Acoustics of Guitars
吉他聲學

國立中央大學光電科學與工程學系

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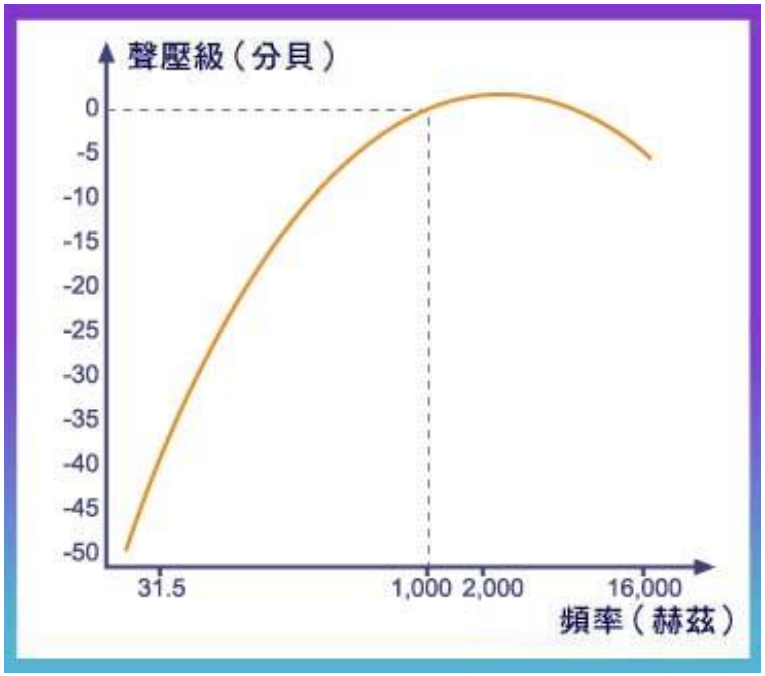
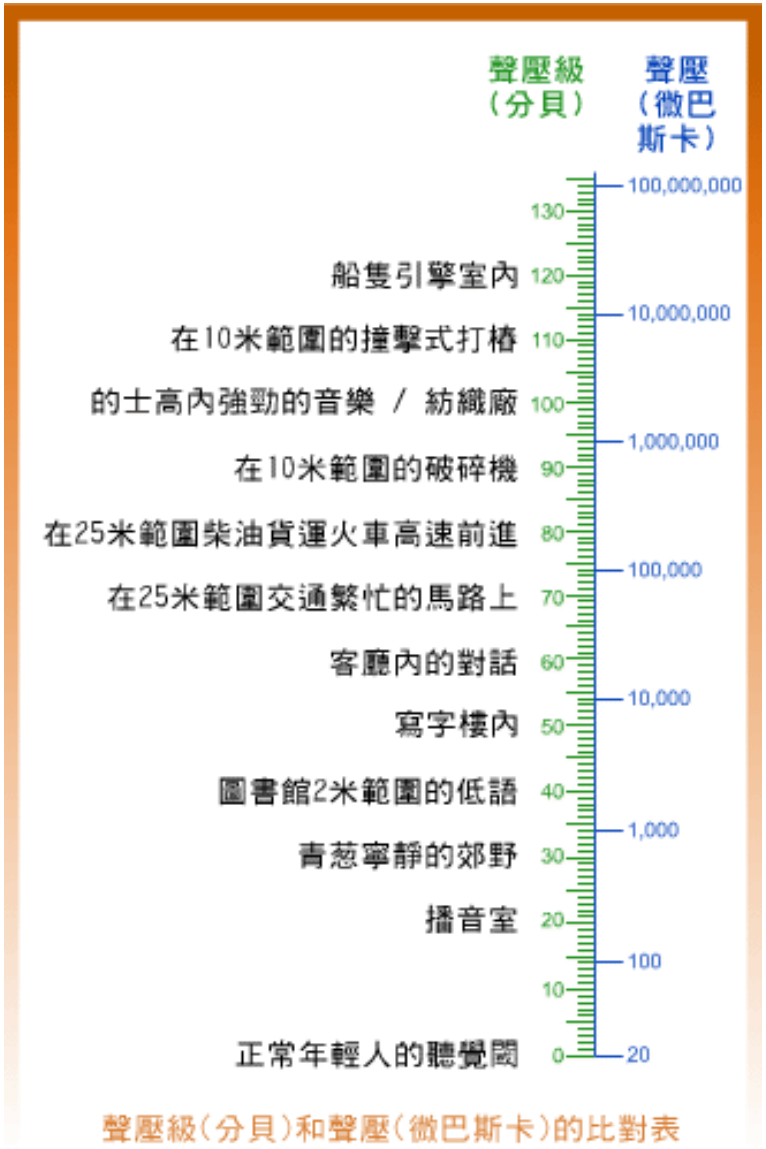
聲音

- 20 μPa 或20 x 10^{-6} Pa作為參考聲壓值，並定義這聲壓水平為0分貝(dB)
- Pa=N/m^2。dB=20x$\log_{10}(\frac{壓力_1}{壓力_2})$
- 人耳20Hz-20kHz



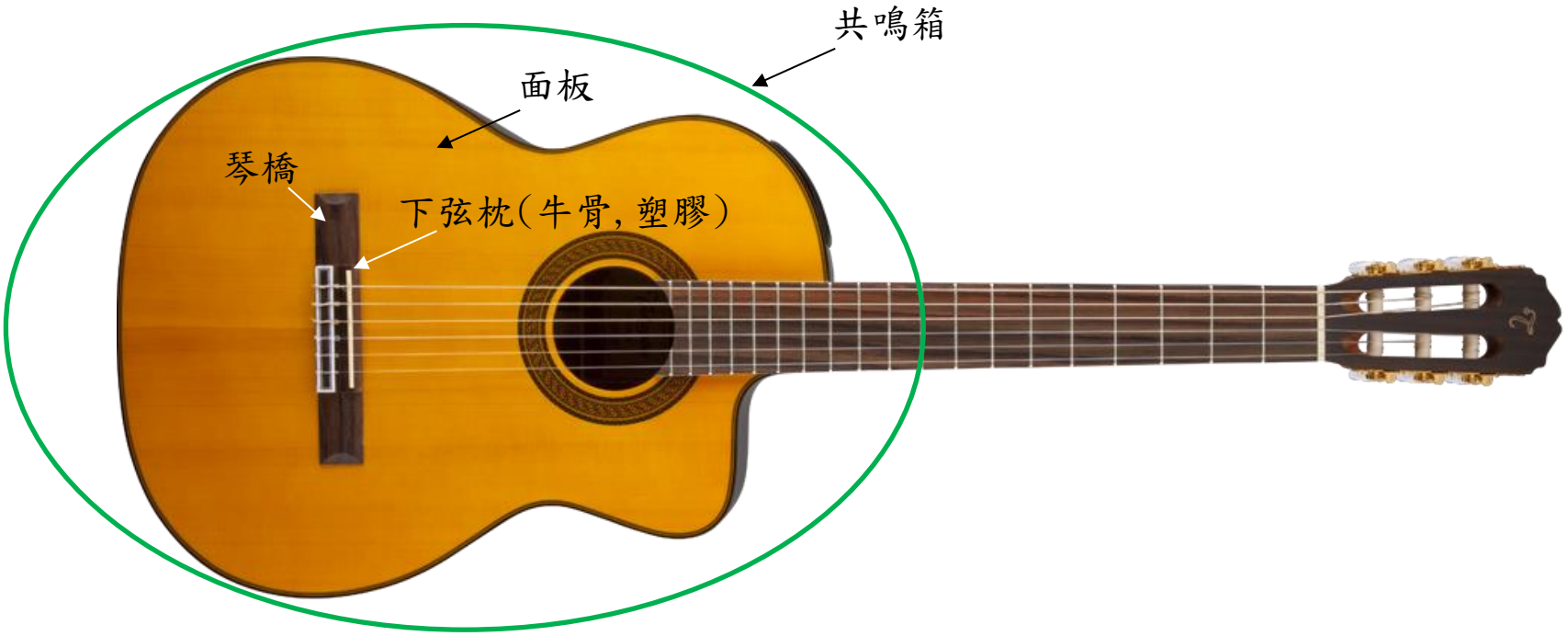


聲壓級(分貝)和聲壓(微巴斯卡)的比對表

吉他聲音的激發

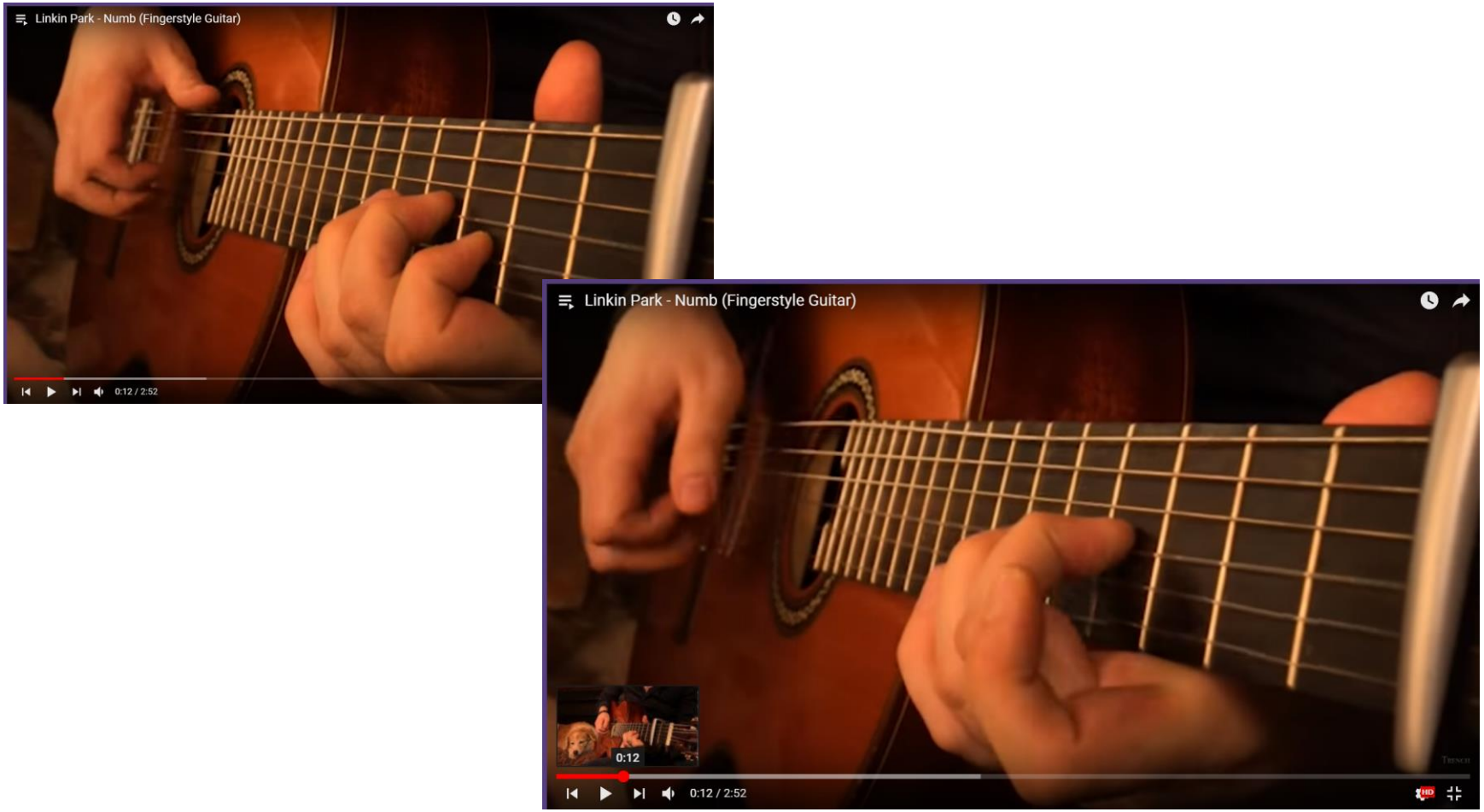
手指撥弦琴振動 ⇨ 振動由下弦枕傳至琴橋 ⇨ 振動傳給面板

⇨ 帶動共鳴箱內的空氣並傳出聲波

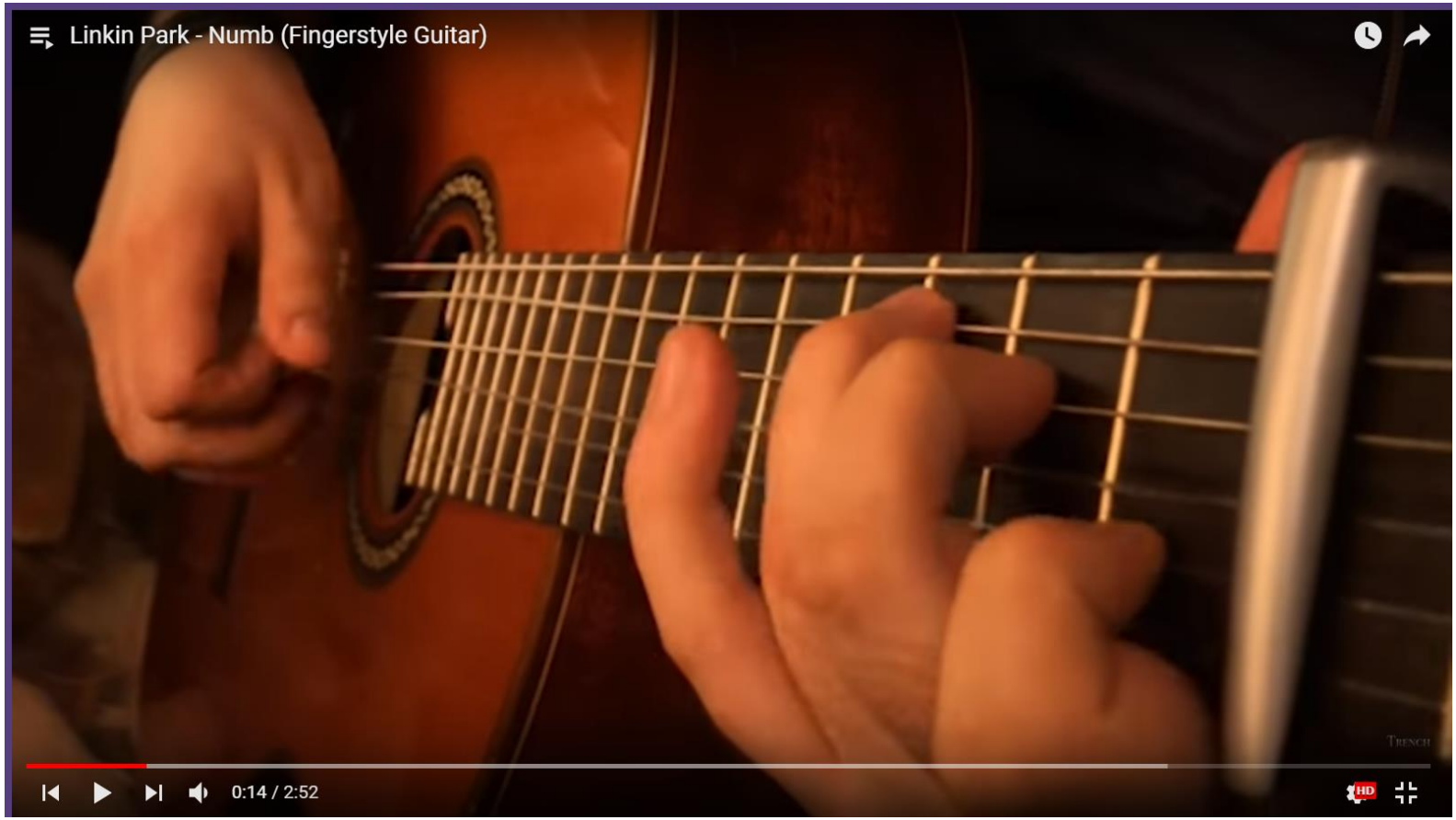


https://www.youtube.com/watch?v=k49gVD-2hio&index=5&list=RD94-KCPOxd2Y

弦的振動

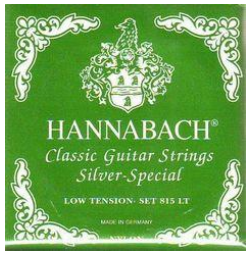

Linkin Park - Numb (Fingerstyle Guitar)
0:12 / 2:52
Linkin Park - Numb (Fingerstyle Guitar)
0:12
0:12 / 2:52

弦的振動



弦

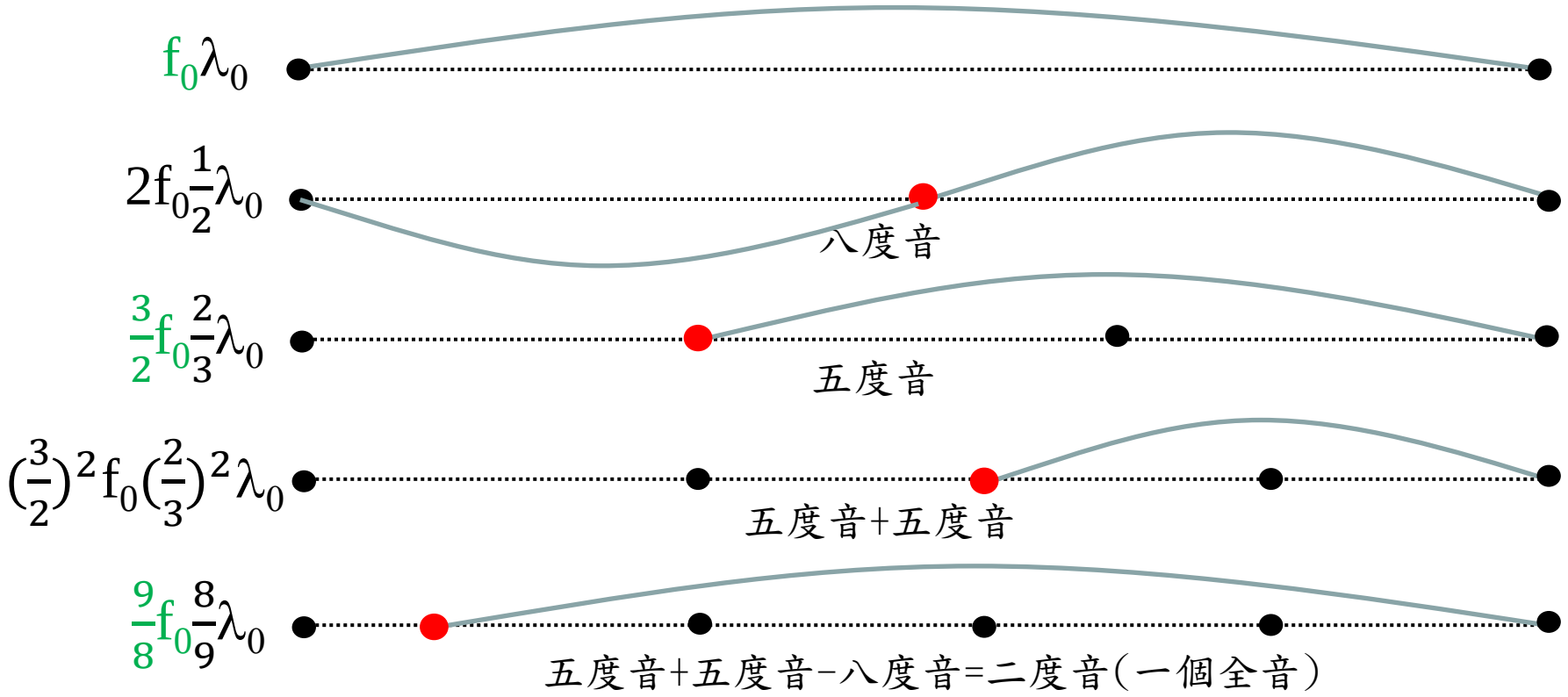
- v波速$=\sqrt{\frac{T張力}{\mu單位長度弦重}}=\sqrt{\frac{T張力(N=kg\ m/s^2)}{m弦重(kg)/L弦長(m)}}=$f頻率λ波長
- $1kg=9.8N$



TENSION	E-1	B-2	G-3	D-4	A-5	E-6	TOTAL TENSION
Folk guitar strings (Elixir)							
Custom Light	.011 / 9.08	.015 / 9.08	.022 / 11.35	.032 / 13.62	.042 / 13.62	.052 / 11.35	68.1kg
Medium Strings	.013 / 12.29	.017 / 11.80	.026 / 15.89	.035 / 16.80	.045 / 15.436	.056 / 13.166	83.4kg
Classical guitar strings (Hannabach)							
Low	7.44	6.32	6.53	6.93	6.63	6.63	40.48kg
Super High	7.95	6.63	6.93	8.46	8.16	8.16	46.29kg

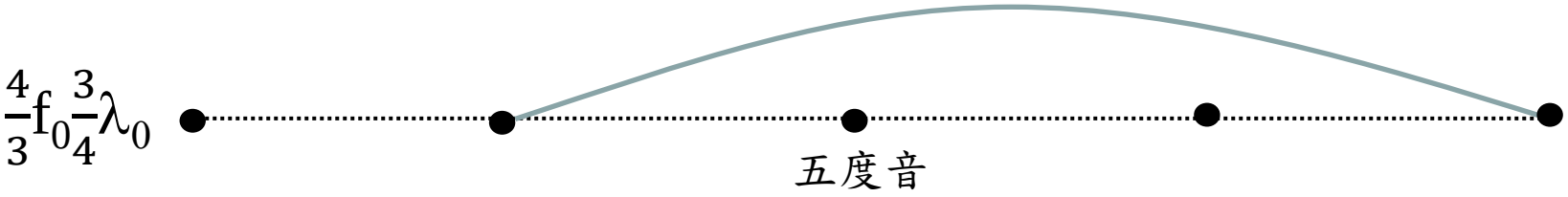
十二平均律 Equal temperament

v=fλ=*constant*, for a string



$(\frac{3}{2})^3f_0 \Rightarrow \frac{27}{8}\times\frac{1}{2}f_0=\frac{27}{16}f_0$ $(\frac{3}{2})^4f_0 \Rightarrow \frac{81}{16}\times\frac{1}{4}f_0=\frac{81}{64}f_0$ $(\frac{3}{2})^5f_0 \Rightarrow \frac{243}{32}\times\frac{1}{4}f_0=\frac{243}{128}f_0$

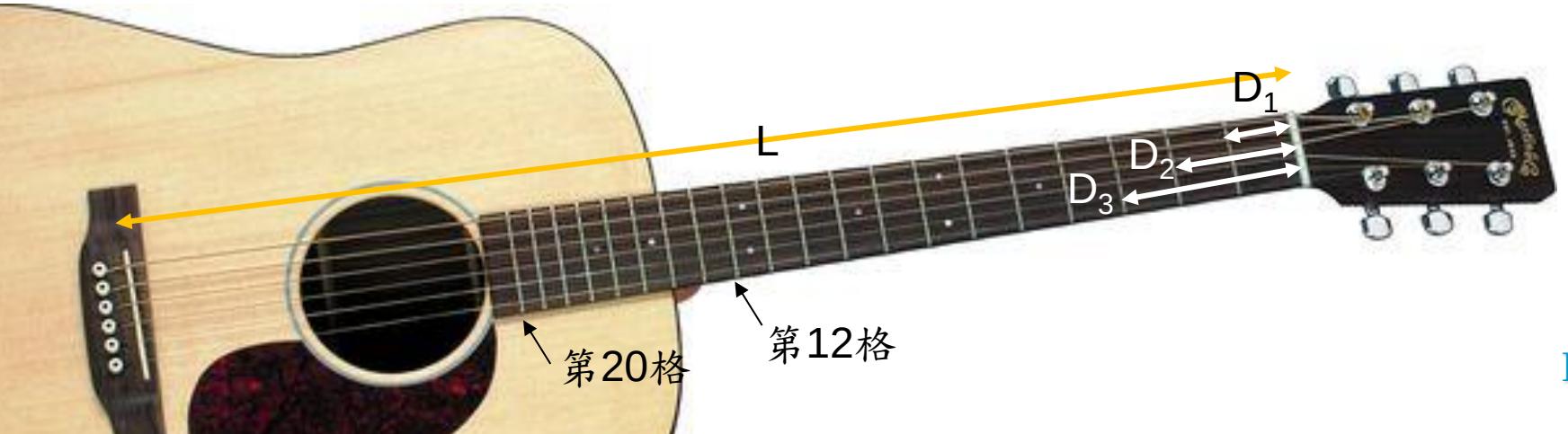
F



C	D	E	F	G	A	B
f_0	$\frac{9}{8}f_0$	$\frac{81}{64}f_0$	$\frac{4}{3}f_0$	$\frac{3}{2}f_0$	$\frac{27}{16}f_0$	$\frac{243}{128}f_0$
f_0	$1.125f_0$	$1.266f_0$	$1.333f_0$	$1.5f_0$	$1.688f_0$	$1.898f_0$

音程

- 兩個相接鄰的音高頻率比為$\sqrt[12]{2}\cong 1.05946$
- 兩個相差八度的音其頻率必定相差一倍。舉例而言，若A為440Hz，則高八度的則為880Hz
- 吉他上琴枕到第i琴格的距離公式$D_i=L(1-\frac{1}{R^i})$, where $R=\sqrt[12]{2}=1.059463094359295$, L是弦長(650mm或630mm)



音高頻率表(Hz)

八度→ 音名↓	0	1	2	3	4	5	6	7	8	9
C	16.352 (−48)			130.81 (−12)	261.63 (0)	523.25 (+12)	1046.5 (+24)	2093.0 (+36)	4186.0 (+48)	8372.0 (+60)
C♯/D♭	17.324 (−47)			138.59 (−11)	277.18 (+1)	554.37 (+13)	1108.7 (+25)			
D	18.354	36.708	73.416 (−22)	**146.83** (−10)	293.66 (+2)	587.33	1174.7	2349.3 (+38)	4698.6 (+50)	9397.3 (+62)
D♯/E♭			77.782 (−21)	155.56 (−9)	311.13 (+3)			2489.0 (+39)	4978.0 (+51)	9956.1 (+63)
E	20.602 (−44)	41.203 (−32)	**82.407** (−20)	164.81 (−8)	**329.63** (+4)	659.26 (+16)	1318.5 (+28)	2637.0 (+40)	5274.0 (+52)	10548 (+64)
F	21.827 (−43)	43.654	87.307	174.61 (−7)	349.23 (+5)	698.46		2793.8	5587.7 (+53)	11175 (+65)
F♯/G♭	23.125 (−42)			185.00 (−6)	369.99 (+6)	739.99 (+18)			5919.9 (+54)	11840 (+66)
G	24.500	48.999	97.999 (−17)	**196.00** (−5)	392.00 (+7)	783.99 (+19)	1568.0 (+31)	3136.0 (+43)	6271.9 (+55)	12544 (+67)
G♯/A♭			103.83 (−16)	207.65 (−4)	415.30 (+8)	830.61 (+20)	1661.2 (+32)	3322.4 (+44)	6644.9 (+56)	13290 (+68)
A	27.500 (−39)	55.000 (−27)	**110.00** (−15)	220.00 (−3)	440.00 (+9)	880.00 (+21)	1760.0 (+33)	3520.0 (+45)	7040.0 (+57)	14080 (+69)
A♯/B♭	29.135 (−38)	58.270	116.54	233.08 (−2)	466.16 (+10)	932.33 (+22)	1864.7 (+34)	3729.3 (+46)	7458.6 (+58)	14917 (+70)
B	30.868 (−37)			**246.94** (−1)	493.88 (+11)	987.77 (+23)	1975.5 (+35)	3951.1 (+47)	7902.1 (+59)	15804 (+71)

第四弦D

第一弦20格

第六弦E

第一弦E

第一弦12格

第三弦G

第五弦A

第二弦B

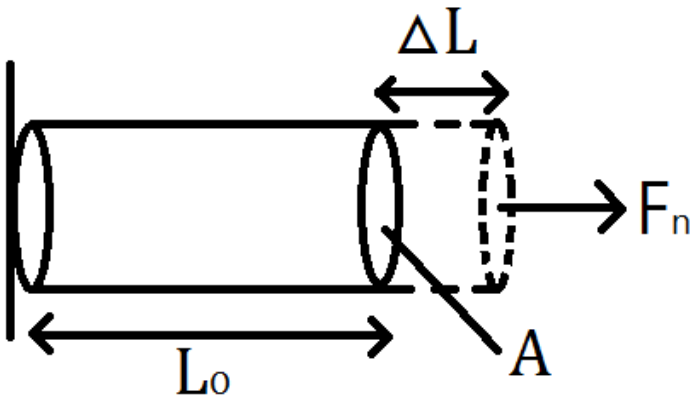
（）括號內為距離中央C（261.63Hz）的半音距離

貝多芬:「吉他像似一個小型的管弦樂團。」

吉他各部位木材材質

- 琴頸－桃花心木(mahogany)、楓木(maple)
- 指版－黑檀木（Ebony）
- 面板－雲杉或稱白松(spruce)、桃花心木(mahogany)、紫檀木(rosewood)、楓木(maple)、黑壇木(ebony)、紅松（cedar）
- 背板、側板－巴西玫瑰木（Brazilian Rosewood 或 Jacaranda）、印度玫瑰木（India Rosewood）
- 下弦枕－塑膠、獸骨 (牛骨、駝骨、象牙) 或是塑鋼與金屬

楊氏係數E



△L：形變量

F_n：正向應力

A：受力面積

L_O：原長

定義：$F_n \equiv E\left(\frac{\Delta L}{L_O}\right)\mathrm{A}$

$$\Rightarrow E \equiv \frac{\frac{F_n}{\mathrm{A}}}{\frac{\Delta L}{L_O}} \equiv \frac{\sigma}{\epsilon}$$

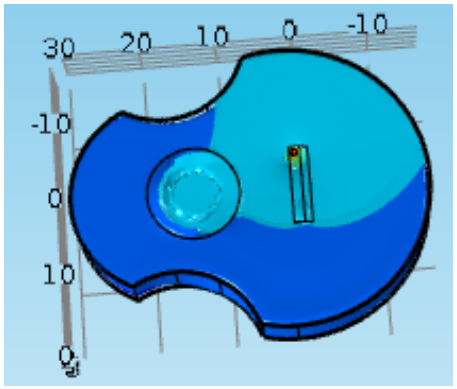
物理量

木材	聲速 (m/s)	楊氏係數(GPa) Pa=N/m²	密度kg/m³
雲杉(Spruce)	4893	9.7	405
桃花心木(Mahogany)	4070	10.6	640
玫瑰木(Rosewood)	3722	11.5	830

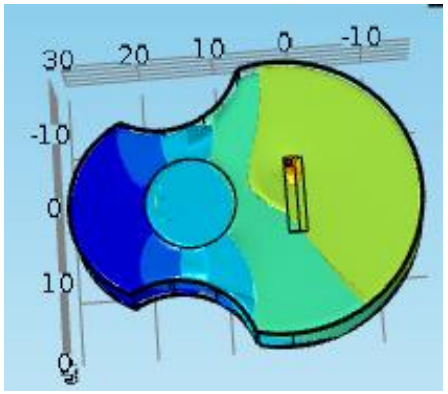
$$v\text{聲速}=\sqrt{\frac{\text{E楊氏係數}}{\rho\text{密度}}}=f\lambda$$

聲壓分佈100~800Hz

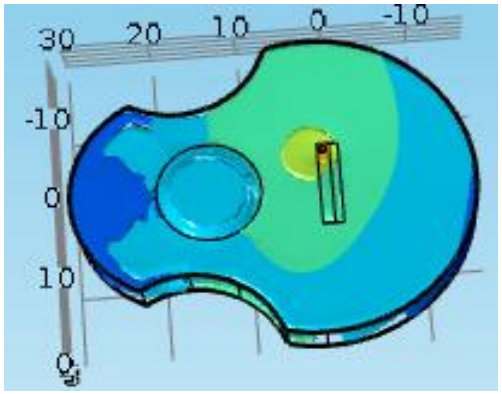
100Hz



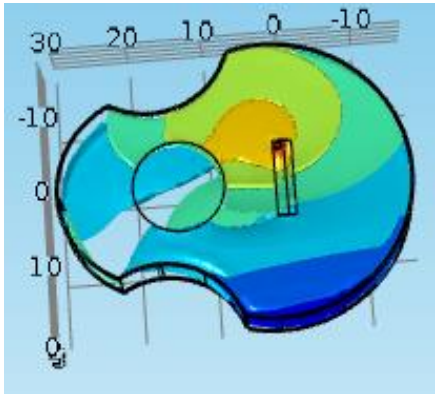
200Hz

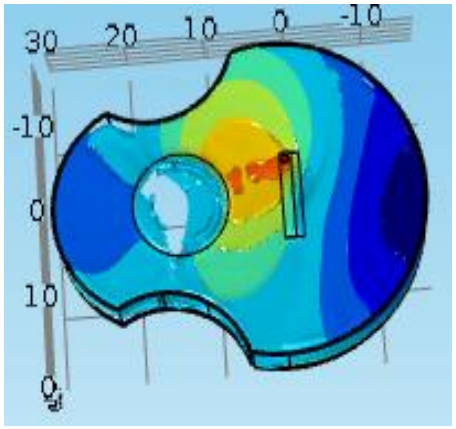


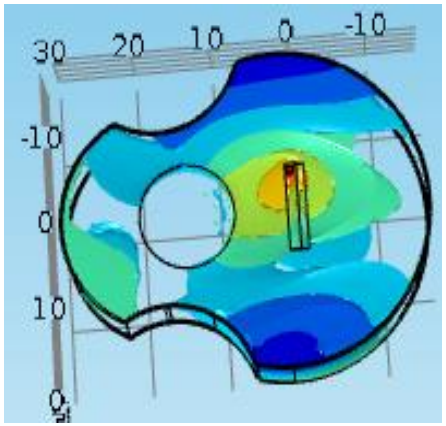
300Hz

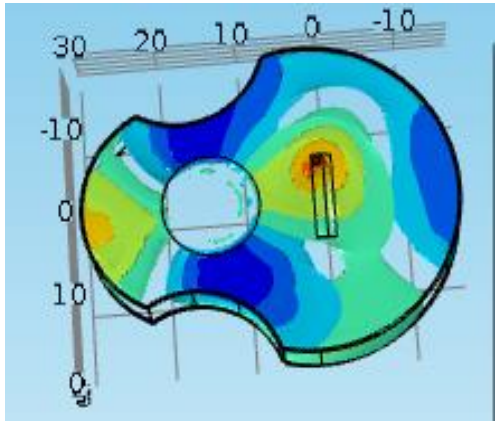


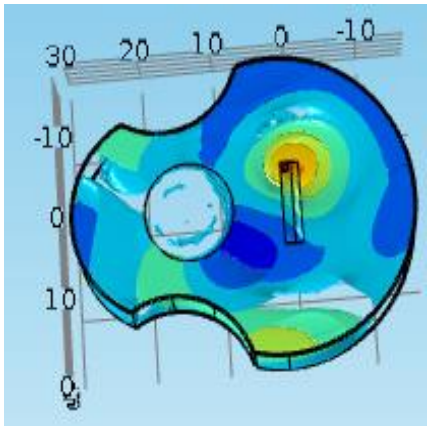
400Hz











500Hz

600Hz

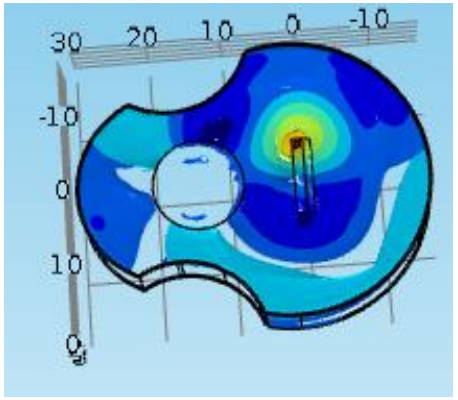
700Hz

800Hz

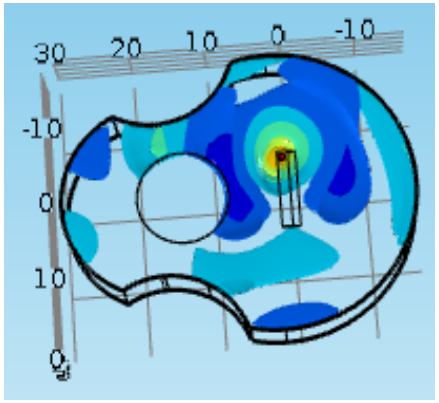
有限元素法Finite-element method

聲壓分佈900~1600Hz

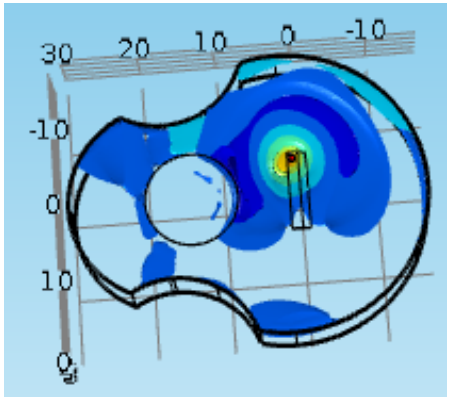
900Hz



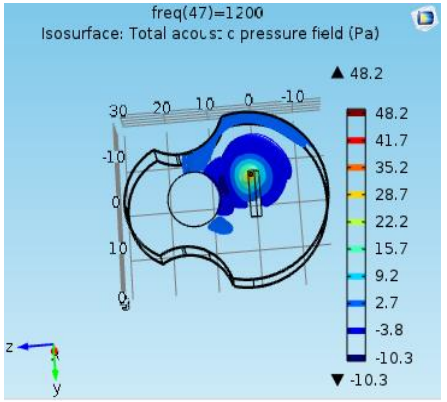
1000Hz

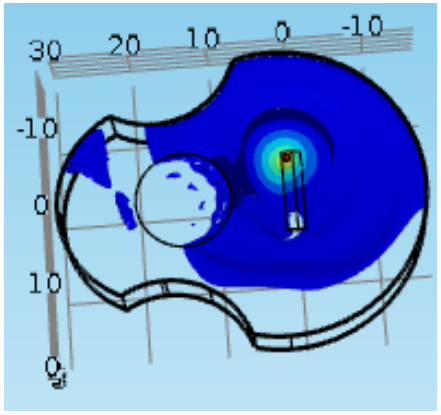


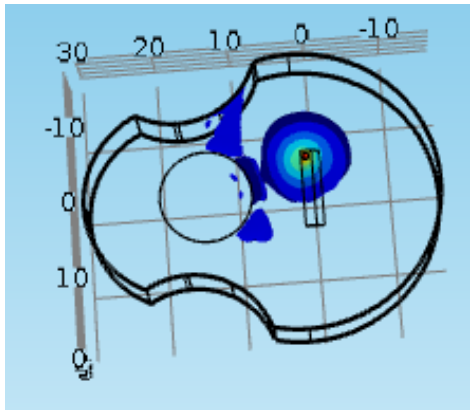
1100Hz

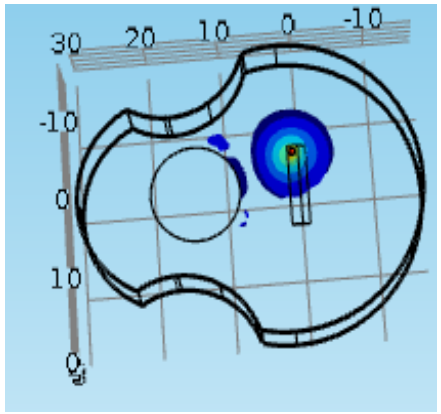


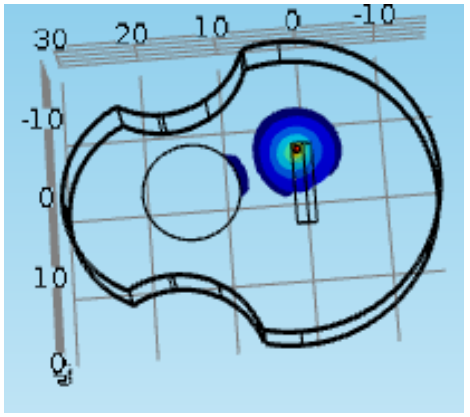
1200Hz











1300Hz

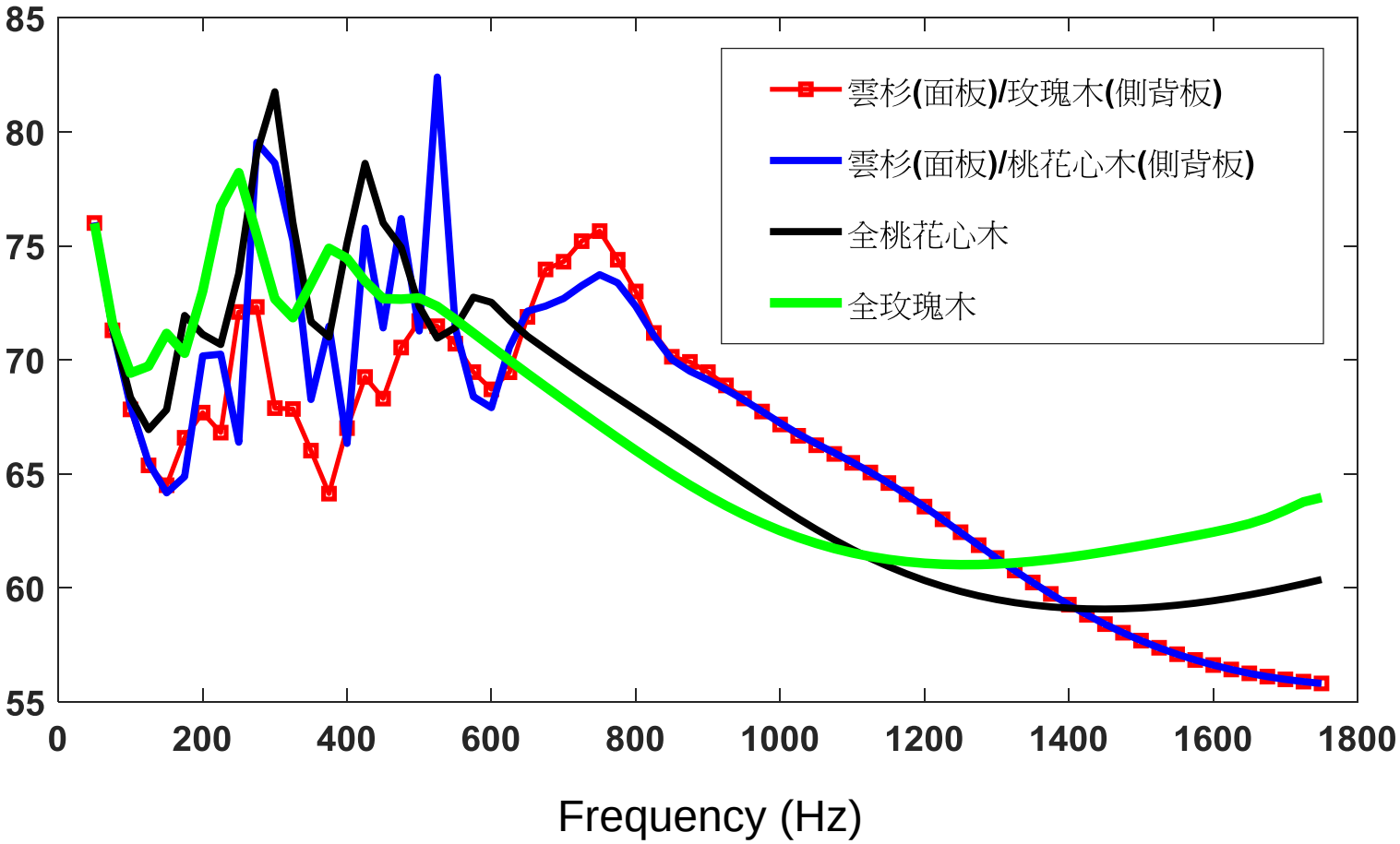
1400Hz

1500Hz

1600Hz

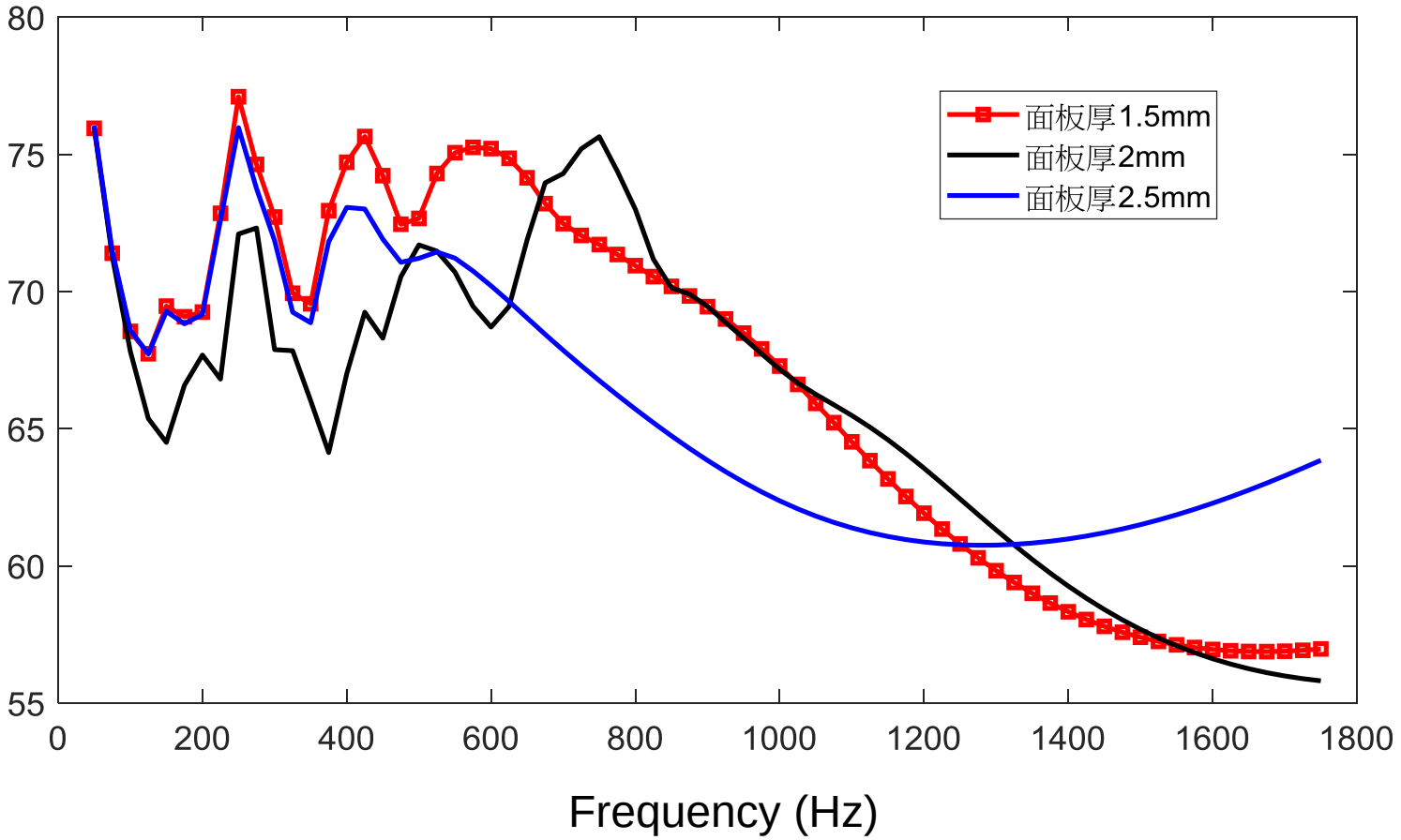
不同木材所吉他的頻率響應

10*Log$_{10}$(Output power/Input power)



雲杉(面板)玫瑰木(側背板)

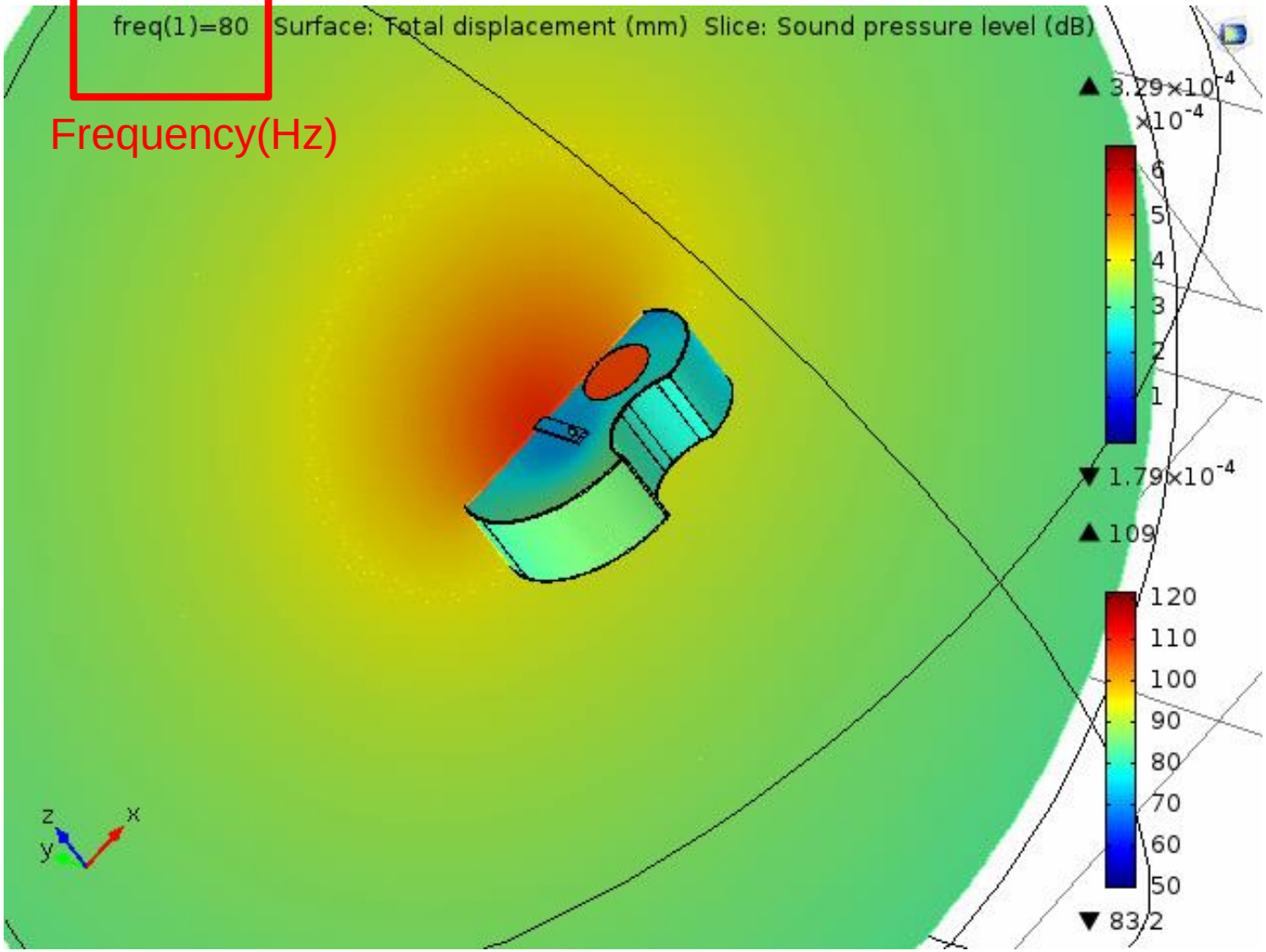
10*Log$_{10}$(Output power/Input power)



分析

- 低頻，聲音在面板上的分佈較廣，依賴面板的震動傳播
- 高頻，能量傳遞受面板與琴枕的阻礙較大，聲音較不依賴面板的震動傳播
- 高頻依賴面板的震動較少，若在琴枕附近設較多的音樑，可使聲音回彈，讓能量更集中，發出更亮更集中的高音。
- 與製琴師的經驗相符，厚板的高頻效果較好。

空氣中的聲壓


freq(1)=80
Surface: Total displacement (mm)
Slice: Sound pressure level (dB)
Frequency(Hz)
▲ 3.29×10^-4
×10^-4
6
5
4
3
2
1
▼ 1.79×10^-4
▲ 109
120
110
100
90
80
70
60
50
▼ 83.2
z
x
y

吉他製作過程









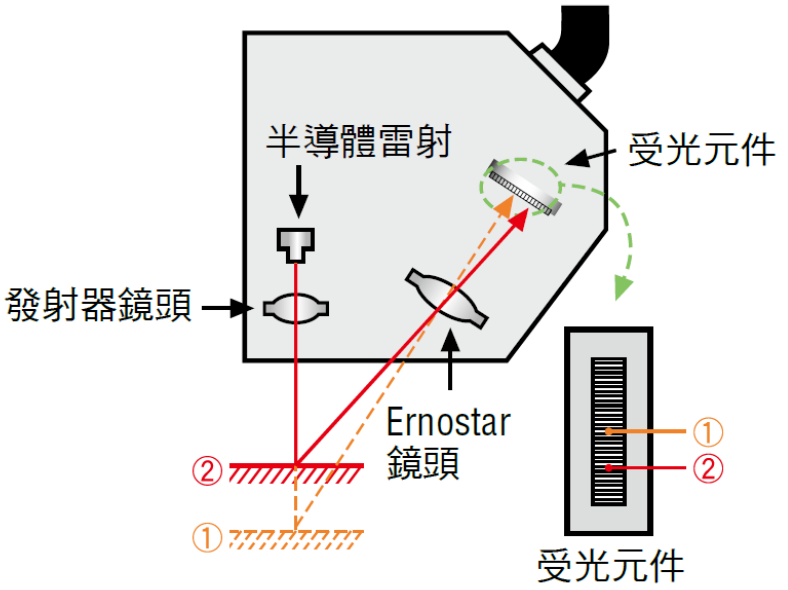
面板音樑

古典吉他

民謠吉他

雷射位移計


LASER ON
LK-H055
RS-CMOS
圓柱形物鏡
寬度是傳統
型號的兩倍


半導體雷射
受光元件
發射器鏡頭
Ernostar
鏡頭
①
②
②
①
受光元件

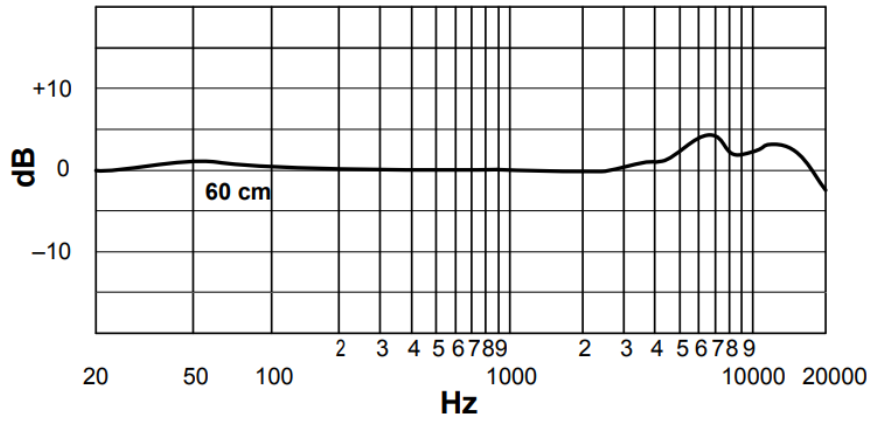
雷射位移計

Microphone

Specifications

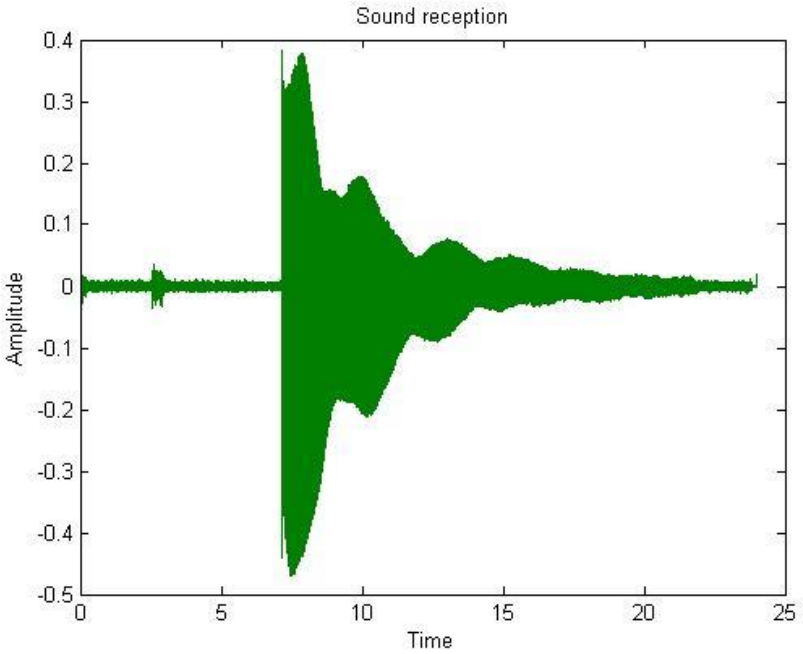
Type	Condenser (externally biased)
Frequency Response	20 to 20,000 Hz
Polar Pattern	Cardioid
Impedance	150 Ω (140 Ω actual)
Sensitivity *(at 1 kHz, open circuit voltage)*	–37 dBV/Pa *1 Pascal=94 dB SPL*
Maximum SPL *(1 kHz at 1% THD)*	1000 Ω load: 133 dB (147 dB, Pad 2500 Ω load: 138 dB (152 dB, Pad
Signal-to-Noise Ratio *(referenced at 94 dB SPL at 1 kHz)*	84.5 dB *S/N ratio is difference between 94 dB SPL and equivalent SPL of self noise, A-weighted*
Dynamic Range *(at 1 kHz)*	1000 Ω load: 123 dB 2500 Ω load: 128 dB
Common Mode Rejection *(20 Hz to 20 kHz)*	> 50 dB
Clipping Level *(20Hz-20 kHz at 1% THD)*	1000 Ω load: 1 dBV 2500 Ω load: 7 dBV *THD of microphone preamplifier when applied input signal is equivalent to cartridge output at specified SPL*
Self Noise *(typical, equivalent SPL, A-weighted)*	9.5 dB
Polarity	Positive pressure on diaphragm produces positive voltage on pin 2 with respect to pin 3
Weight	Net: 642 g (22.6 oz.)
Switch	Attenuator: -15 dB Low Frequency Response: flat; -6 dB/octave below 115 Hz; -18 dB/octave below 80 Hz
Connector	Three-pin professional audio (XLR), male
Power Requirements	48 Vdc phantom, 5.4 mA

Frequency response

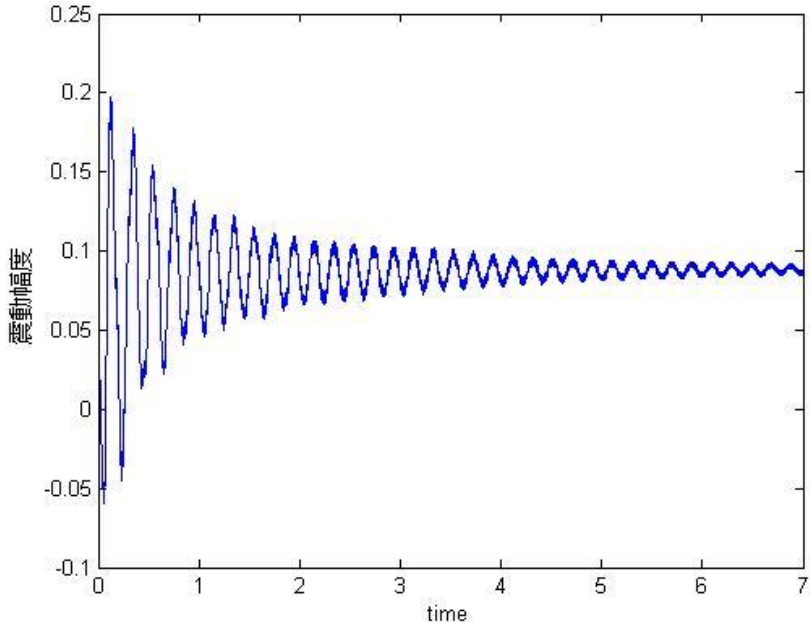


tory - NCU

收音的強度-時間
雷射儀的振福-時間

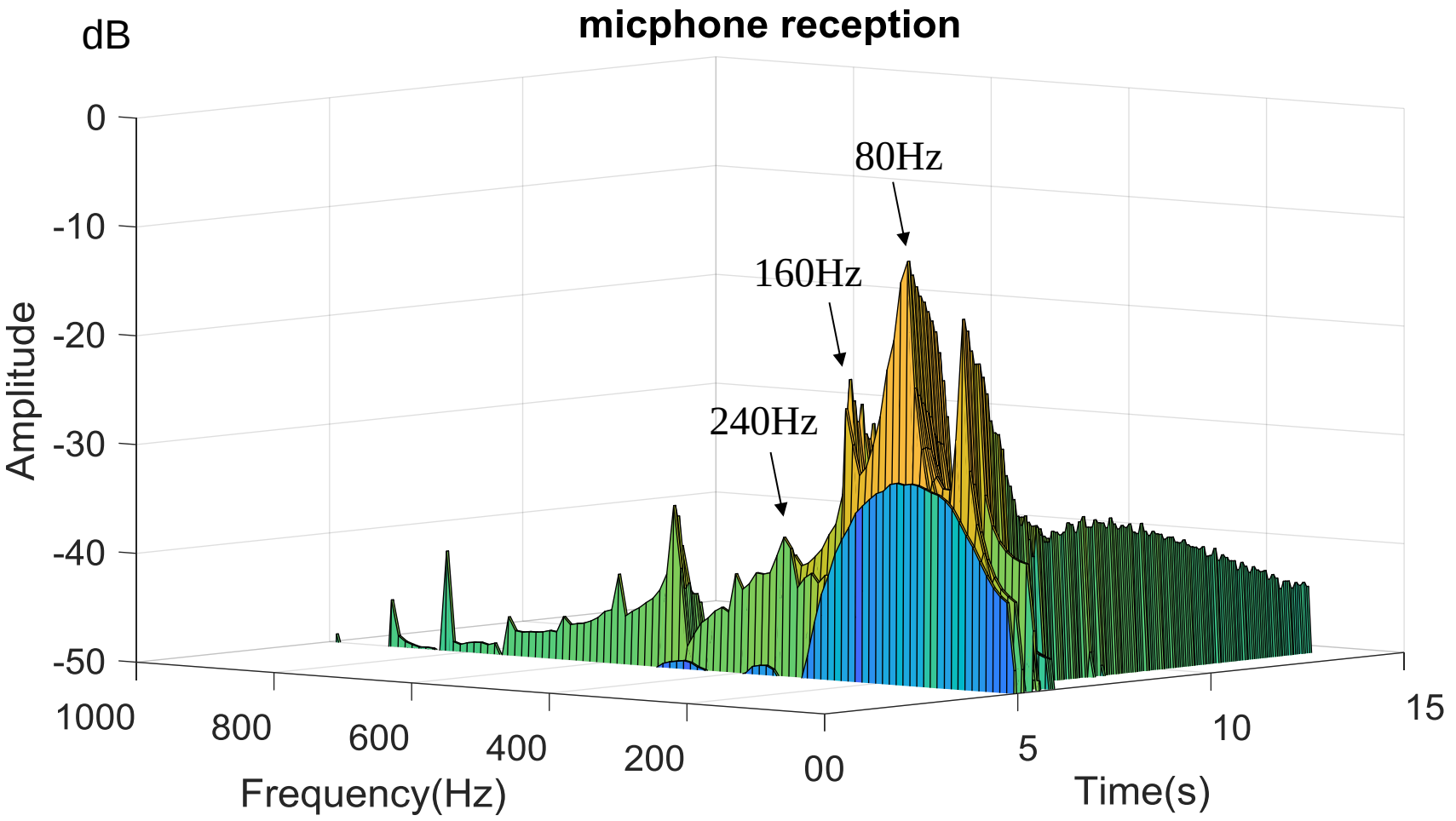


麥克風收音

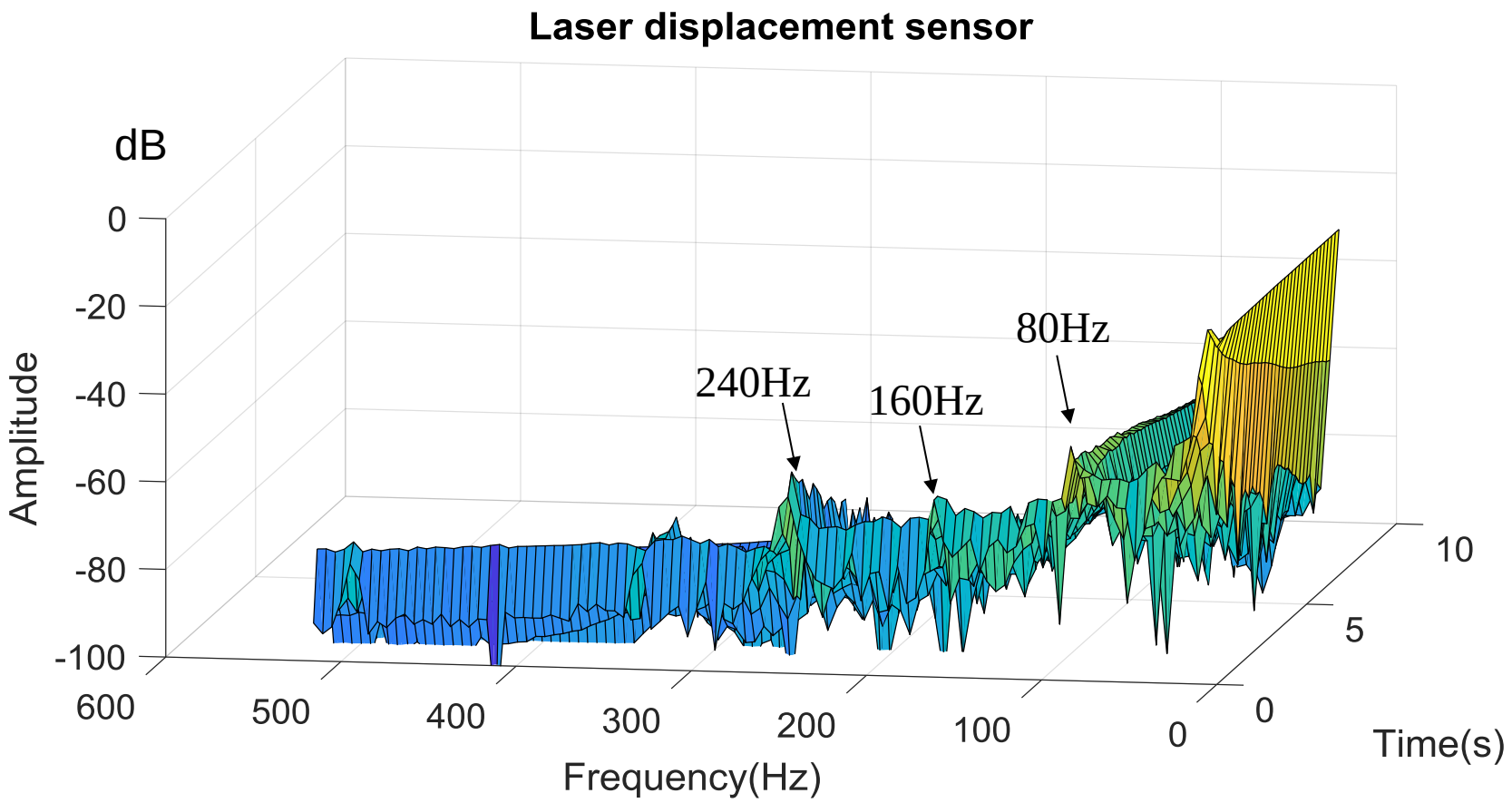


雷射位移計

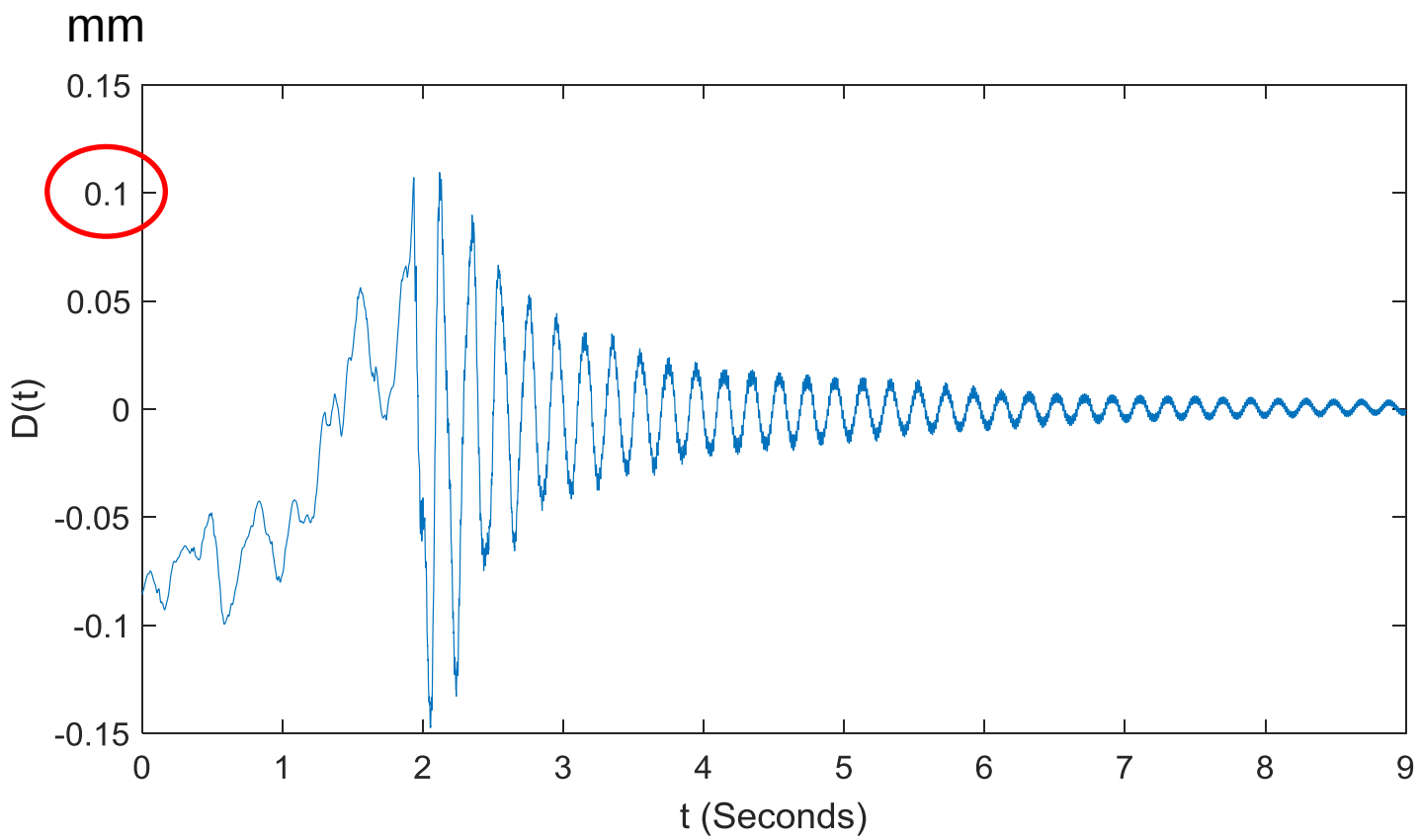
麥克風頻譜分析


micphone reception
dB
0
-10
-20
-30
-40
-50
Amplitude
80Hz
160Hz
240Hz
1000
800
600
400
200
00
Frequency(Hz)
5
10
15
Time(s)

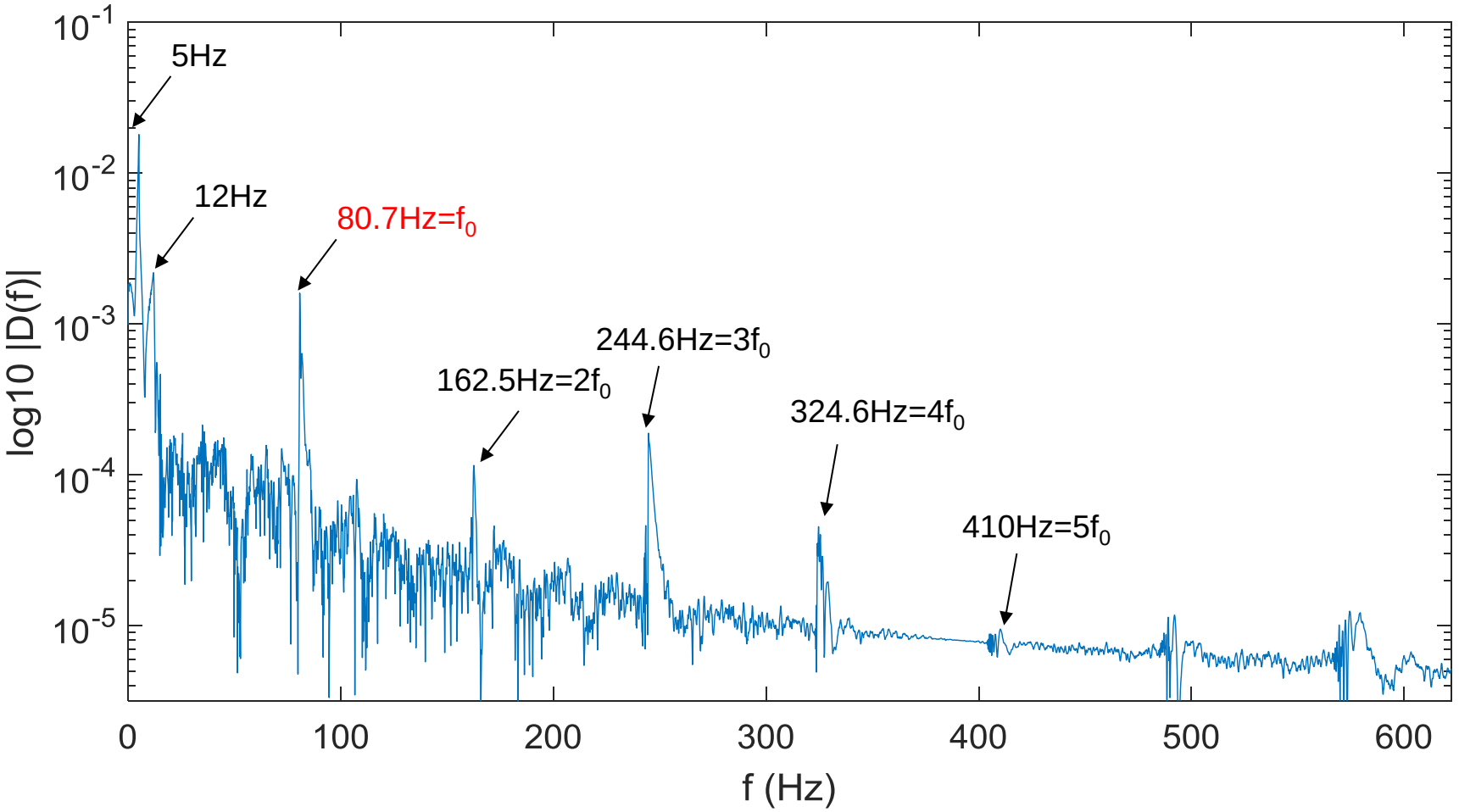
Laser displacement sensor



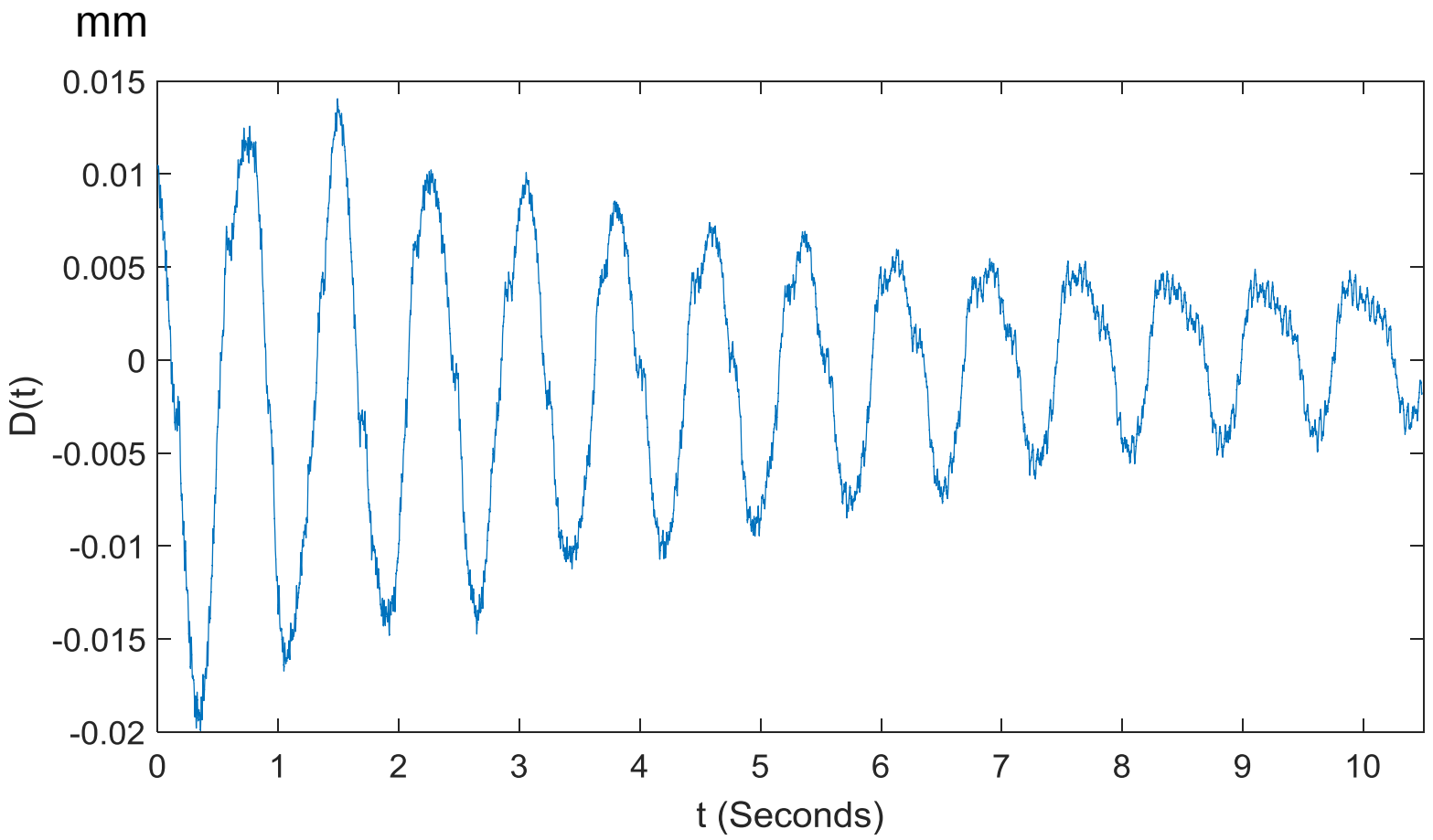
面板的振動



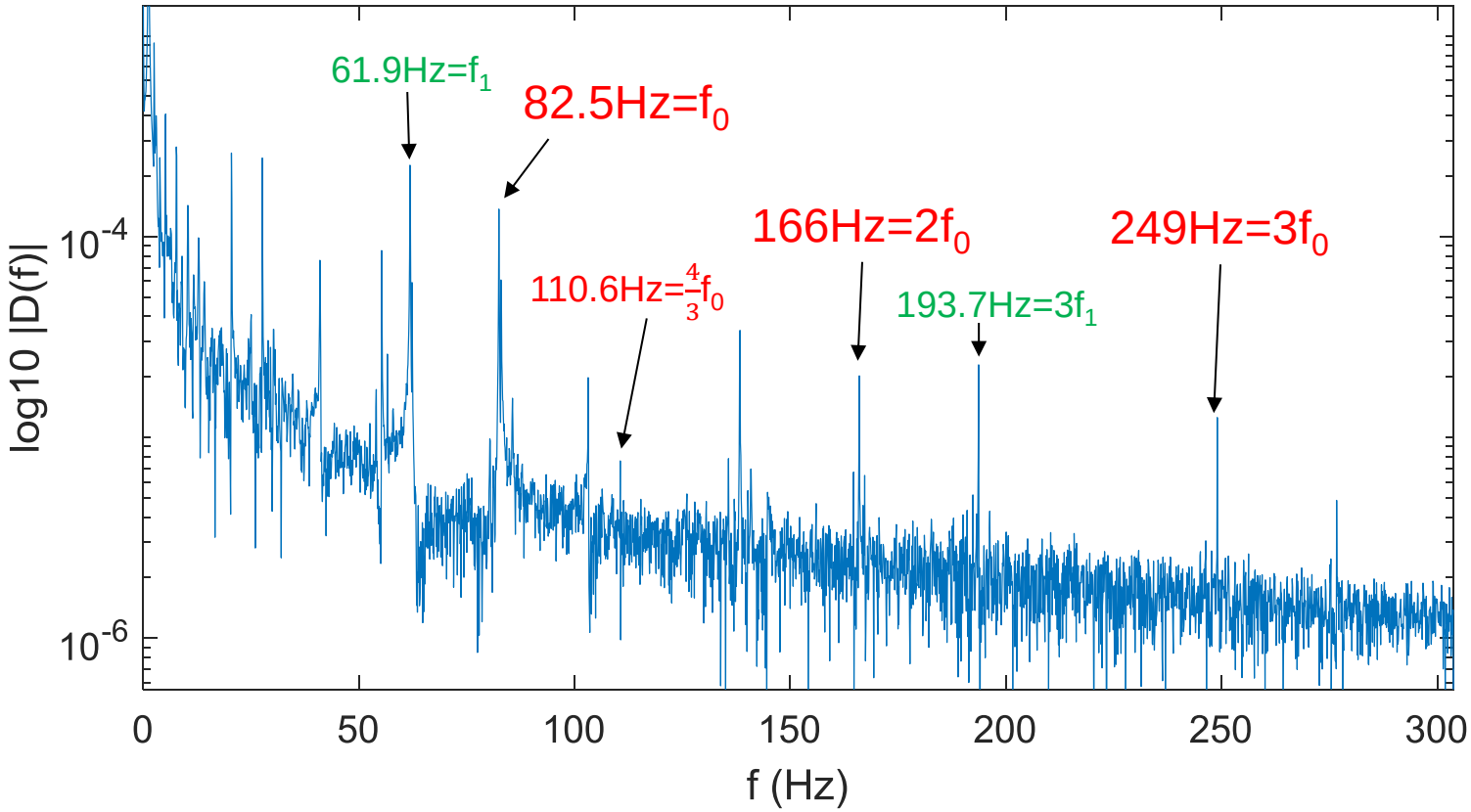
頻譜分析


5Hz
12Hz
80.7Hz=f0
162.5Hz=2f0
244.6Hz=3f0
324.6Hz=4f0
410Hz=5f0
log10 |D(f)|
f (Hz)
10-1
10-2
10-3
10-4
10-5
0
100
200
300
400
500
600

面板的振動

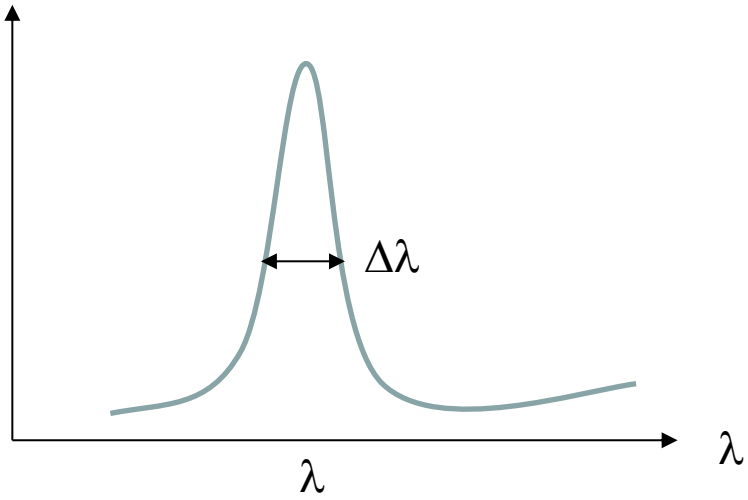

mm
0.015
0.01
0.005
0
-0.005
-0.01
-0.015
-0.02
D(t)
0
1
2
3
4
5
6
7
8
9
10
t (Seconds)

頻譜分析


61.9Hz=f1
82.5Hz=f0
166Hz=2f0
249Hz=3f0
110.6Hz=4/3f0
193.7Hz=3f1
log10 |D(f)|
10-4
10-6
0
50
100
150
200
250
300
f (Hz)

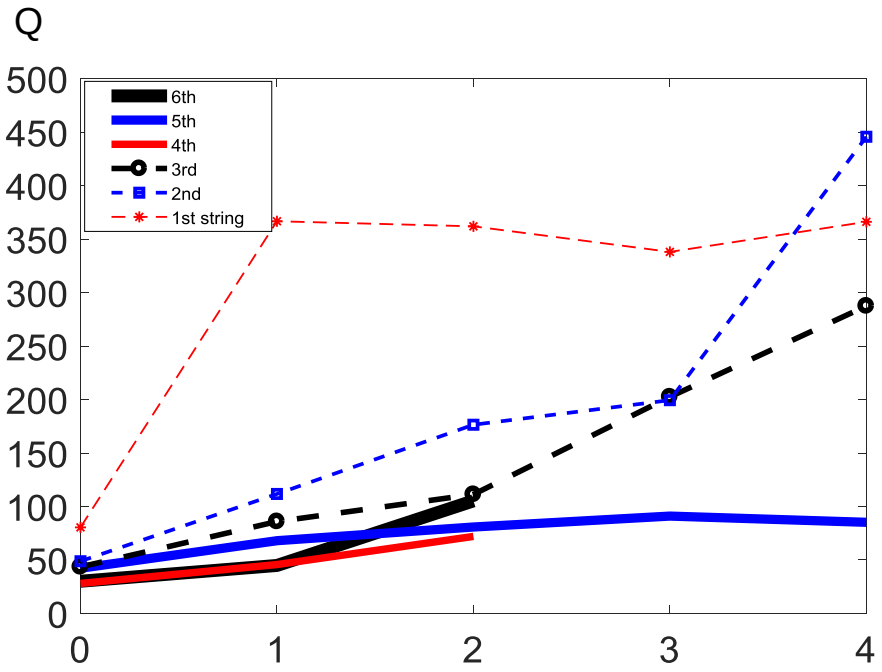
Q-factor

$$\text{Q factor} = \frac{\lambda}{\Delta\lambda} = \frac{f}{\Delta f}$$

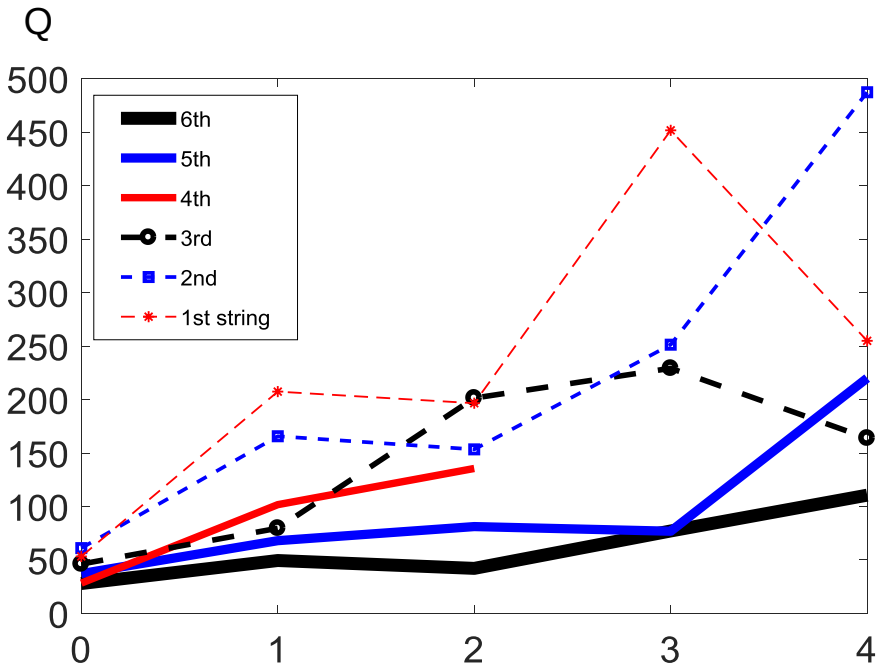


民謠吉他新舊弦Q值

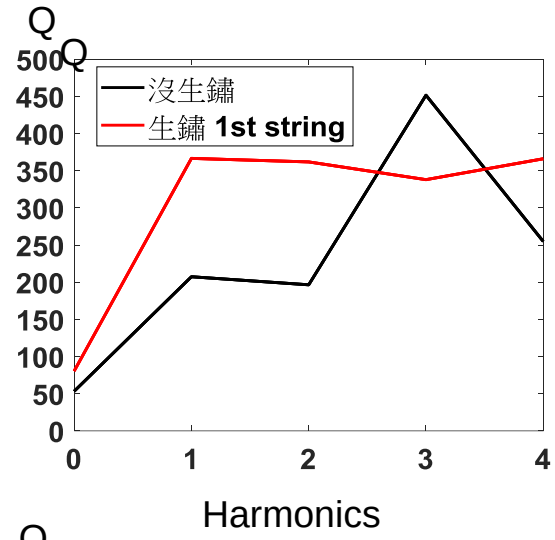
舊弦

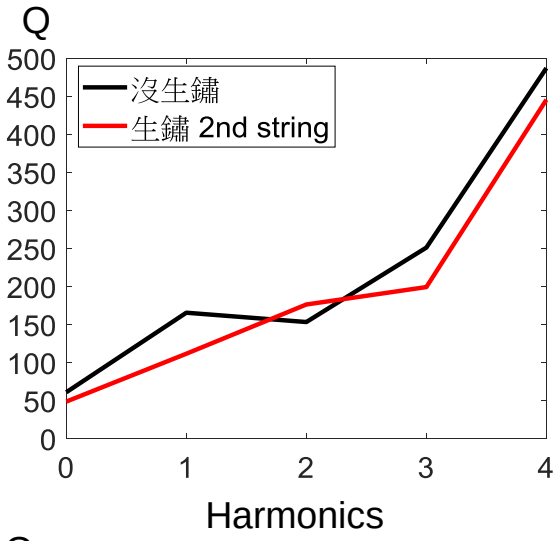


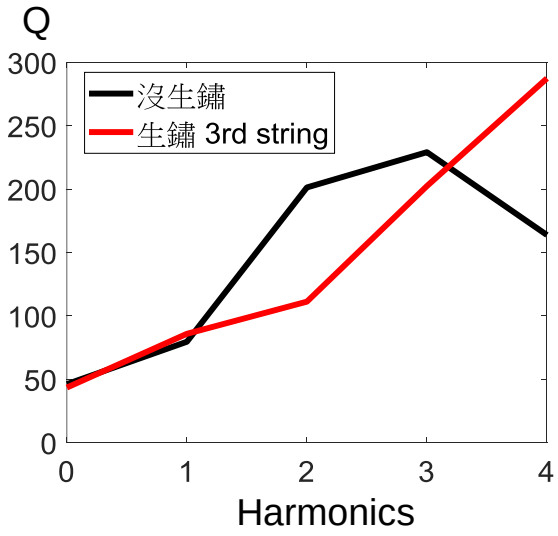
新弦

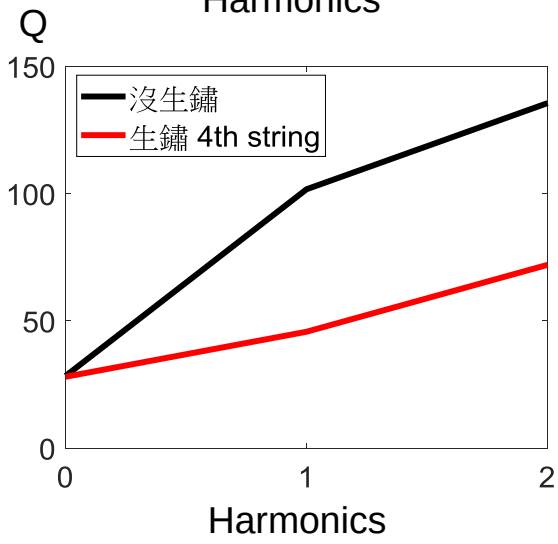


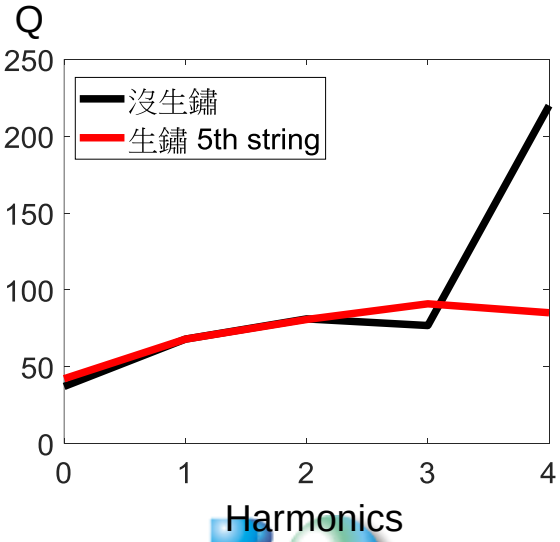
新弦與舊弦Q值

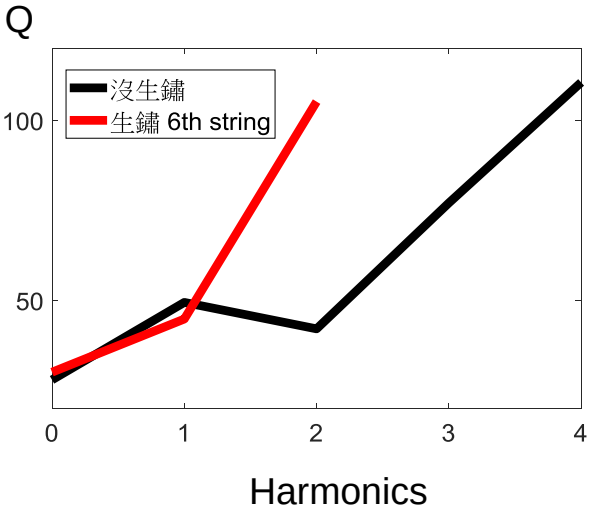

Q
沒生鏽
生鏽 1st string
Harmonics


Q
沒生鏽
生鏽 2nd string
Harmonics

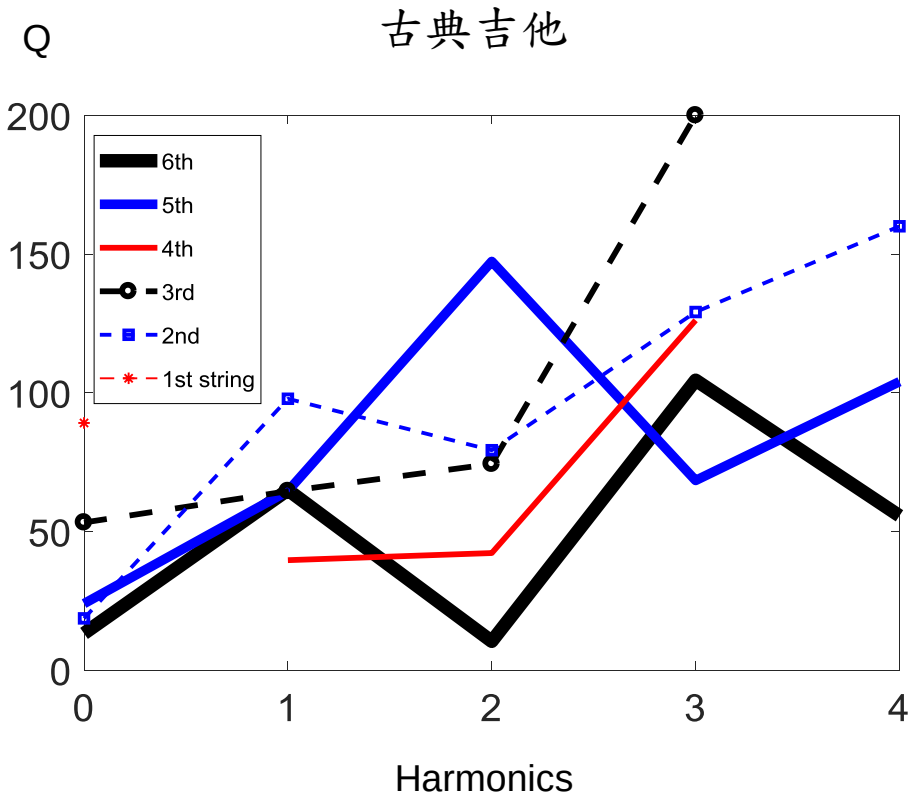

Q
沒生鏽
生鏽 3rd string
Harmonics

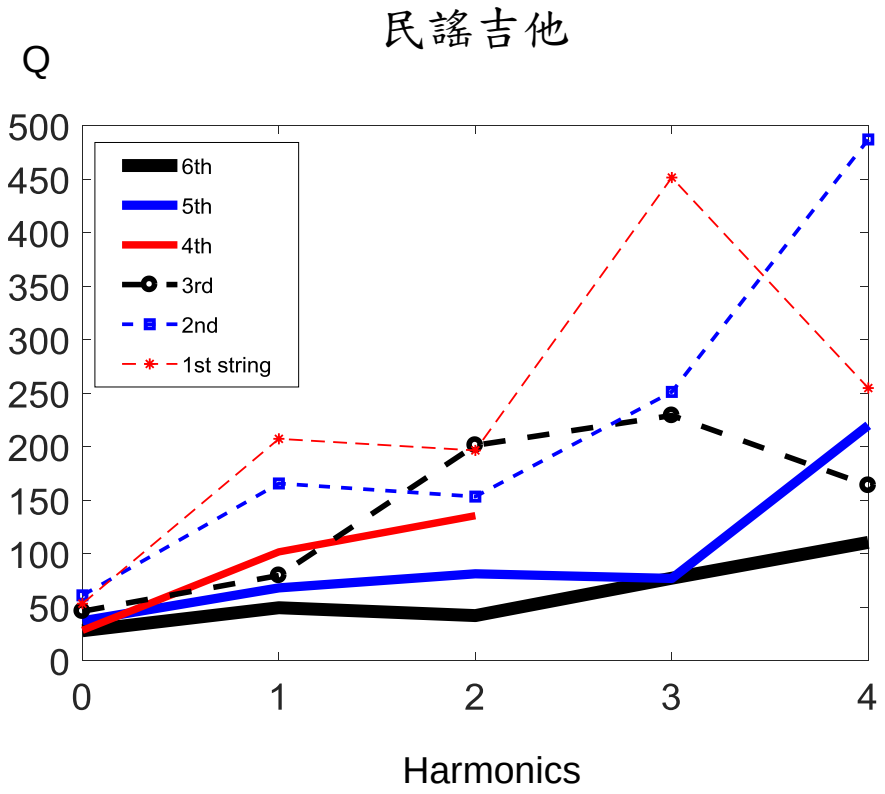

Q
沒生鏽
生鏽 4th string
Harmonics


Q
沒生鏽
生鏽 5th string
Harmonics


Q
沒生鏽
生鏽 6th string
Harmonics

古典/民謠吉他Q值


古典吉他
Q
200
150
100
50
0
0
1
2
3
4
6th
5th
4th
3rd
2nd
1st string
Harmonics


民謠吉他
Q
500
450
400
350
300
250
200
150
100
50
0
0
1
2
3
4
6th
5th
4th
3rd
2nd
1st string
Harmonics

Conclusion

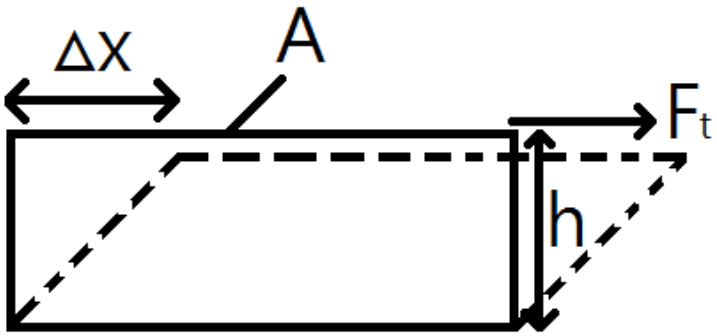
- Localized high frequency vibration
- Reduction of ultra-low frequency vibration
- Reflection of high-frequency sound

Thanks for your attention

- 全音（tone）半音（semitone）

MIDI音高標記

- MIDI標準A440的號碼則指定為69
- $\mathrm{p} = 69 + 12 \times log_2(\frac{f}{440})$
- $f = 440 \times 2^{(p-69)/12}$
- 每一個八度大小都是12，半音之間則相差1



Δx ：形變量

F_t ：剪應力

A：塊材頂部面積

h：塊材原高度

$F_t \propto \left(\frac{\Delta x}{h}\right)\mathrm{A}$　　定義：$F_t \equiv S\left(\frac{\Delta x}{h}\right)\mathrm{A}$

$$\Rightarrow S \equiv \frac{\frac{F_t}{\mathrm{A}}}{\frac{\Delta x}{h}} \equiv \frac{\tau}{\gamma}$$

Conclusion&future

Conclusion:

在實際測量吉他木板振動之前，必須先充分了解其發聲原理及構造，涉及材料力學的參數可用於測量結果後的探討。

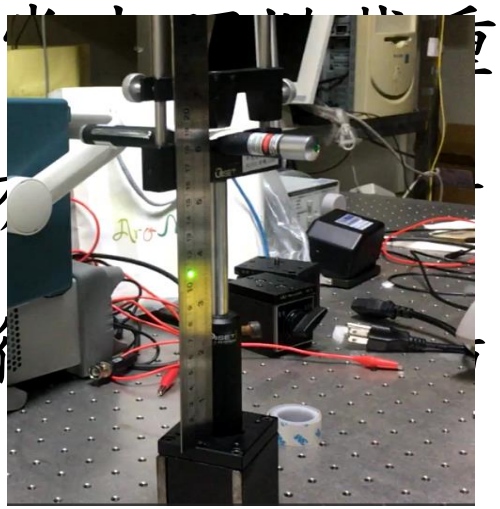
Future:

1. 學會如何使用matlab中instrument control的USB功能。
2. 了解雷射位移感測器的規格及使用方法。

Work：matlab練習

我所搭設的實驗

- 左邊部分為光源與量尺刻度
- 右邊部分為面鏡與待測木頭
- 重物，木板 度



回來的雷 進

氏係數

常數測量部分

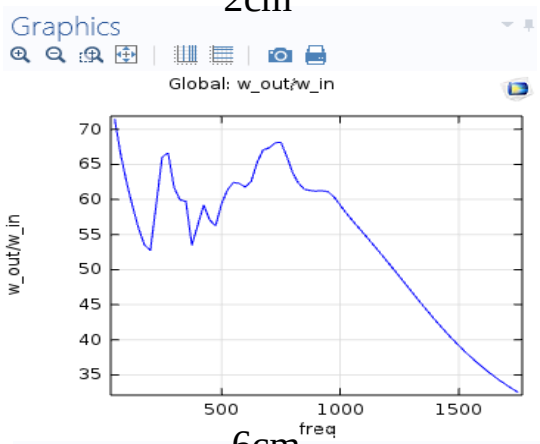
常數名稱	代表符號	長度值 (cm)		
面鏡底座直徑	a	3.15		
木頭厚度	t	0.277		
木頭寬度	w	6.82		
測量的木頭長度	L	16.2		
雷射光源與面鏡距離	d	100		

計算結果

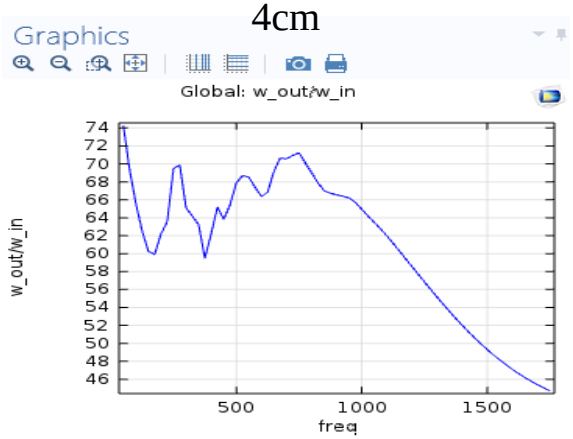
重量(g)	尺上刻度(s) (單位:cm)	尺上刻度-初始(s´-s)	凹陷高度(H)	揚氏係數(Y) (單位:Gpa)
0	11.5	0	0	None
100	11.8	0.3	0.004725	15.96
200	12.0	0.5	0.007875	18.71
300	12.3	0.8	0.0126	17.39
400	12.6	1.1	0.017325	16.79
				平均:17.21

結果

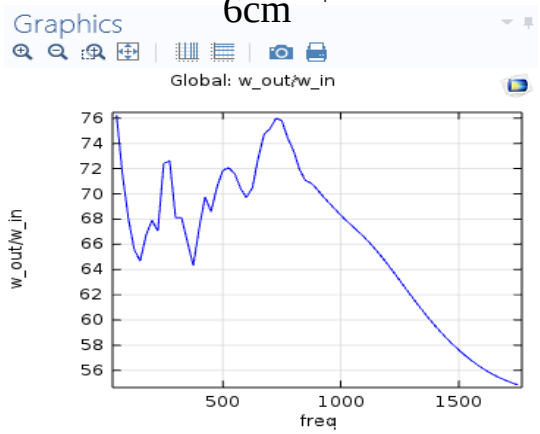
2cm



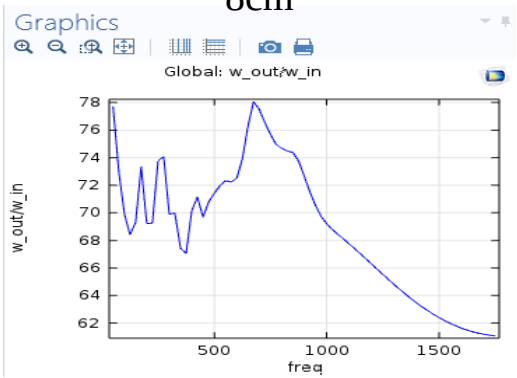
4cm



6cm



8cm



側背板為玫瑰木　面板為雲杉木

- 面板厚度依序為:　(1)0.05cm (2)0.1cm (3)0.15cm(4)0.2cm(5)0.25cm
- 小結論:

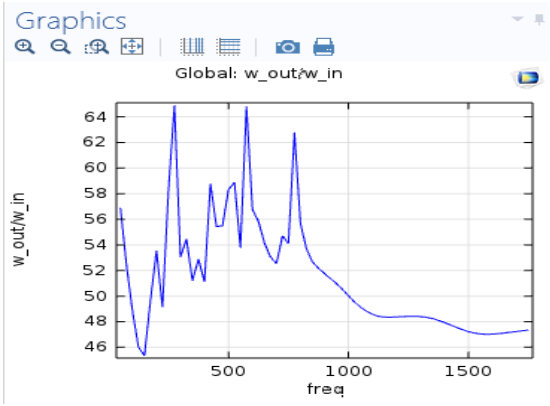
1.隨厚度增加峰值分佈有向低頻移動的趨勢.

2.隨厚度增加高頻的強度衰減的越快.

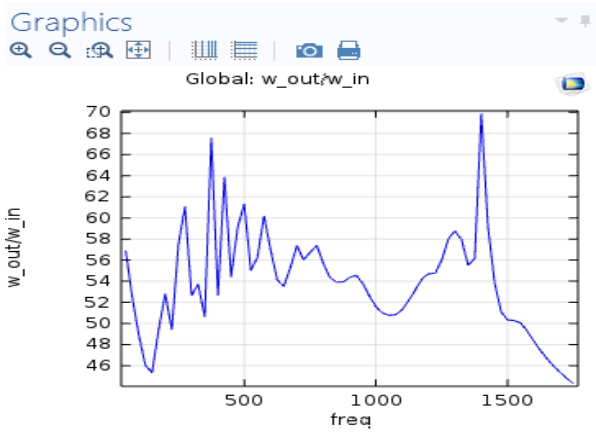
3.厚度越薄的狀況下中低頻出現的peak越多，可能意味著擁有高Q值的頻率越多.

結果

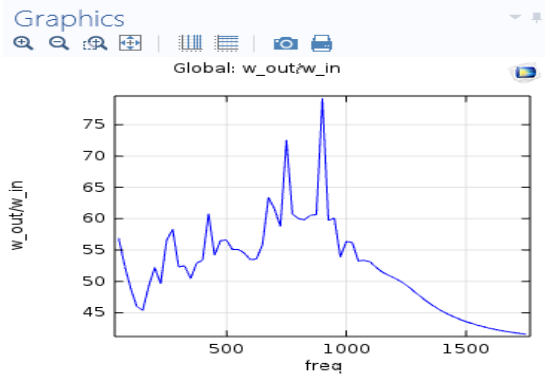
0.05cm



0.1cm



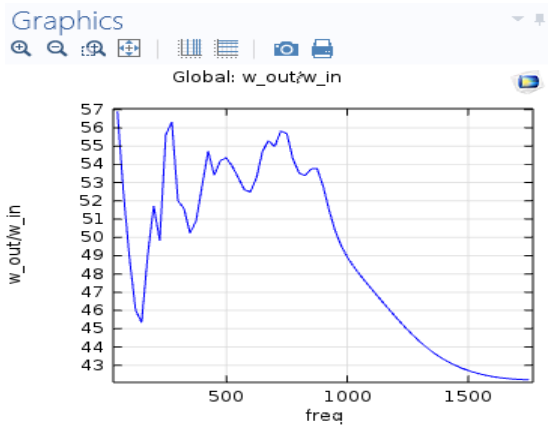
0.15cm

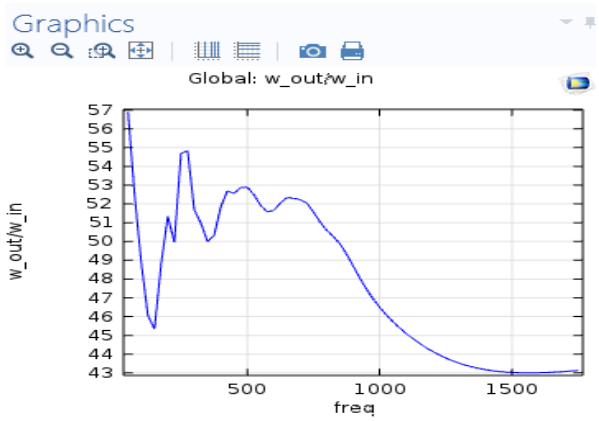


結果

0.2cm

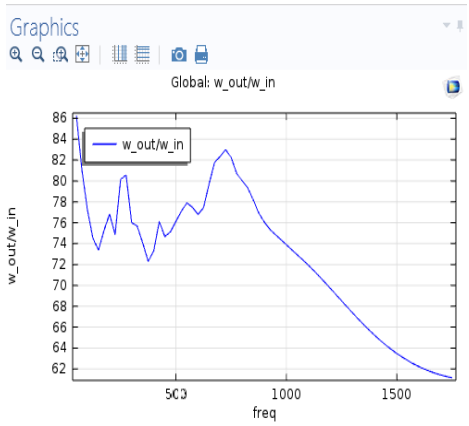
0.25cm

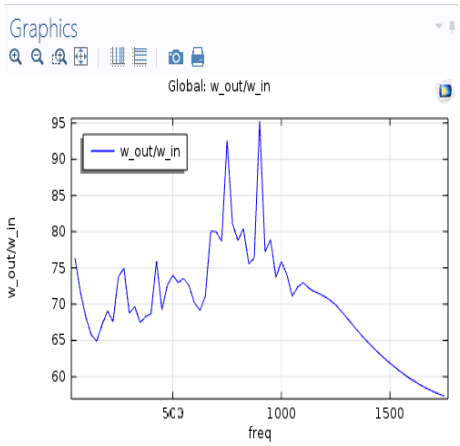


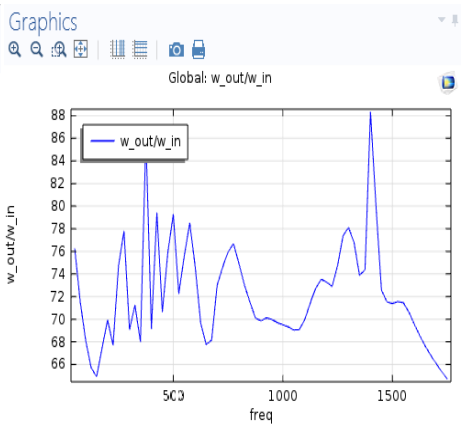


面板雲杉 側背板玫瑰 全吉他表面輸出

- 改變面板厚度
- 0.2cm



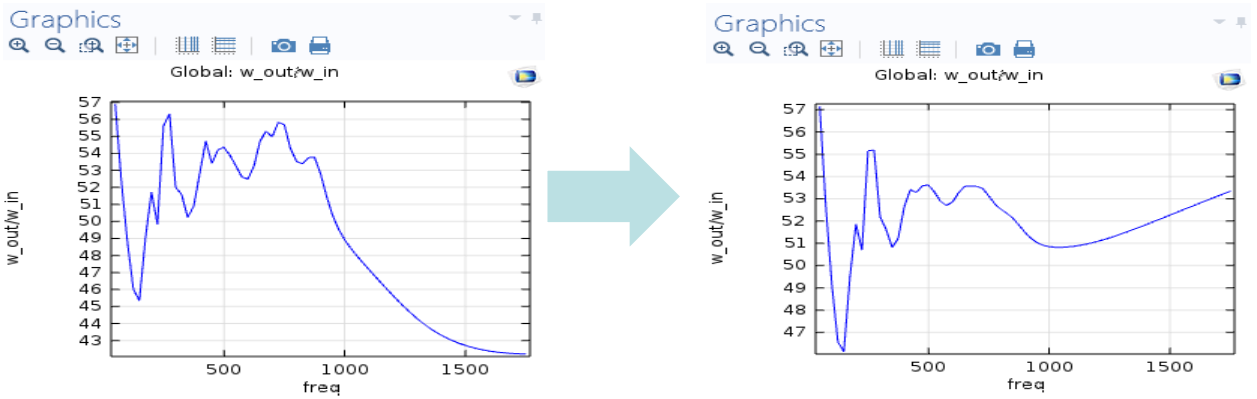


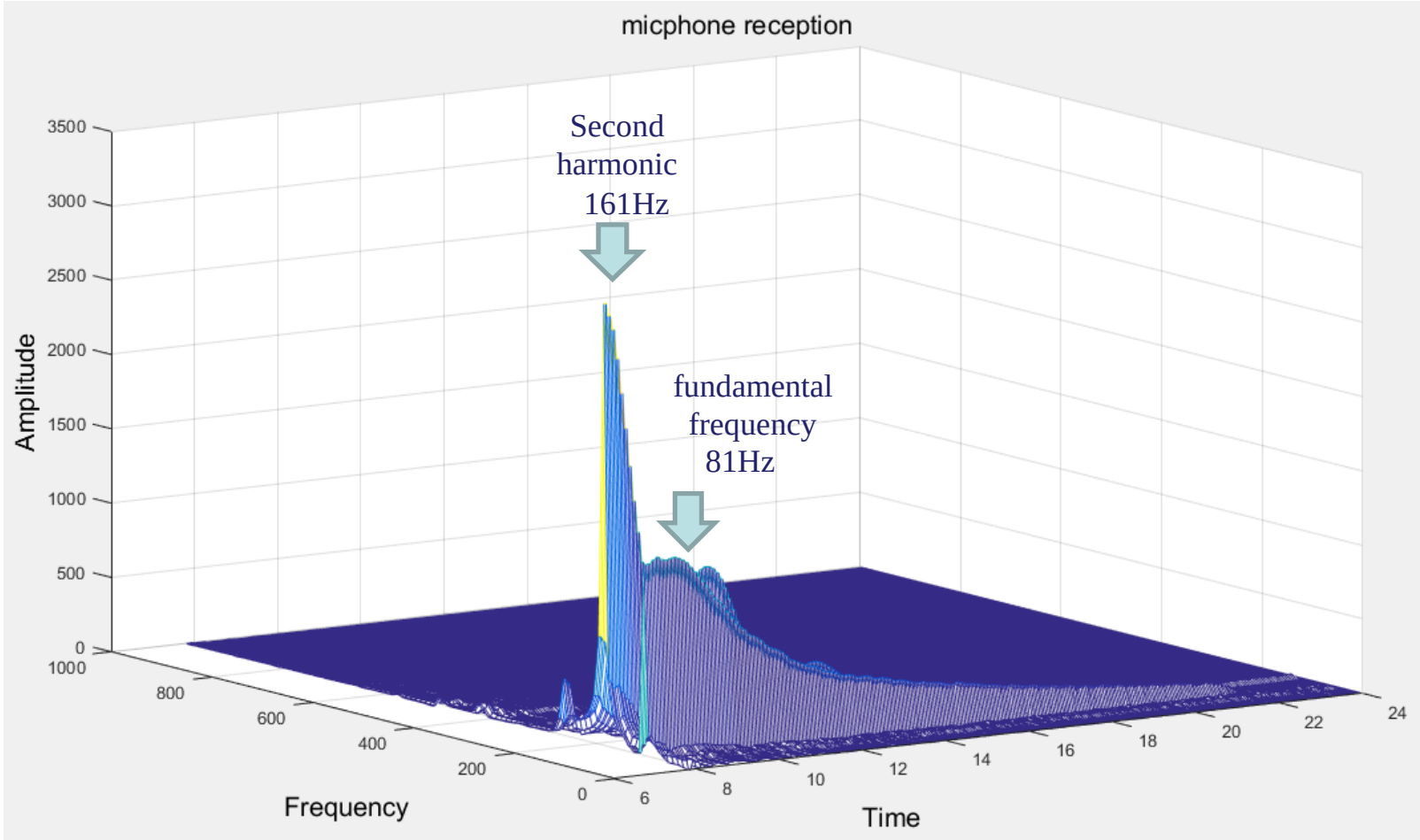


1cm

面板上的高頻呢?

- 左圖為輸出端僅有面板，右圖為輸出端包含琴枕部分




micphone reception
Second
harmonic
161Hz
fundamental
frequency
81Hz
Amplitude
3500
3000
2500
2000
1500
1000
500
0
1000
800
600
400
200
0
Frequency
6
8
10
12
14
16
18
20
22
24
Time