Fluctuation-Induced Perpetual Motion in Nonequilibrium Systems

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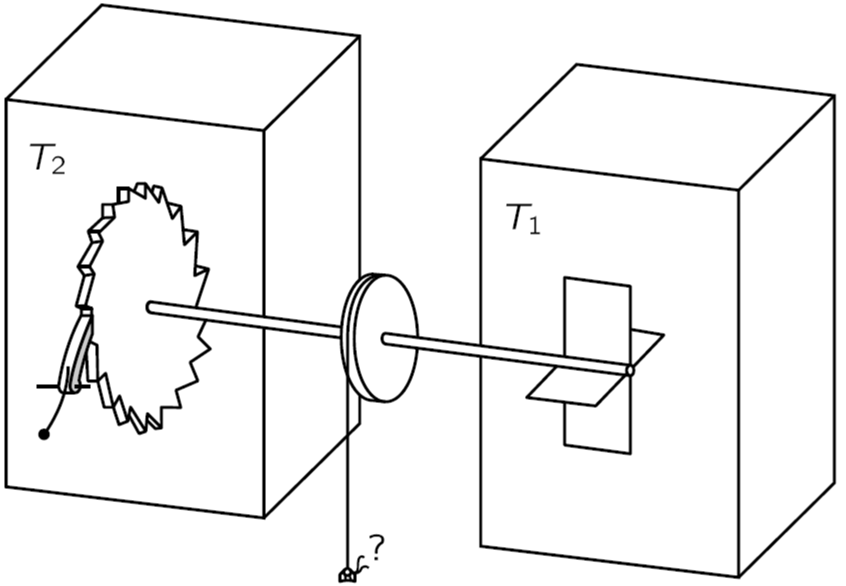


學士班演講, October 3rd, 2017

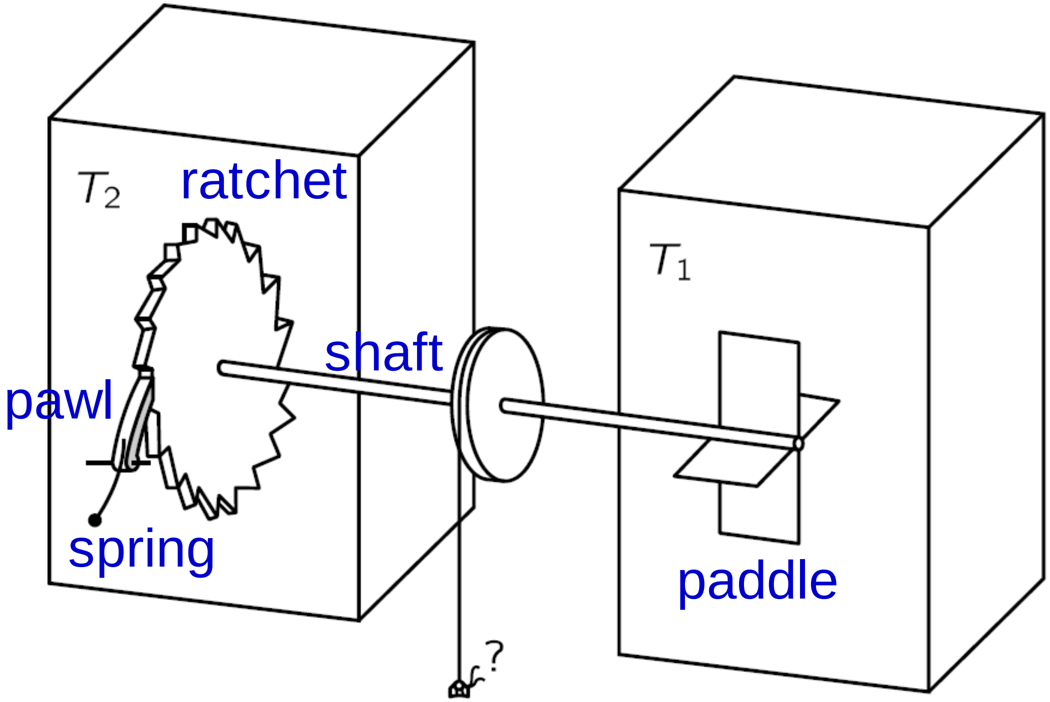
Feynman Ratchet

Small device: collisions from irregular motion of molecules have effects

Asymmetric: asymmetric toothed wheel (ratchet) with pawl

→ Irreversibility: Directional, perpetual motion agitated by Brownian motion

Nonequilibrium ($T_1 \neq T_2$) is required

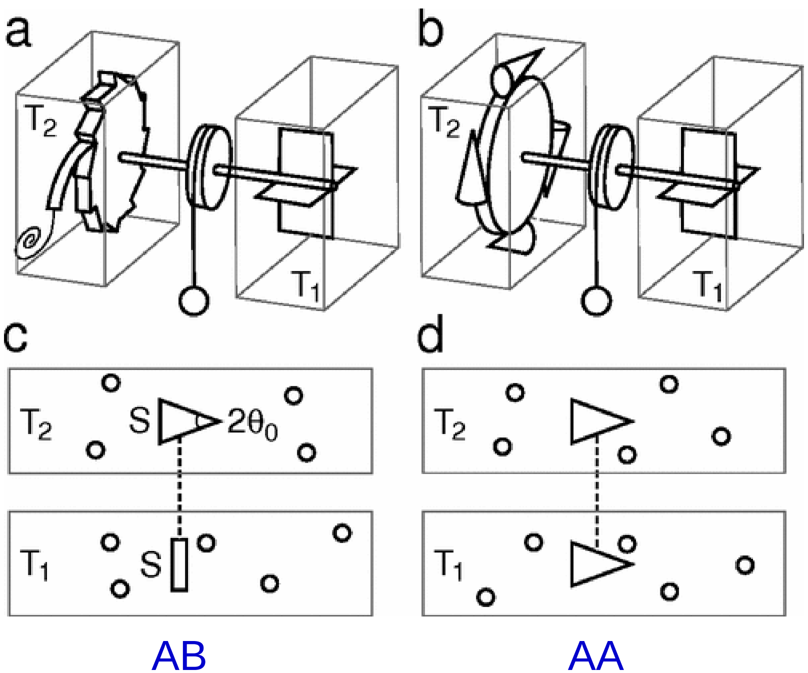


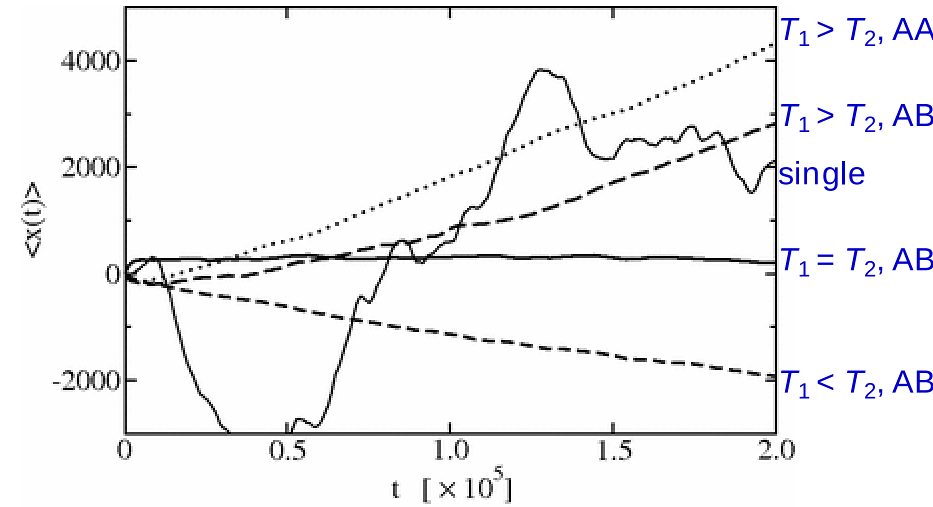
The Feynman Lectures on Physics, Vol I

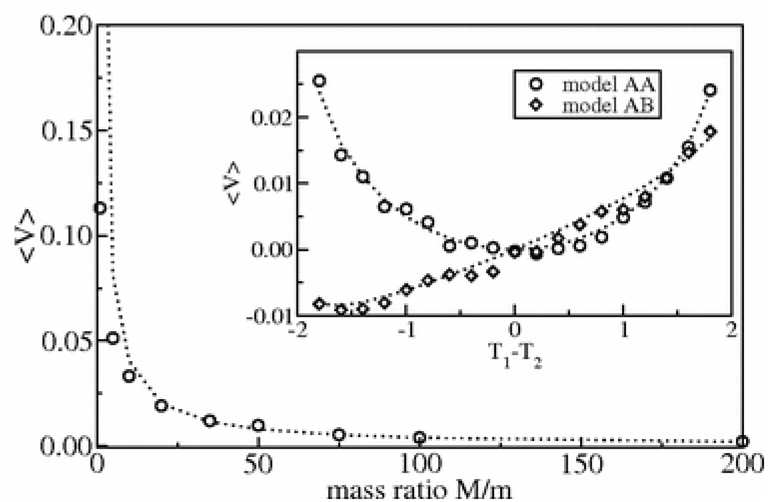
Molecular Simulation of Feynman Ratchet

Two requirements for directional motion

- coupled asymmetric particles
- $T_1 \neq T_2$





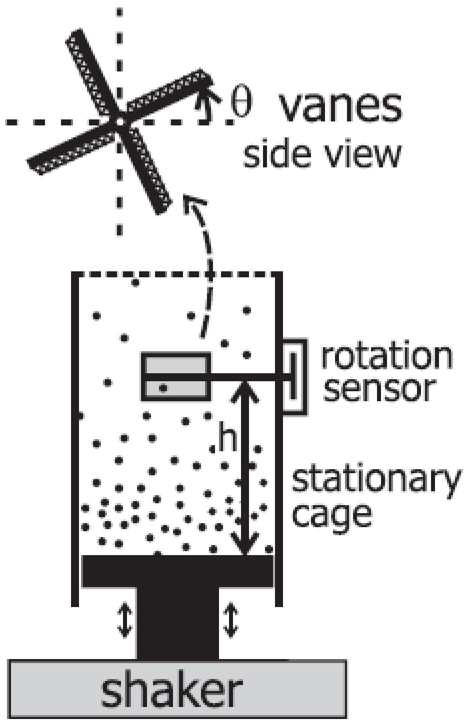


C. Van den Broeck *et al.*, *PRL* **93**, 090601 (2004)

Experimental Realization

Macroscopic Feynman ratchet

- *T* at vanes (made by a granular gas) >> *T* at pawl (room temperature)



P. Eshuis *et al.*, *PRL* **104**, 248001 (2010)

Basis of Brownian Motion

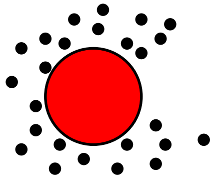
Eq. of motion: microscopic object bombarded by irregular motion of molecules

$$m\ddot{x} = -\gamma\dot{x} + \xi$$

damping random

physics is simplified by Langevin eq.

the same microscopic origin

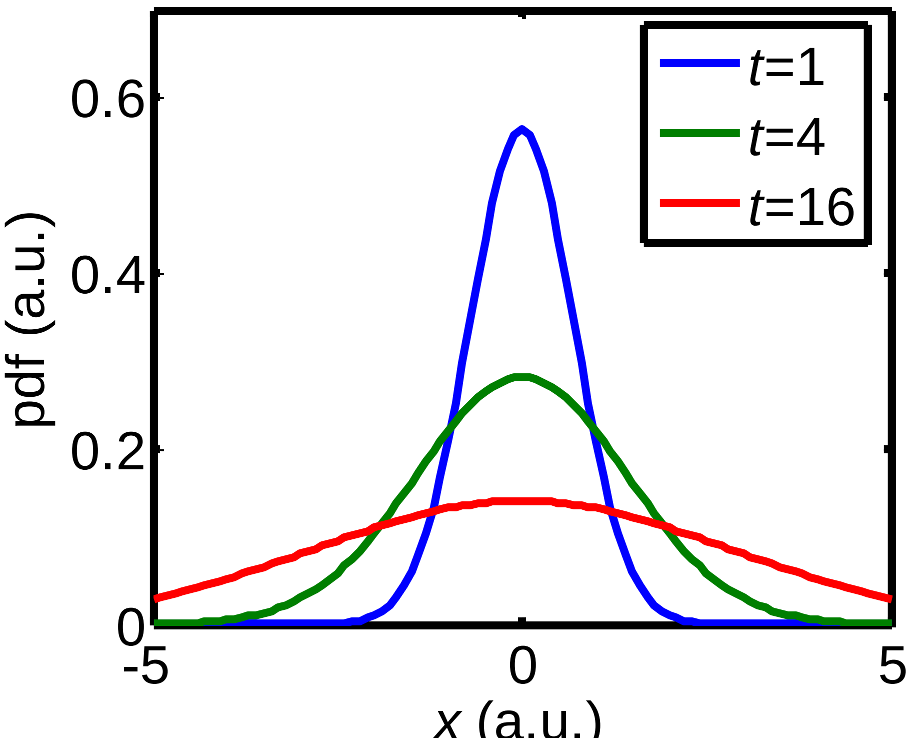


Characteristic of random (thermal) force

$$\langle \xi(t) \rangle = 0$$

$$\langle \xi(t)\xi(t') \rangle = 2k_{\mathrm{B}}T\gamma\delta(t-t')$$

Fluctuation-dissipation theorem (FDT)



Diffusion behaviors

$$\langle x \rangle = 0$$

$$\langle x^2 \rangle = \frac{2k_{\mathrm{B}}T}{\gamma} t$$

A Modern Course in Statistical Physics by L. E. Reichl

Trapped Brownian Particle

1D Brownian motion with harmonic confinement

$$m\ddot{x} = -\gamma\dot{x} - kx + \xi$$

$$\left\langle \xi(t)\xi(t') \right\rangle = 2k_{\mathrm{B}}T\gamma\delta(t-t')$$

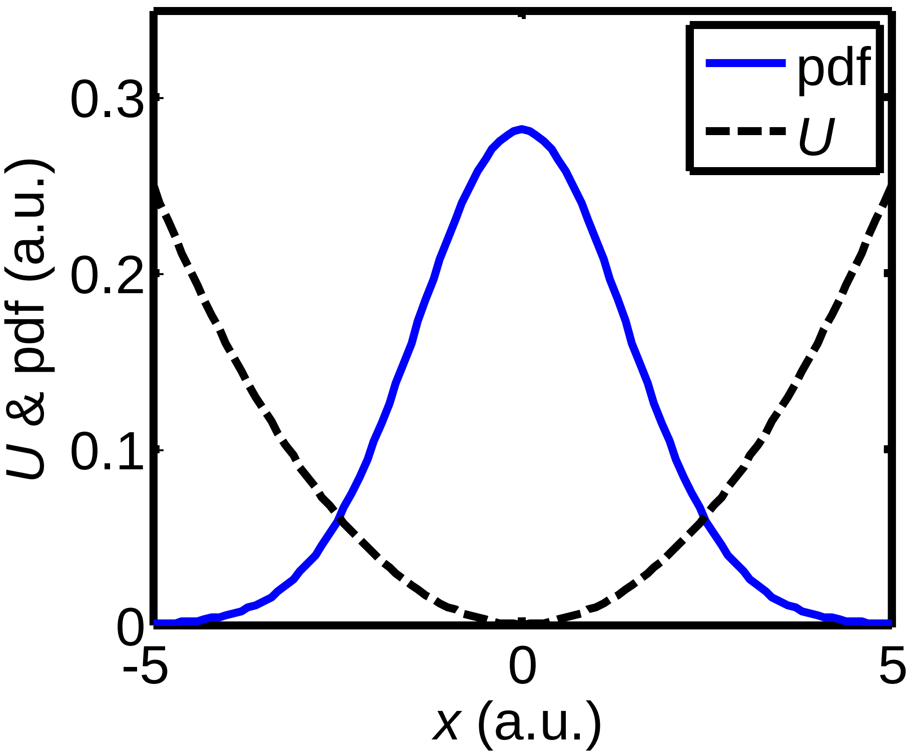
Overdamped system

$$-\gamma\dot{x} - kx + \xi = 0$$

Equilibrium distribution

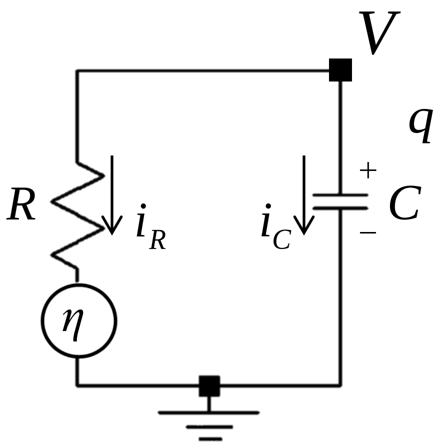
$$P_{\mathrm{eq}}(x) \sim \exp\left(-\frac{\frac{1}{2}kx^2}{k_{\mathrm{B}}T}\right)$$

Behaviors of diffusion vs. drift



A Modern Course in Statistical Physics by L. E. Reichl

Thermal Voltage Noise in RC Circuit



$$q = CV$$

$$i_C + i_R = 0$$

$$C\dot{V} + \frac{V - \eta}{R} = 0$$

Observable: *V*(*t*)

Eq. of motion

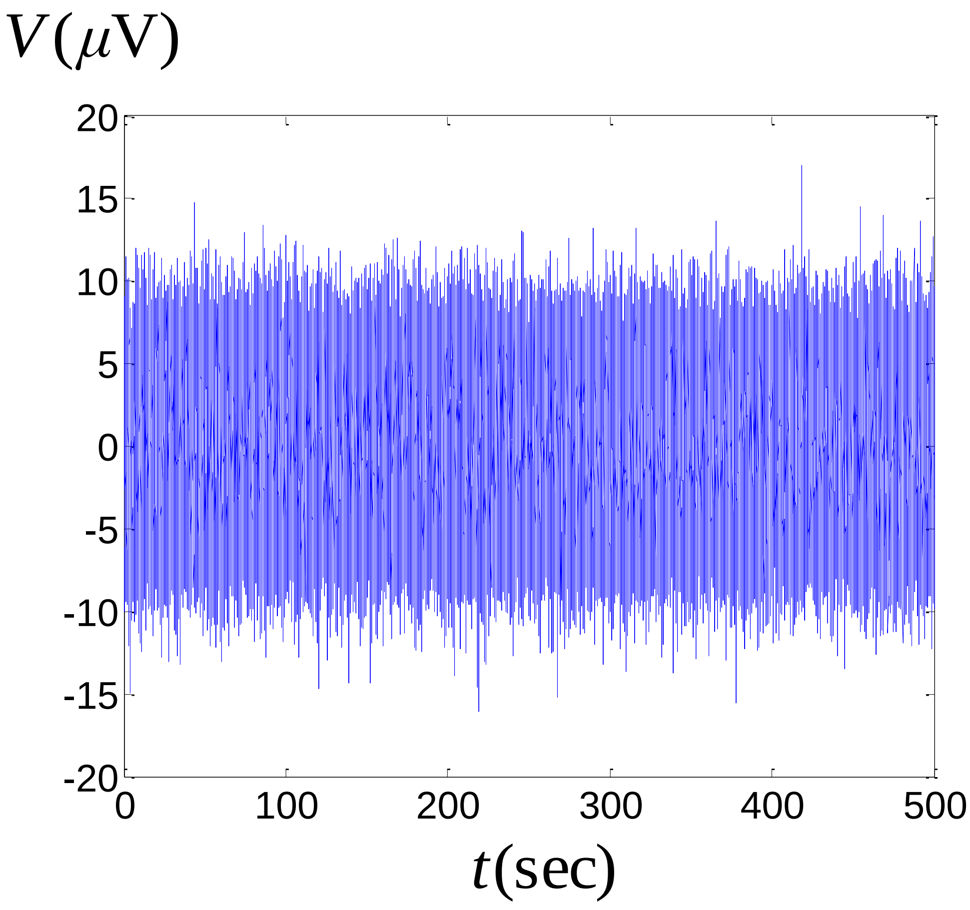
$$RC\dot{V} + V - \eta = 0$$

Johnson-Nyquist noise in conductor

$$\langle \eta(t) \rangle = 0$$

$$\langle \eta(t)\eta(t') \rangle = 2k_\mathrm{B}TR\delta(t - t')$$

white (temporal uncorrelated)

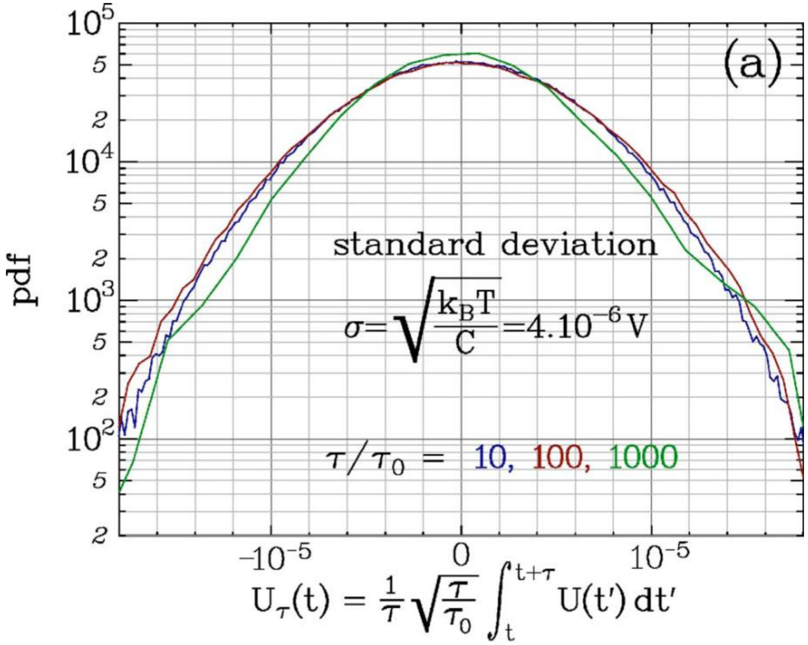


J. B. Johnson, *Phys. Rev.* **32**, 97 (1928)

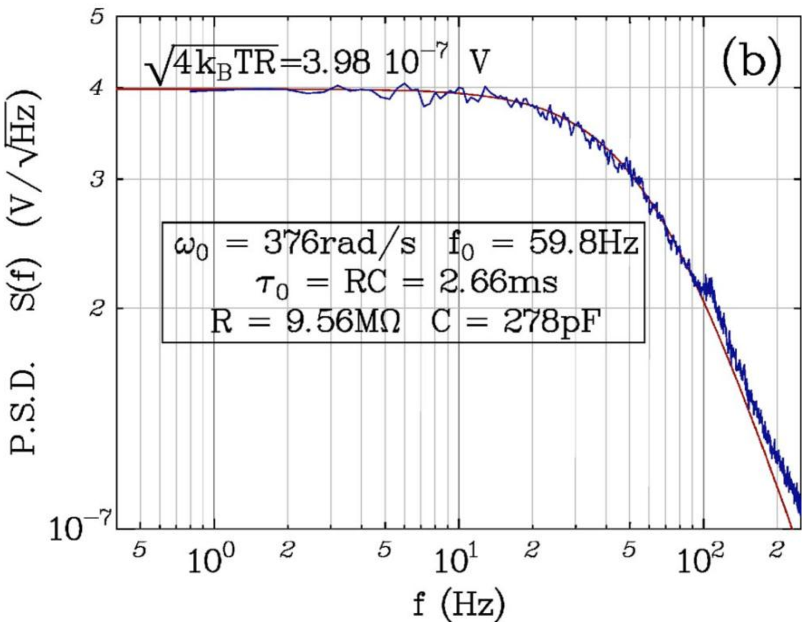
H. Nyquist, *Phys. Rev.* **32**, 110 (1928)

Basic Characterizations in Equilibrium

Histogram of voltage (noise)



Power spectral density



Gaussian

$$P_{\rm eq}(V) \sim \exp\left(-\frac{\frac{1}{2}CV^2}{k_{\rm B}T}\right)$$

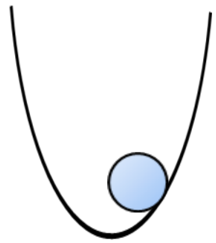
Lorentzian

$$S_V(\omega) = 4k_{\rm B}TR\frac{1}{1+R^2C^2\omega^2}$$

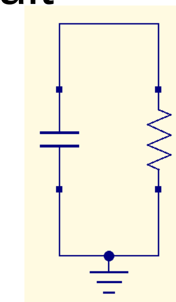
N. Garnier *et al.*, *PRE* **71**, 060101(R) (2005)

Analogies and Comparisons

Trapped Brownian particle



RC circuit



- Equation of motion (Langevin equation)

$$m\ddot{x} = -\gamma\dot{x} - kx + \xi$$

$$L\ddot{q} = -R\dot{q} - \frac{1}{C}q + \eta$$

- Random (thermal) force (satisfy FDT)

$$\langle \xi(t)\xi(t') \rangle = 2k_{\mathrm{B}}T\gamma\delta(t-t')$$

$$\langle \eta(t)\eta(t') \rangle = 2k_{\mathrm{B}}TR\delta(t-t')$$

- Overdamped system

$$\gamma\dot{x} = -kx + \xi$$

$$R\dot{q} = -\frac{1}{C}q + \eta$$

- Experimental observable

$$x(t)$$

$$V(t)$$

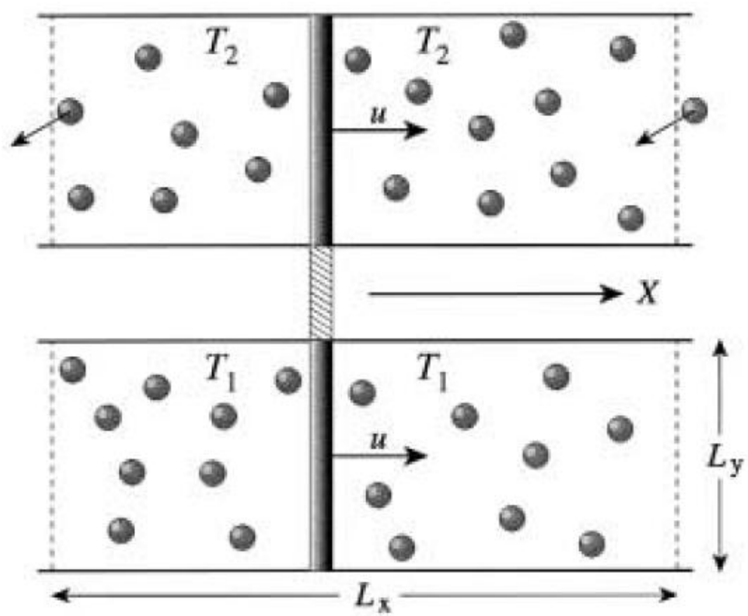
- Advantage

Easy to couple!

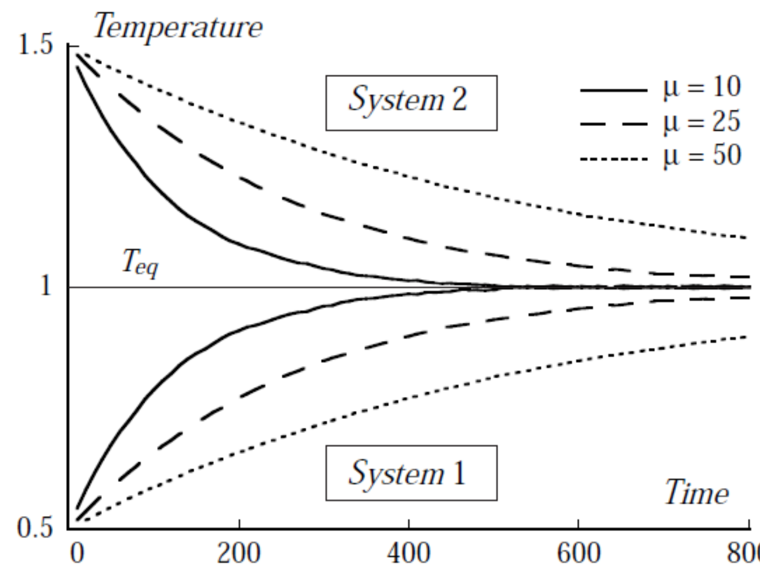
R. Van Zon *et al.*, *PRL* **92**, 130601 (2004)

Single Piston Affected by Two Heat Baths

A tiny piston contact to two heat baths at different temperatures (nonequilibrium)



Relaxation of temperature difference:



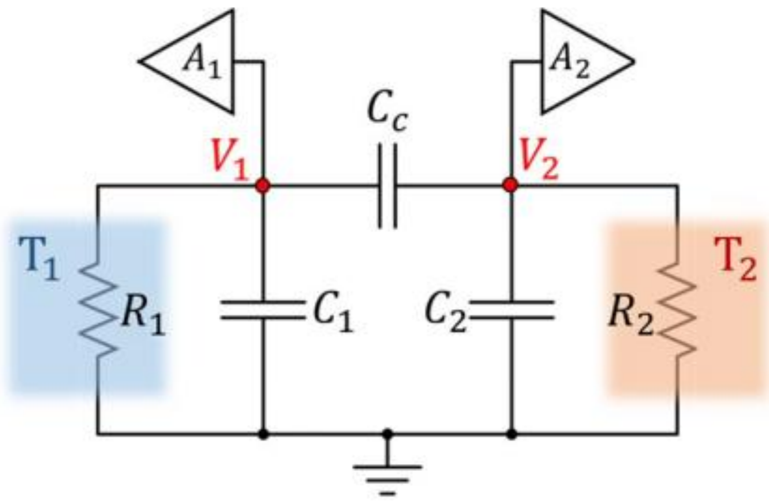
Heat conductivity:

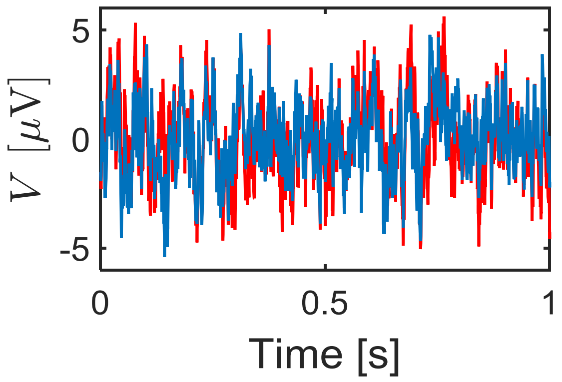
$$\left\langle \dot{Q}_{1\to 2} \right\rangle = \frac{k_B}{M}\frac{\gamma_1\gamma_2}{\gamma_1+\gamma_2}(T_1 - T_2)$$

C. Van der Broeck *et al.*, *Europhys. Lett.* **56**, 771 (2001)

Coupled RC Circuit

- Two RC circuits coupled by a capacitor
- Two resistors in contact with different thermal baths: nonequilibrium steady state (NESS)
- Observables: $V_1(t)$ and $V_2(t)$





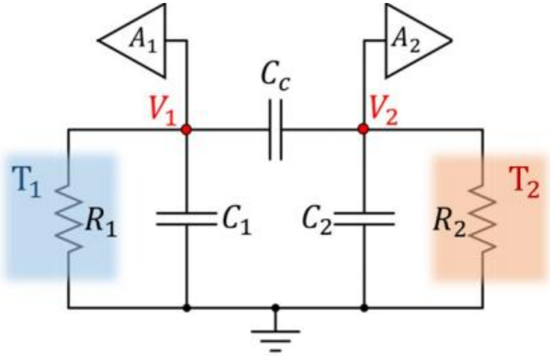
S. Ciliberto *et al.*, *PRL* **110**, 180601 (2013)

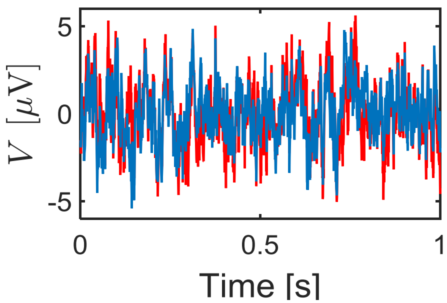
Experimental Setup

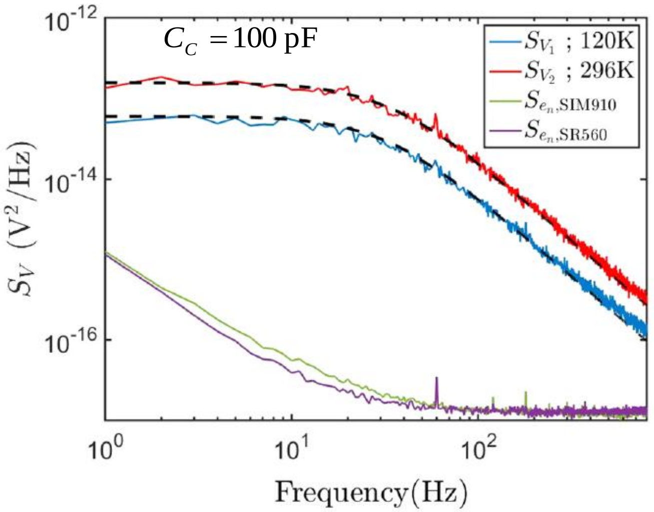
- Parameters

$$R_1 = 9.01\,\mathrm{M\Omega},\ C_1 = 488\ \mathrm{pF}$$
$$R_2 = 9.51\,\mathrm{M\Omega},\ C_2 = 420\ \mathrm{pF}$$
$$T_2 = 296\ \mathrm{K}$$
$$T_1\text{: vary from } 120\ \mathrm{K} \text{ to } 296\ \mathrm{K}$$
$$C_C\text{: vary from } 100\ \mathrm{pF} \text{ to } 10\ \mathrm{nF}$$

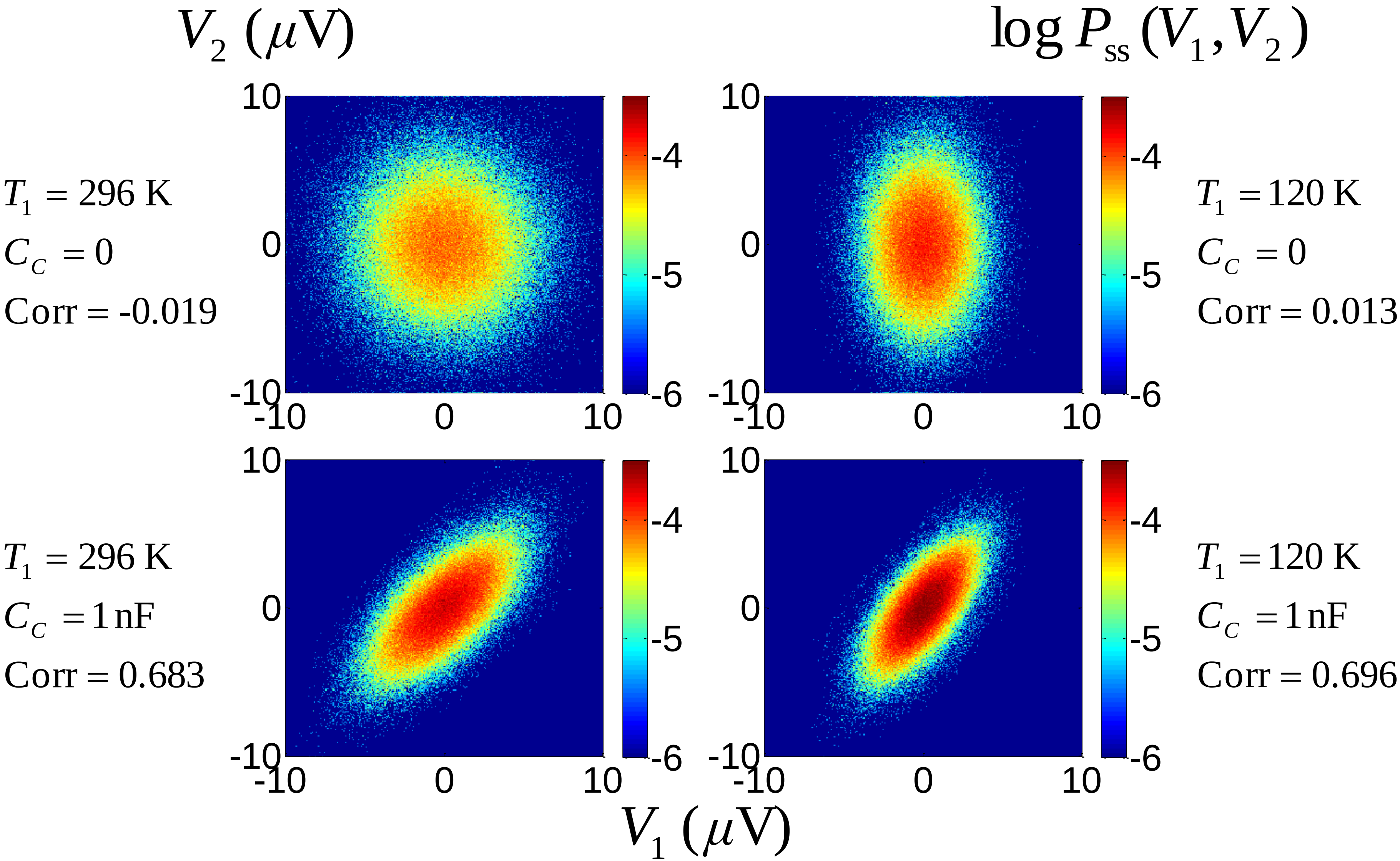
- Measurement of voltage time traces
  - Amplification: 10^4 (cf. noise amplitude of few μV)
  - Sampling rate: 2048 Hz (cf. correlation time of few ms)
  - Samples: 10^5–10^6





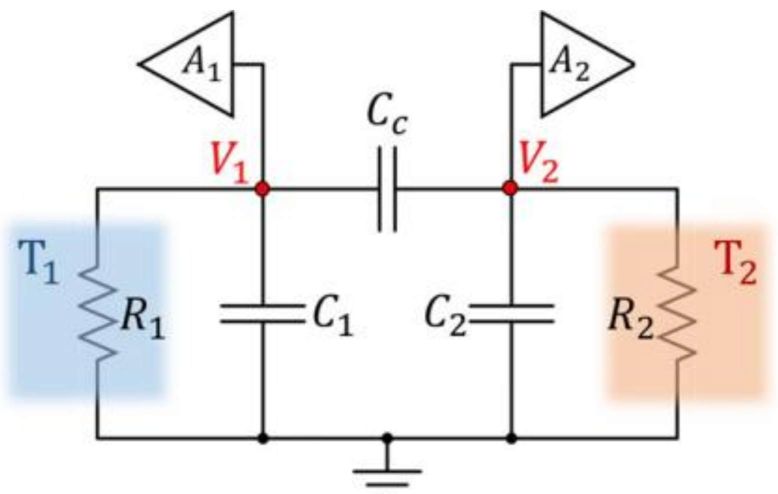


Distribution of Voltages

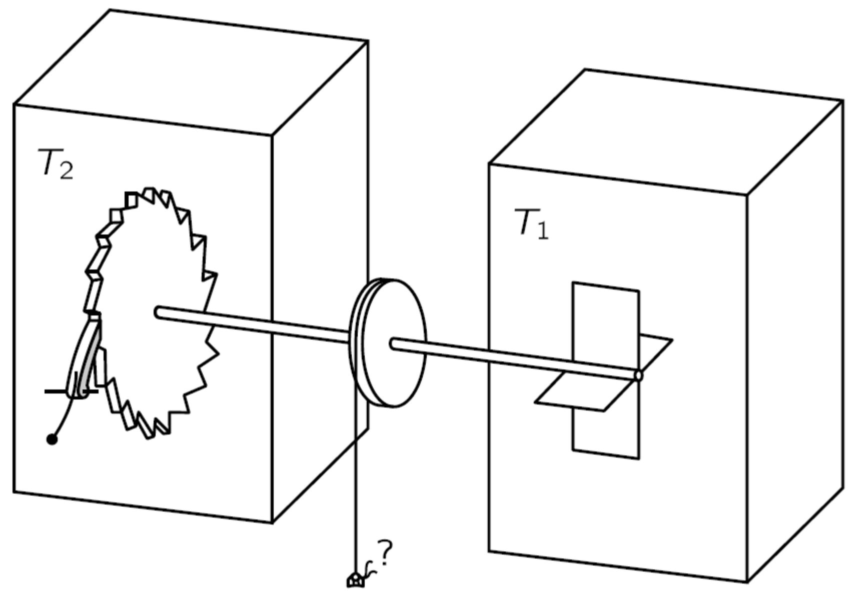


Similarity between Coupled RC Circuit and Feynman Ratchet

Coupled RC circuit



Feynman ratchet



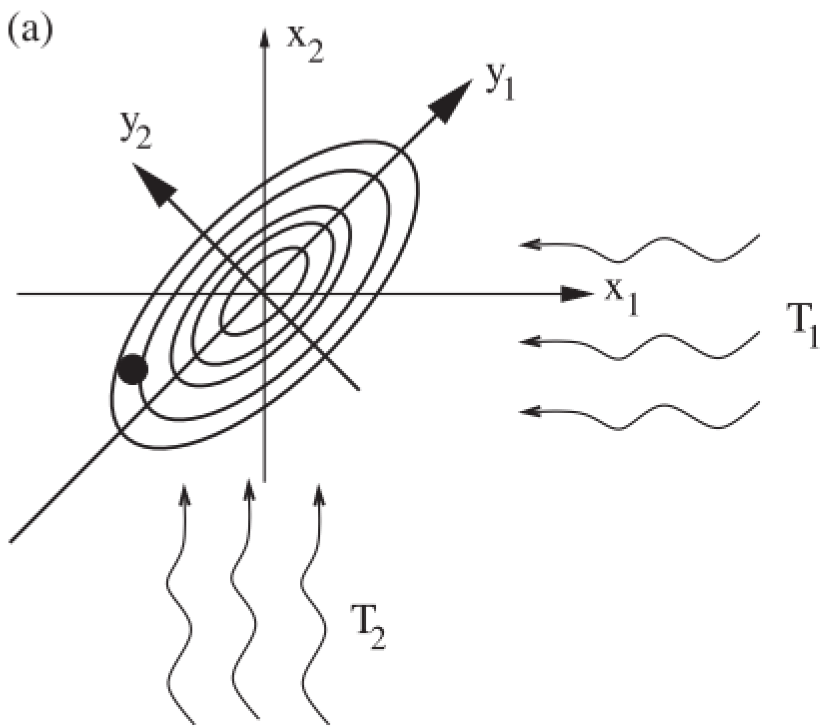
Brownian Gyrator

A Brownian particle confined in 2D harmonic potential

Two requirements for gyrating motion

- asymmetric harmonic potential (different stiffness along y_1 and y_2)
- agitated by T_1 and T_2 along x_1 and x_2, respectively ($T_1 \neq T_2$)

$$U(\vec{x}) = \sum_{i=1}^{2} \frac{u_i}{2} y_i^2$$



Eq. of motion (analytically solvable)

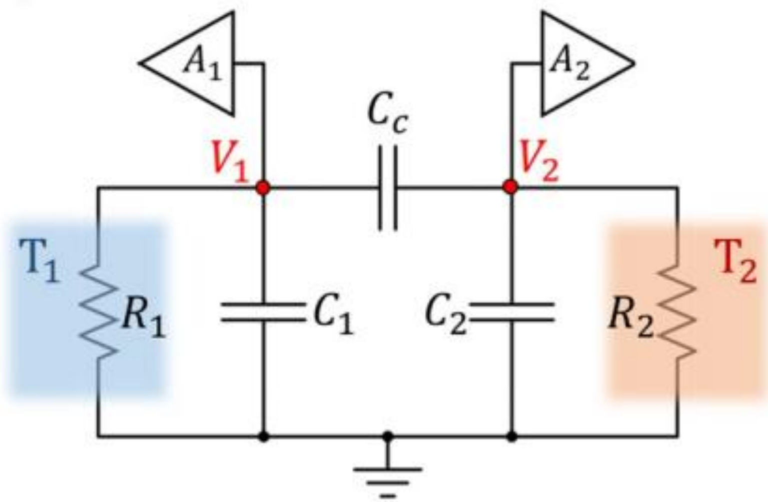
$$\gamma_i \dot{x}_i = -\frac{\partial U(\vec{x})}{\partial x_i} + \xi_i$$

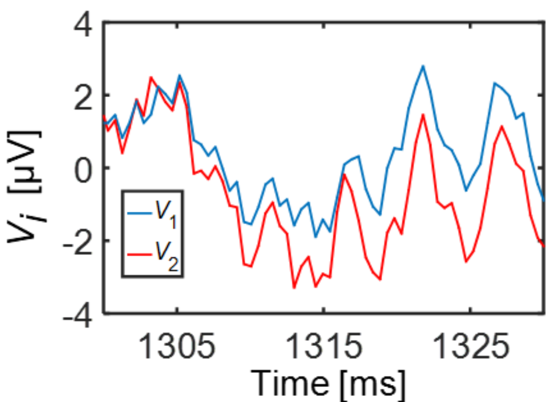
$$\left\langle \xi_i(t) \xi_j(t') \right\rangle = 2 k_{\mathrm{B}} T_i \gamma_i \delta_{ij} \delta(t - t')$$

R. Filliger *et al.*, *PRL* **99**, 230602 (2007)

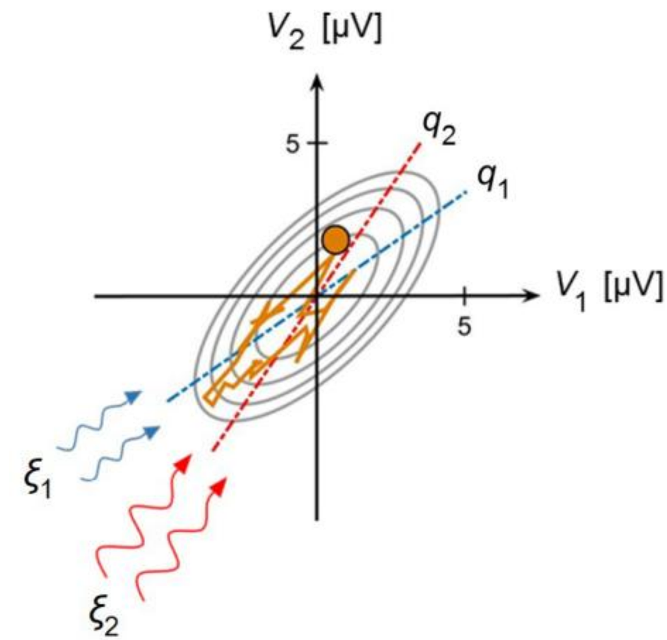
Mapping

Fluctuating voltages in coupled RC circuit





A virtual Brownian particle moving in 2D space (V_1, V_2)

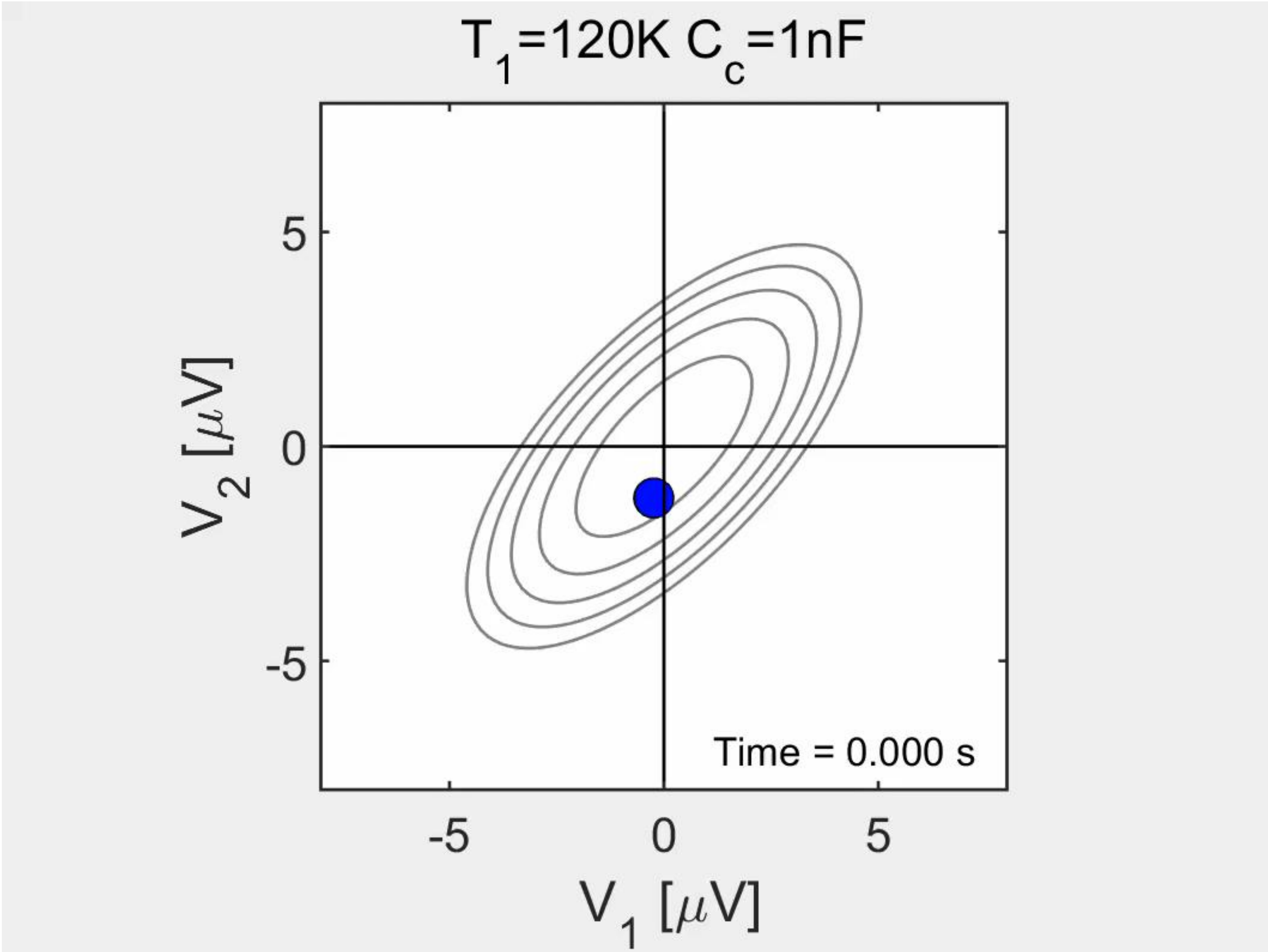


Confined potential:

$$U(V_1,V_2) = \frac{C_1}{2}V_1^2 + \frac{C_2}{2}V_2^2 + \frac{C_C}{2}(V_1 - V_2)^2$$

K.-H. Chiang *et al.*, *PRE* **96**, 032123 (2017)

Visualization of Brownian Gyrating Motion

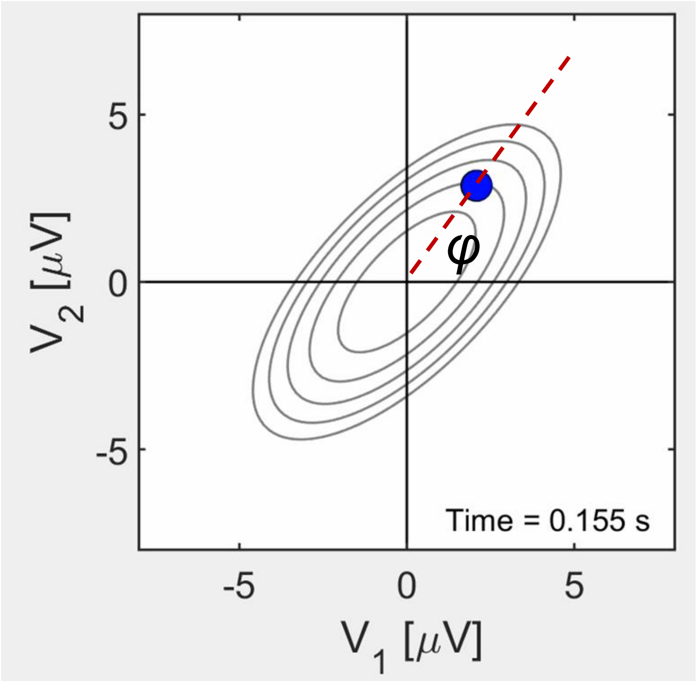


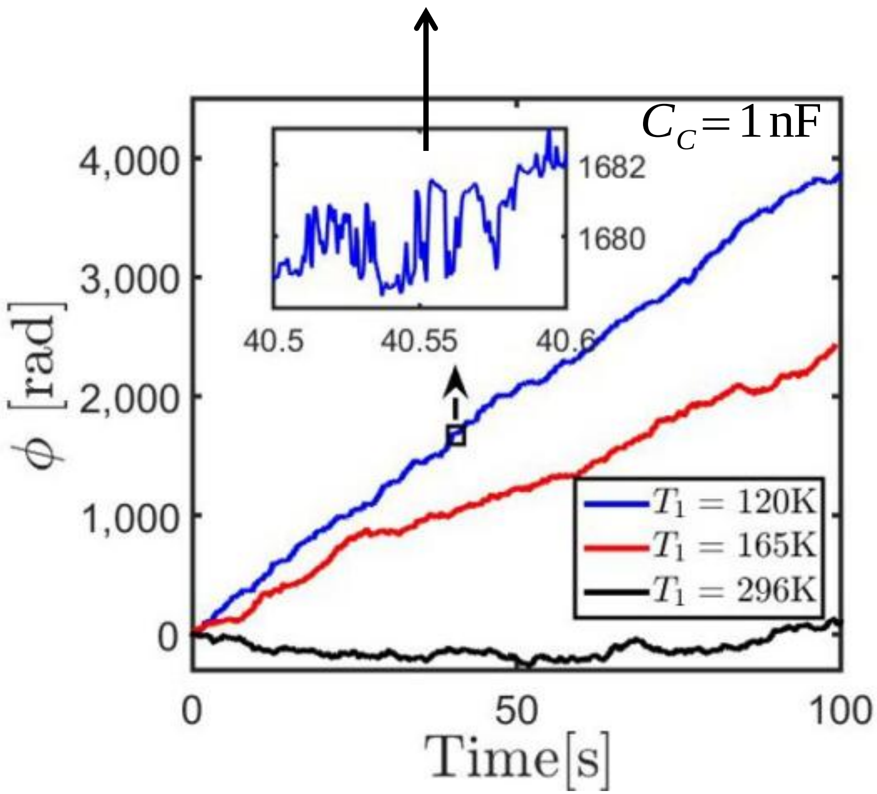
Steady Circulation Motion

Angle of position vector

$$\varphi \equiv \tan^{-1}\frac{V_2}{V_1}$$

Fluctuating behavior





Visualization of Steady Circulation Motion

Probability flux

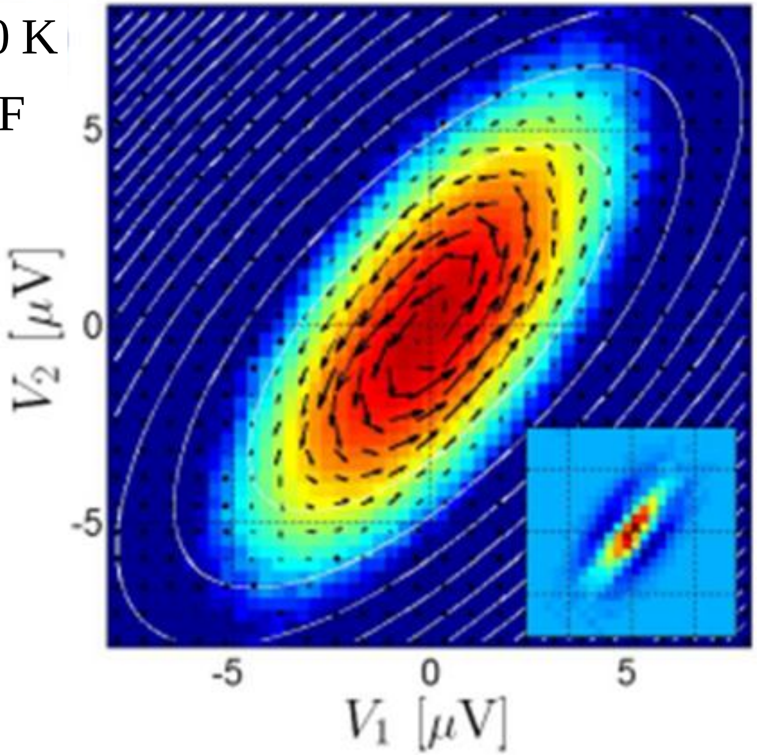
$$\vec{J}_{\rm ss}(\vec{V}) \equiv \frac{1}{2}\sum_{\vec{V}'}\left[\frac{\vec{V}'-\vec{V}}{\Delta t}P_{\Delta t}(\vec{V}',\vec{V})+\frac{\vec{V}-\vec{V}'}{\Delta t}P_{\Delta t}(\vec{V},\vec{V}')\right] \equiv P_{\rm ss}(\vec{V})\vec{v}_{\rm flow}(\vec{V})$$

out flux ↑ (first term) in flux ↑ (second term)

U. Seifert, *Rep. Prog. Phys.* **75**, 126001 (2012)

$T_1 = 120$ K

$C_C = 1$ nF



conservation of prob. density

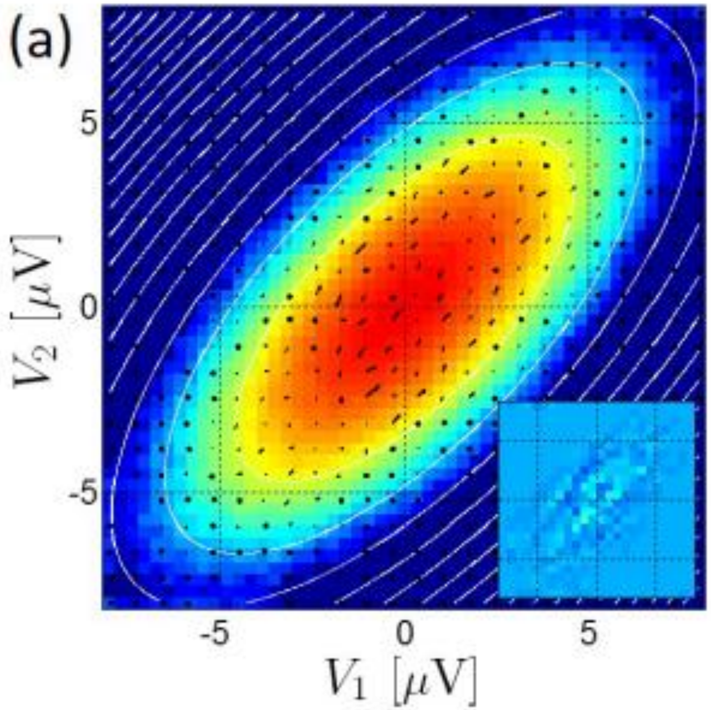
$$\frac{dP_{\rm ss}}{dt} = \nabla P_{\rm ss}\cdot\vec{v}_{\rm flow} + \frac{\partial P_{\rm ss}}{\partial t} = 0$$

$$\frac{\partial P_{\rm ss}}{\partial t} = 0$$

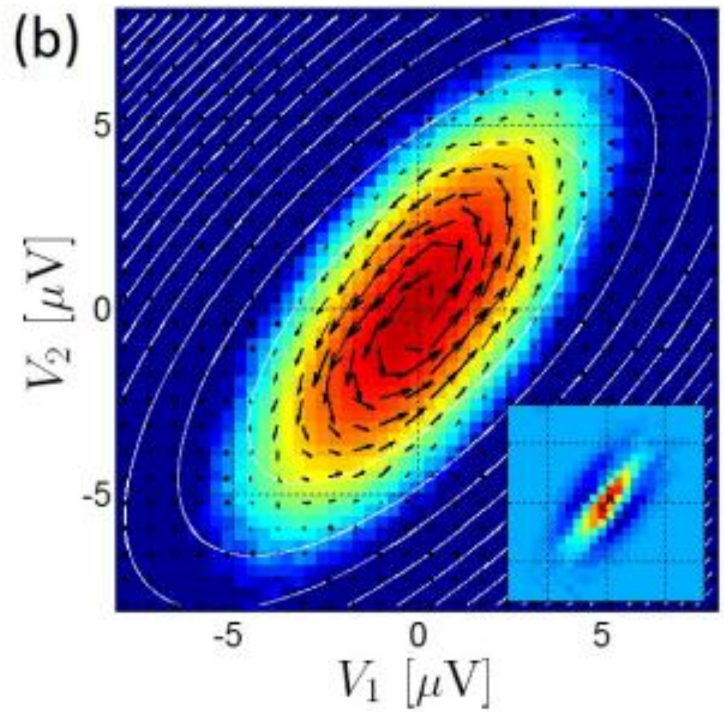
$$\Rightarrow \vec{J}_{\rm ss} \perp \nabla P_{\rm ss}$$

Equilibrium vs. NESS

Equilibrium (T_1 = 296 K)



NESS (T_1 = 120 K)



In equilibrium

- Detailed balance between any two states
- $P_{\mathrm{eq}}(\vec{V}) \sim \exp\left(-\frac{U(\vec{V})}{k_{\mathrm{B}}T}\right)$

In NESS

- Breaking of detailed balance
- Perpetual motion caused by unbalanced diffusion and drifting force

Theoretical Analysis

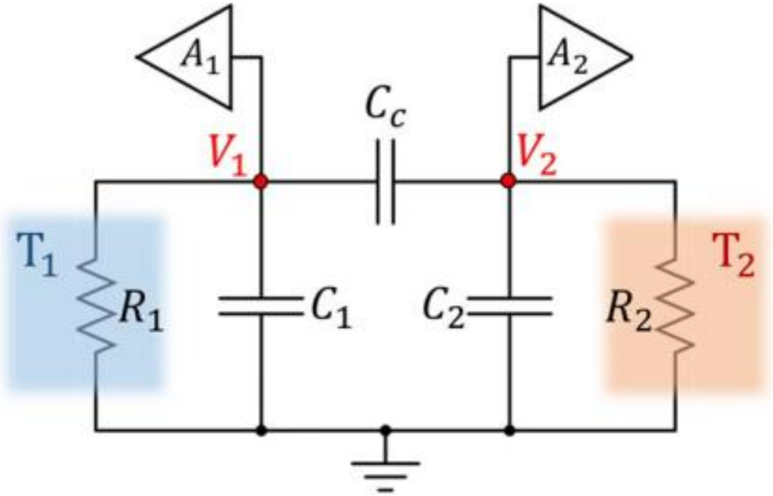
Eq. of motion (coupled Langevin eqs.)

$$R_1(C_1 + C_C)\dot{V}_1 - R_1 C_C \dot{V}_2 = -V_1 + \xi_1$$
$$R_2(C_2 + C_C)\dot{V}_2 - R_2 C_C \dot{V}_1 = -V_2 + \xi_2$$

$$\left\langle \xi_i(t)\xi_j(t') \right\rangle = 2k_{\mathrm{B}} T_i R_i \delta_{ij} \delta(t - t')$$

Thermal noises: white and uncorrelated

Linear coupling: FDT analysis is applicable



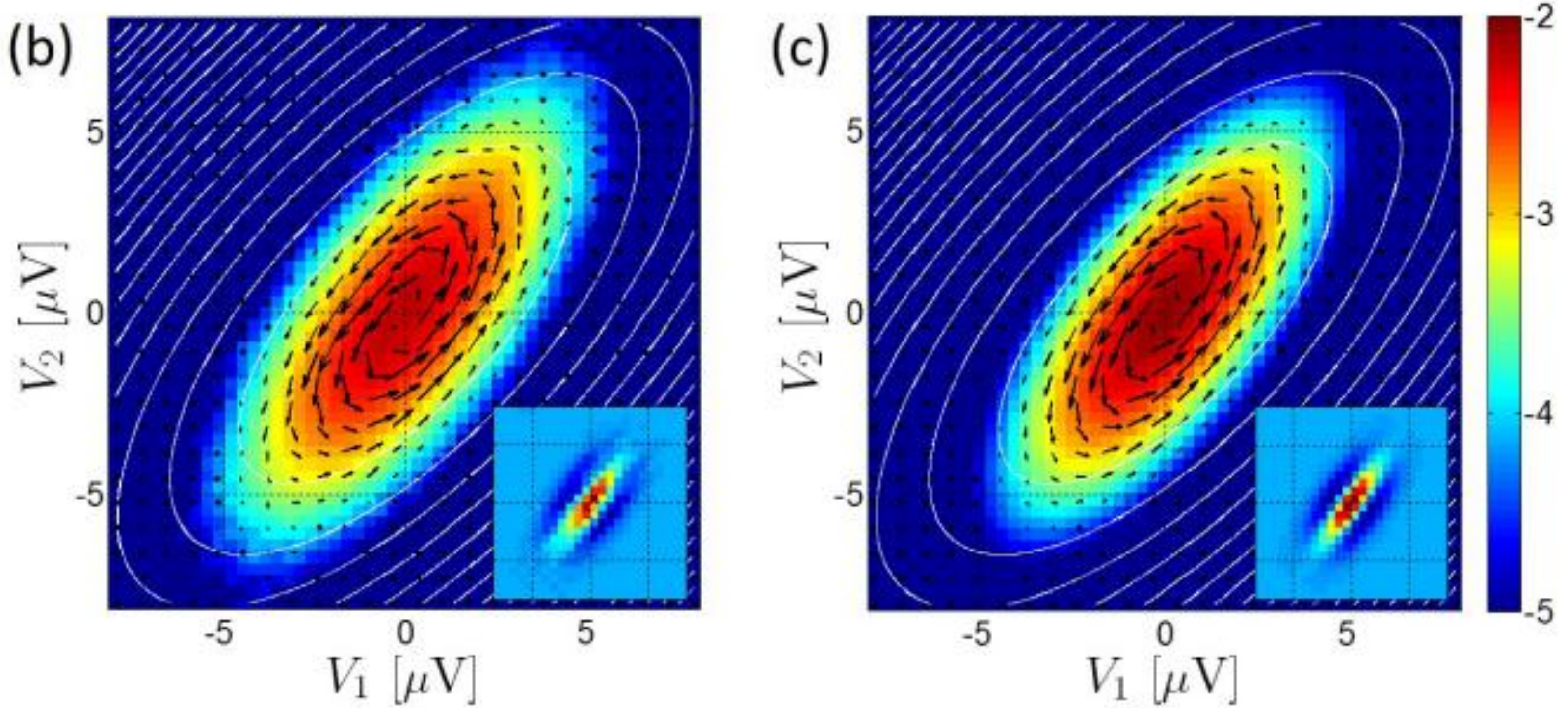
Equivalent Fokker-Planck eq. for state distribution and prob. flux

Comparison to FDT Analysis

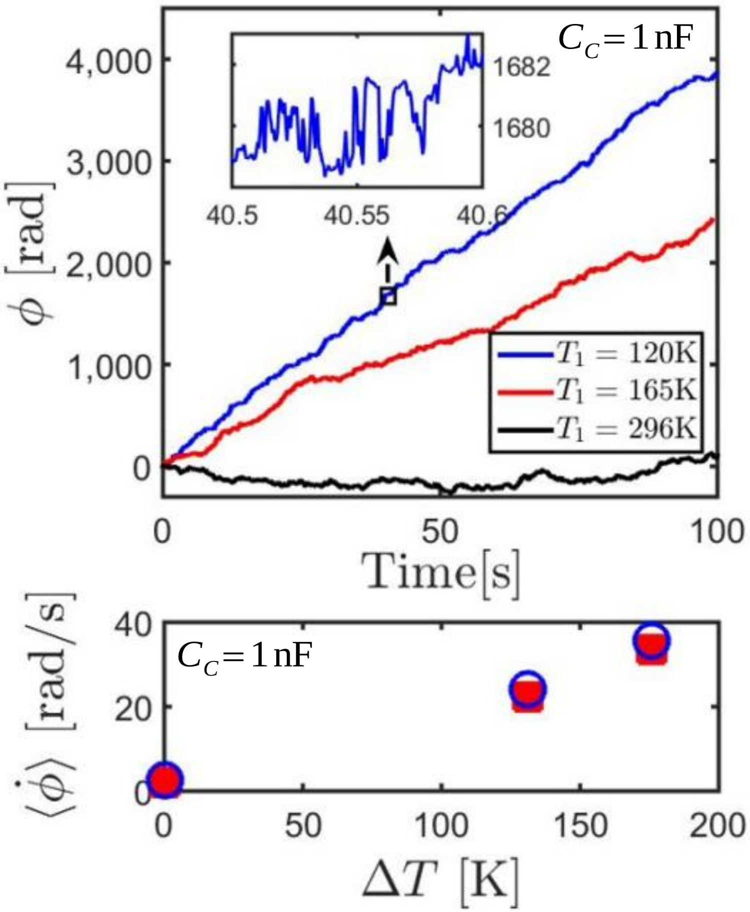
$T_1 = 120\ \mathrm{K}$

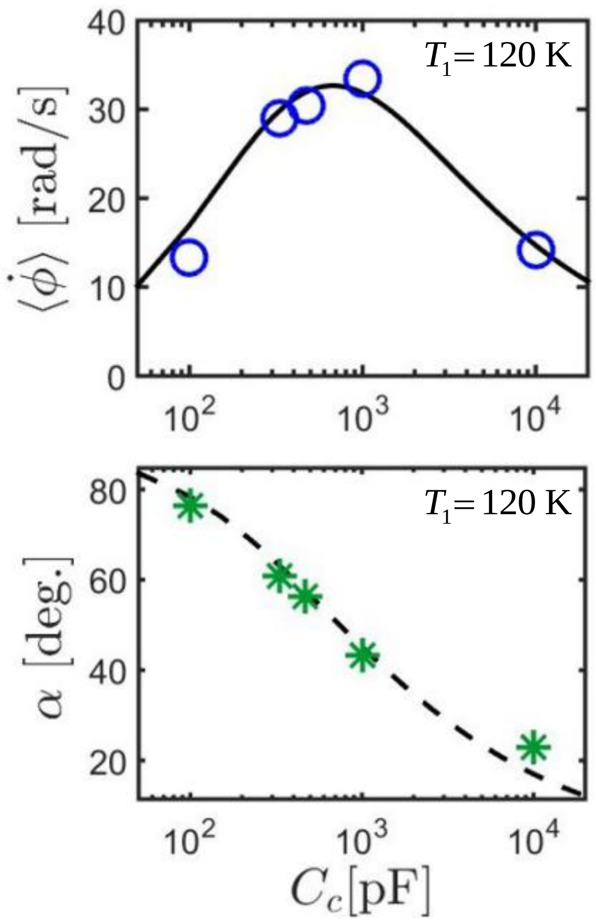
$C_C = 1\ \mathrm{nF}$

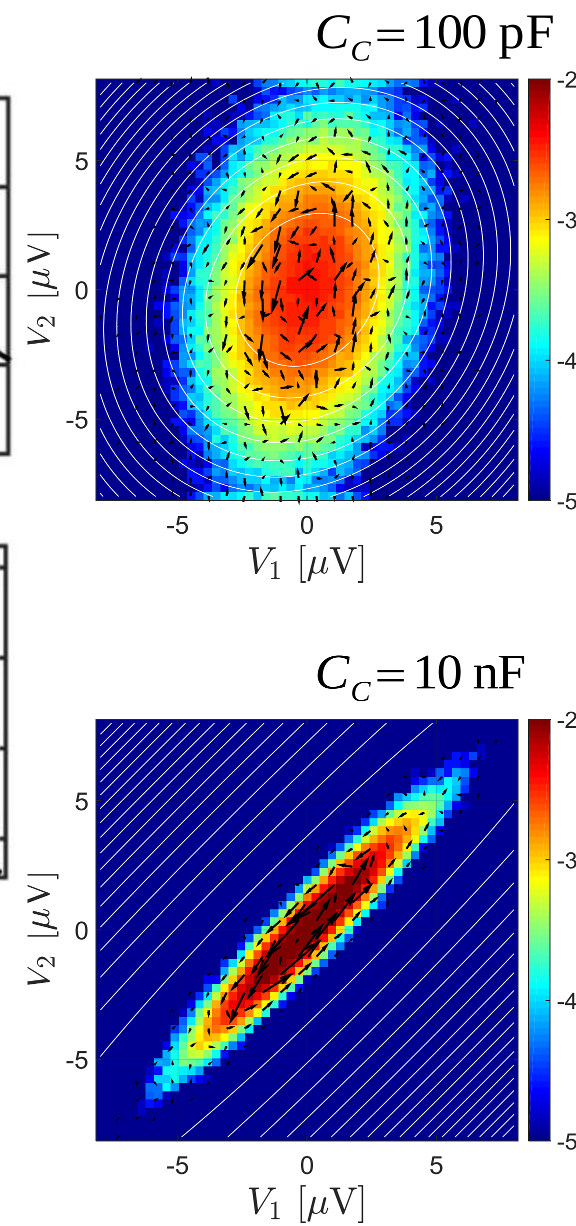
Experimental result | Analytical result



Phase of Circulation and Perpetual Speed







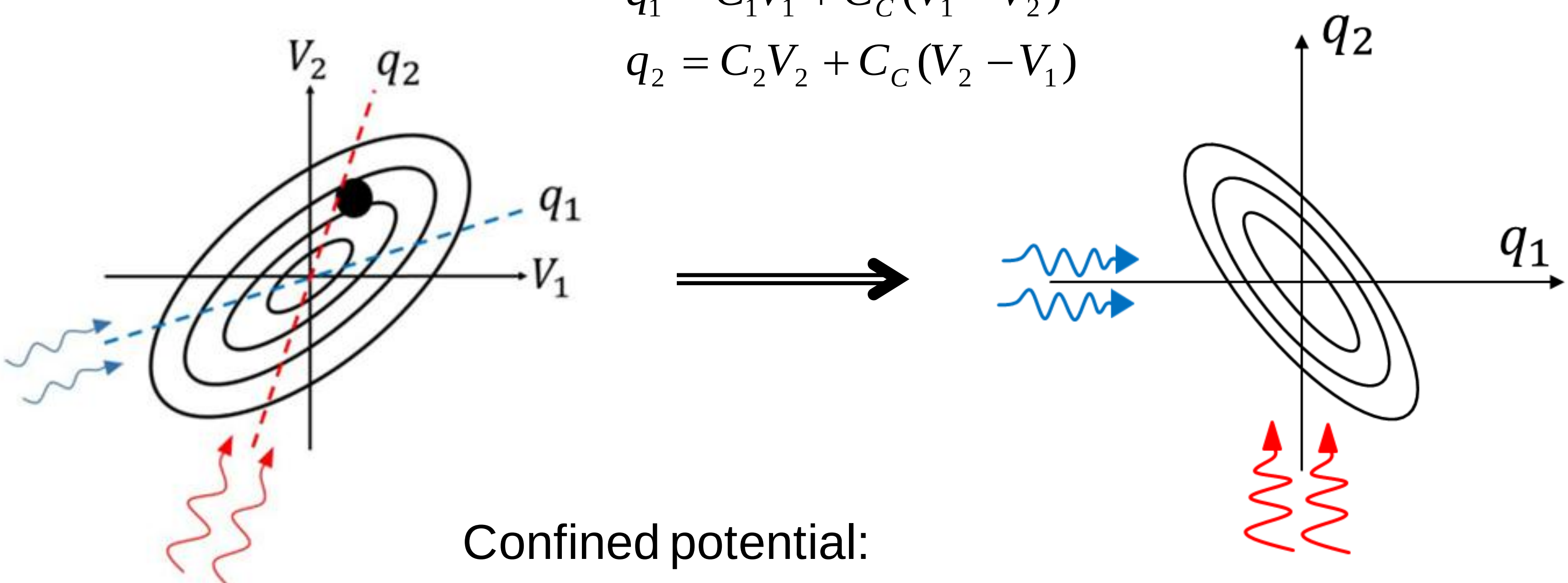
$$\varphi = \tan^{-1}\frac{V_2}{V_1}$$

α : leading angle of elliptical circulation

Change of Variables (Observables)

charges accumulated on two nodes

$$q_1 = C_1V_1 + C_C(V_1 - V_2)$$
$$q_2 = C_2V_2 + C_C(V_2 - V_1)$$



Confined potential:

$$U(q_1, q_2) = \frac{1}{2X}[(C_1 + C_C)q_1^2 + (C_2 + C_C)q_2^2 + 2C_C q_1 q_2]$$
$$X = C_1C_C + C_2C_C + C_1C_2$$

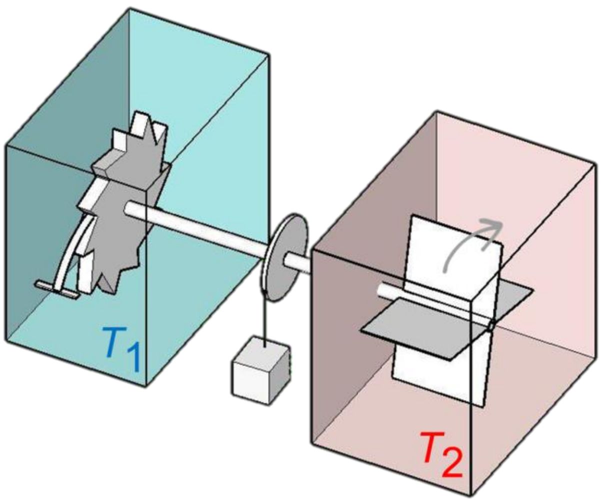
Equation of motion:

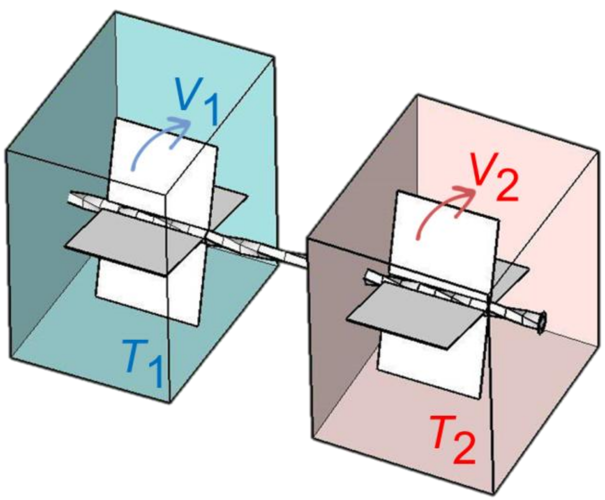
$$R_i\dot{q}_i = -\frac{\partial U(\vec{q})}{\partial q_i} + \xi_i$$

electrical Brownian gyrator

A Feynman Ratchet?

System	Feynman ratchet	coupled RC circuit
Asymmetry	clockwise vs. counterclockwise	In-phase vs. out-of-phase
Rectified motion	DC	AC





Summary: Nonequilibrium Coupled RC

- Coupled RC circuit in NESS driven by temperature difference is well characterized and understood in terms of noise characteristics, voltage correlation, and heat transfer rate
  - Distribution of entropy production satisfies FT
- The device experiences regular, perpetual (gyrating) motion
  - Experimental realization of Brownian gyrator (Brownian motor, Feynman ratchet) at room temperature
- Possibility of making autonomous heat engine and refrigerator is under investigation

Collaborations

Group:

Kuan-Hsun Chiang

Xing Zhang

PI:

Prof. Chi-Lun Lee

Prof. Pik-Yin Lai

Prof. Yonggun Jun

Department of Physics,

National Central University

Thank You!