# Why Math Introduction to Math Finance and Courses

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Department of Mathematics National Central University

at Interdisciplinary Program of Science

September 18, 2018

### Where to work

You can be a

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### Where to work

You can be a

• Actuary;



### Where to work

You can be a

- Actuary;
- Data Analyst;

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- Hedge fund company: Renaissance Technologies (Top 3, US\$84 billion on 2018);
- Bank;
- Pharmaceutical industry;
- Science Park.

#### Stock Price

#### NASDAQ:AAPL 656.08 🛦 +7.97 (+1.23%) Open: 650.08 High: 656.315 Low: 649.9 Close: 656.08

August 20, 2012



Figure: Path of Brownian Motion

## Path of Brownian Motion



#### Brownian Motion

• In 1827, botanist Robert Brown observed the movement of pollen immersed in water.

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#### Figure: Brownian Motion

# Brownian Motion B(t)



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• 
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- Stochastic Integral  $\int_0^t f(s) dB(s)$ .

### Black–Scholes Equation

$$\frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} + rS \frac{\partial V}{\partial S} - rV = 0$$



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- $\sigma$ : the standard deviation of the stock's returns;
- t: a time in years.

# Languages

• SQL

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- SQL.
- Python, C++...



- SQL.
- Python, C++...
- Chinese, English, Japanese...

## Calculus

#### Contents

• Limit;

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### Calculus

#### ${\rm Contents}$

- Limit;
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- Limit;
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- Limit;
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## Calculus

- Limit;
- Continuity;
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- Integral;
- Series;

## Calculus

- Limit;
- Continuity;
- Derivatives;
- Integral;
- Series;
- Taylor Series.

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## Calculus

#### Contents

- Limit;
- Continuity;
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- Integral;
- Series;
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Probability, Statistics, Financial Math.

# Analysis

- Set Theory;
- Topology on Euclidean Space;

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- Set Theory;
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- Metric Space;

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- Set Theory;
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- $\bullet\,$  Metric Space; ,
- More on Limits and Continuity ;

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- Set Theory;
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- Set Theory;
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Probability, Statistics, Financial Math.

# Probability

Contents

• Probability Space;

# Probability

- Probability Space;
- Independence;

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# Probability

- Probability Space;
- Independence;
- Strong Law of Large Numbers;

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# Probability

- Probability Space;
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- Central Limit Theorem;

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# Probability

- Probability Space;
- Independence;
- Strong Law of Large Numbers;
- Central Limit Theorem;
- Martingale.

### Financial Math

Contents

• Optional Pricing;

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- Optional Pricing;
- Actuarial;

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- Optional Pricing;
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- Optional Pricing;
- Actuarial;
- Insurance;
- High-Frequency Trading.