon_{\tau,t-1} + \upsilon_{\tau,t}$ and $h_{\tau,\rho,t} = \omega_{\tau,0} + \omega_1 \upsilon^2_{\tau,t-1} + \omega_2 h_{\tau,\rho,t-1}$.
b. The numbers in parentheses are t-statistics. 0.000 indicates a very small value.
c. $\Delta\rho_{\tau,t}$ (10-min) denotes change in conditional correlation coefficient for the (10-min) series, etc.
d. $\bar{R}$ is the adjusted $R$-squared, *SSE* is the sum of squared errors, *LB* (10) is the Ljung-Box statistics testing for autocorrelation up to the 10th lag. The critical values of the chi-squared distribution for the 10%, 5%, and 1% levels are 16.0, 18.3, and 23.2, respectively.
*** Indicate statistical significance at the 1% level.
** Indicate statistical significance at the 5% level.

indices have been commonly driven by certain market forces in a relatively stable fashion. The variations are still subject to macroeconomic news, announcements, and dynamic social/political factors. Interestingly, the mean equation of $\Delta\rho_{\tau,ij,t}$ for intraday daily data consistently reveals an MA(1) pattern; no particular pattern is shown on the coefficient of the daily series. It is generally recognized that an MA(1) process is equivalent to AR($\infty$), meaning that the $\Delta\rho_{\tau,ij,t}$ is highly correlated in the high-frequency data. The correlation coefficients exhibit even higher values in the post-2001 crisis period. Although the pattern is rather stable, the message derived from the GARCH coefficients indicates that the correlation coefficients are time varying. By comparing values of the adjusted $R$-squared ($\bar{R}$) and the sum of squared errors (*SSE*), we find that the explanatory power increased and the *SSE* decreased after the crisis. Note that the post-crisis period coincides with a persistent downturn in production in the US economy [52].

It is of interesting to note that the evidence in this section is consistent with the theoretical results from phase correlations between DJIA and Nasdaq. Wu et al. [53] found that the distributions of phase differences between DJIA and Nasdaq show an impressive change of phase correlation after the events of 911, 2001, and the scenario persisted in later trading activities. The phenomenon has been attributable to speedy communications and a greater sensitivity to investors' psychology and to socio-political events after the 911 shock to stock markets.

## 6. Scaling analysis

To gain more insight into understanding the collective behaviors revealed by activities in stock markets, we perform scaling analysis on the DJIA and the Nasdaq indices at different scales. To elucidate, let us define the probability distribution $P$ as a normalized distribution (i.e., the total probability is equal to one) of a measure $Z$, which satisfies

$$\int_{-\infty}^{\infty} P(Z_{\tau,t}) \mathrm{d}Z_{\tau,t} = 1 \tag{10}$$

where $Z_{\tau,t}$ is the measure of stock return, and $\tau$ (=10-min, 20-min, ...etc.) is a multiple of the primary time sampling unit $\Delta t$. Fig. 6(a) and (b) depict the probability distributions $P(Z_{\tau,t})$ of the intraday frequencies for both DJIA and Nasdaq return changes $Z_{\tau,t}$ observed at five different time intervals $\tau$, ranging from 10 to 1950-min, in which the opening intervals have been included. These distributions are scale-dependent, and the shorter the time interval, the narrower the width of the distribution. It has been reported that a properly normalized version of return can have its probability distribution behave as a rescaled-like distribution, such that probability distributions of normalized returns for different time scales can converge into a single curve [15,53]. The probability distribution of the normalized return can be described well by the double-exponential distribution at not-too-long $t$ [19]. The double-exponential distribution of return at not-too-long times $t$ is a universal, ubiquitous feature of financial time series and was observed for different countries, stock-market

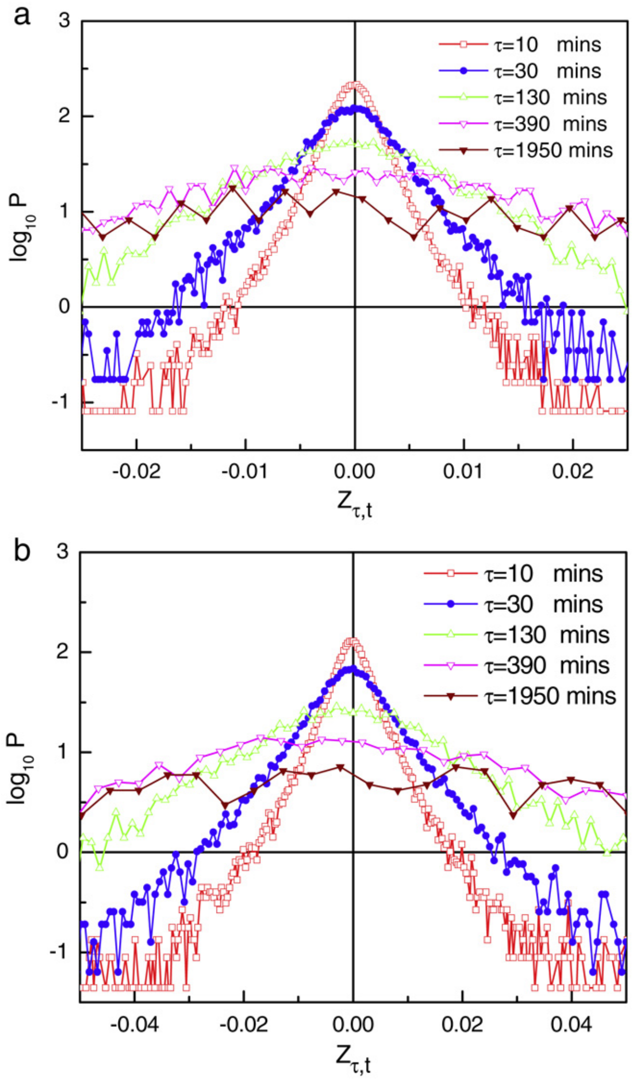


**Fig. 6.** Probability distributions of return changes of (a) DJIA, and (b) Nasdaq for intraday data with time sampling intervals of multiples of 10-min.

indices, and individual stocks. According to Ref. [19], the central part of the curves can be fitted by the scaling form using a Bessel function, where 99% of probability resides and statistics are good, followed by power laws in the far tails, where data statistics are often poor. These features can be modeled by the Heston model with stochastic volatility [54].

As suggested by Mantegna and Stanley [13,14], it is possible to map probability distributions with different time sampling scales into a single curve by performing scaling analysis. We plot $P(Z_{\tau,t} = 0)$ of two indices against the time sampling intervals $\tau$ as shown in Fig. 7(a). Within a truncated time scale, the distributions of $P(Z_{\tau,t} = 0)$ in relation to $\tau$ plotted in logarithmic scale are linear [14]; the best-fitting straight lines (also plotted in Fig. 7(a)) obey the following [53]:

$$\log_{10} P(Z_{\tau,t} = 0) = c - \frac{1}{\alpha} \log_{10} \tau, \tag{11}$$

where $c$ is a constant, and $\alpha$ is a quantity characterizing the class of distribution. It has been shown that the $\alpha$ value in Eq. (11) can be used to determine the stability of a distribution, which, in turn, enables us to determine the stability of the process under consideration (see Ref. [14]). By measuring the slope of the fitting straight line, we obtain $\alpha \approx 1.84$ for the DJIA and $\alpha \approx 1.75$ for the Nasdaq; both of the $\alpha$ values are greater than 1.4 [13] and less than or equal to 2 ($\alpha \leq 2$), satisfying the condition for stable Lévy distributions [15].

Note that the stable non-Gaussian type of the probability distributions is a stochastic process with infinite variance characterized by distributions with power-law tails. Power-law distributions also imply a lack of a characteristic scale. We then rescale the probability distribution function $P(Z_{\tau,t})$ and return changes $Z_{\tau,t}$ as suggested by Mantegna and Stanley [13].

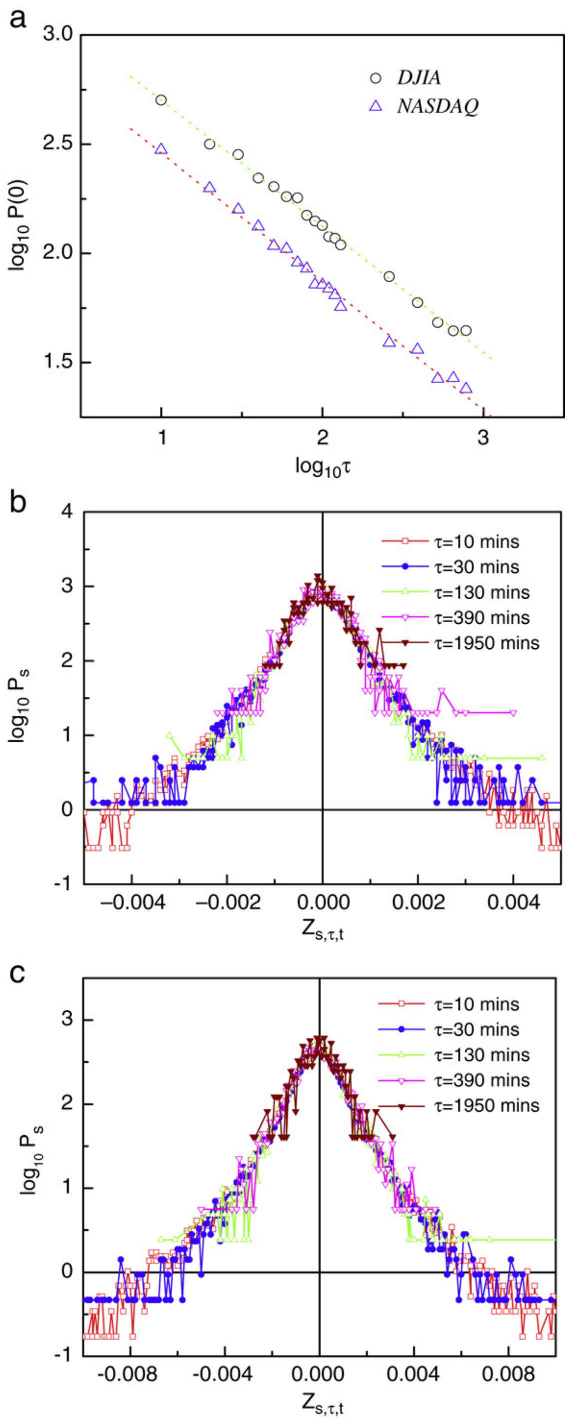


**Fig. 7.** (a) Probability of return variation $P(Z_{\tau,t} = 0)$ as a function of the time sampling intervals $\tau$. The slope of the best-fit straight line is $-0.54 \pm 0.01$ for the DJIA, and $-0.57 \pm 0.01$ for the Nasdaq. Scaled plot of the probability distributions with (b) $\alpha = 1.84$ for the DJIA, and (c) $\alpha = 1.75$ for the Nasdaq.

It follows

$$Z_{s,\tau,t} = \frac{Z_{\tau,t}}{\tau^{1/\alpha}}, \qquad P_s\left(Z_{s,\tau,t}\right) = \frac{P(Z_{\tau,t})}{\tau^{-1/\alpha}} \tag{12}$$

where the subscript $s$ is used to denote scaled quantities. Fig. 7(b) and (c) show the scaled plots of the probability distributions with $\alpha = 1.84$ for the DJIA and $\alpha = 1.75$ for the Nasdaq. Apparently, probability distributions of time scales can coincide with each other very well.

We can now argue that for a stock market showing that the collective behavior of a large number of investors, either DJIA (with $\alpha = 1.84$) or Nasdaq (with $\alpha = 1.75$) can converge to a stable state by itself after a shock, without relying on external intervention. This also implies that stock markets such as the DJIA and the Nasdaq are considered to have mature, self-governing capacity for maintaining their own stability. Meanwhile, the scaling analysis indicates that the DJIA, with a slightly higher $\alpha$ value than that of the Nasdaq, is equipped with a relatively stronger self-governing mechanism.

The same analysis has also been performed by excluding the opening interval of the 9:30 data, with the results of $\alpha = 1.88$ for DJIA and $\alpha = 1.85$ for Nasdaq. According to stability analysis of probability distribution [14], a value of $1 \le \alpha \le 2$ implies a stable distribution, and $\alpha = 2$ corresponds to the Gaussian distribution. System with a $\alpha$ value closer to 2 may exhibit a distribution with rarer large-fluctuation. From this perspective, opening intervals are likely to perturb a distribution to have more larger fluctuation and lead to smaller values of $\alpha$. This evidence can be observed from Fig. 4(a), in which the average of the opening interval has larger return.

## 7. Conclusions and discussion

In this paper, we investigate the statistical properties of high-frequency data on stock returns. We find that both Nasdaq and DJIA have excessively high returns at opening intervals, although the Nasdaq, on average, has a higher return than DJIA. The higher returns in Nasdaq are associated with higher volatilities across all the intervals in each trading day. Our evidence also shows that the high-frequency-return variances for a given scale produce a U-shaped curve, and the curvature is increasing with the increasing scales.

Our AR(1)-GARCH(1, 1) result shows that for both 10-min and 30-min return horizons, DJIA returns are negatively autocorrelated, whereas Nasdaq are positively autocorrelated. These results imply that investors in DJIA follow a positive feedback strategy, whereas a negative feedback strategy in Nasdaq.

By examining the dynamic correlation coefficients between DJIA and Nasdaq returns over time, a series of positive correlations are found and fluctuate mainly ranging from 0.6 to 0.8. The statistics show that the correlation coefficients are time varying, reflecting some sort of dynamic portfolio allocations among different financial assets. By inspecting the time series path of conditional correlation coefficients, we find that the variations of the coefficients are declining and appear to be more stable over the post-2001 period. This suggests that both markets are driven by some common factors, such as systematic risk, macroeconomic announcements, Fed policy, or investor psychology etc. This also implies that the benefit of diversifying by holding a portfolio of DJIA and Nasdaq stocks declined.

By checking the conditions of a stable Lévy distribution, we find that both the DJIA and the Nasdaq can converge to stable equilibrium after the shocks without needing external intervention. This implies that both markets are characterizing with self-governing mechanisms, especially the DJIA market.

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