On the Formation of Field-Aligned Nonlinear Electrostatic Waves in Space Plasma

T. C. Tsai and L.H. Lyu

Institute of Space Science, National Central University, Chung-Li, Taiwan, R.O.C.
e-mail: tctsai@jupiter.ss.ncu.edu.tw;   lyu@jupiter.ss.ncu.edu.tw

Introduction

Field-aligned nonlinear electrostatic waves and counterstreaming electrons have been observed in the upstream and downstream of Earth's bow shock\(^1\), upstream of the interplanetary shock\(^2\), and upstream of solar wind termination shock\(^3\). Field-aligned nonlinear electrostatic waves or solitons have also been observed in the boundary layers of the Earth magnetosphere\(^4-9\) and in the downward field-aligned current region\(^10-12\).

A low-noise, high-precision, implicit Vlasov simulation code has been developed. This simulation code allowed us to simulate non-uniform plasma distributions with non-periodic and non-reflection boundary conditions.

This implicit Vlasov code is the best simulation code to study evolution of nonlinear electrostatic waves in non-uniform plasma.

Electrostatic shock is the simplest example that can help us to understand the generation, propagation, and interference of field-aligned nonlinear electrostatic waves in a non-uniform plasma medium.

Simulation Model

Vlasov Simulation:

\[
\frac{\partial f_a}{\partial t} + v_x \frac{\partial f_a}{\partial x} + e_a \frac{e}{m_a} E_x \frac{\partial f_a}{\partial v_x} = 0
\]

\[
\frac{\partial E_x}{\partial t} = -\frac{1}{\varepsilon_0} \sum_a e_a \int v_x f_a dv_x
\]

where \(\alpha = i, e\).

Initial conditions:

\[
\frac{\partial E_x}{\partial x} = \frac{1}{\varepsilon_0} \sum_a e_a \int f_a dv_x = \frac{e}{\varepsilon_0} [n_i(x) - n_e(x)]
\]

For convenience, we choose \(E_x(x) = 0\), \(n_i(x) = n_e(x) = n(x)\), and \(V_{ix}(x) = V_{ex}(x) = V_x(x)\).

The initial distribution functions are given by

\[
f_a(x, v_x) = \frac{n(x)}{\sqrt{2\pi} \sigma_a(x)} \exp\left\{-\frac{[v_x - V_x(x)]^2}{2\sigma_a^2(x)}\right\}
\]

where \(\sigma_a(x) = \sqrt{k_B T_a(x) / m_a}\).
\[ T_\alpha(x) = \frac{p_\alpha(x)}{k_B n(x)} \]
\[ n(x) = \frac{n_2 + n_1}{2} + \frac{n_2 - n_1}{2} \tanh\left(\frac{x}{L}\right) \]
\[ T_\alpha(x) = \frac{T_{\alpha 2} + T_{\alpha 1}}{2} + \frac{T_{\alpha 2} - T_{\alpha 1}}{2} \tanh\left(\frac{x}{L}\right) \]

where subscripts ‘1’ and ‘2’ denote the initial upstream and downstream values, respectively.

The above fluid variables satisfy the following shock jump conditions
\[ [n V_x] = 0 \]
\[ [(m_i + m_e)n V_x^2 + nk_B(T_i + T_e)] = 0 \]
\[ \left[ \frac{1}{2} (m_i + m_e) V_x^3 + \frac{5}{2} nk_B(T_i + T_e) V_x \right] = 0 \]

where \([A] = A_2 A_1\). For a given set of electron-ion temperature ratios, \(T_{el}/T_{i1}, T_{el}/T_{i2}\) and upstream Mach number
\[ M_S = \frac{V_{x1}}{\sqrt{(5/3) k_B (T_{el} + T_{i1})/(m_e + m_i)}} , \]
we can determine the density jump and temperature jumps of the electrostatic shock.

**Boundary conditions:**
Uniform boundary conditions are used for both field and plasma distributions.

**Example of Simulation Results**
\(M_S = 10, \frac{m_i}{m_e} = 1836\)
\(T_{el}/T_{i1} = 1, T_{el}/T_{i2} = 0.2\)
Generation of nonlinear ES waves due to electron-electron two-stream instabilities

Upper-left panels:
Solutions of the following dispersion relation
\[
1 - \frac{\omega^2_{pa}}{(\omega - kV_{a0})^2 - k^2 C_{sa0}^2} = 0
\]
\[\alpha = ea \quad \alpha = eb \quad \alpha = i\]

Lower-left panels:
Electrons distribution functions
\[f_{ea} \quad f_{eb} \quad \text{(fitting curves)}\]
\[f_e \quad \text{(curve obtained from simulation)}\]

Right columns:
Solutions of the following dispersion relation
\[
1 - \sum_{\alpha} \frac{\omega^2_{pa}}{(\omega - kV_{a0})^2 - k^2 C_{sa0}^2} = 0
\]
▲ : Simulation results

Region (A)
How to determine growth rate of ES waves in region (C)
Non-uniform leakage electrons can lead to non-uniform development of electron-electron two-stream instabilities in the upstream region.

Theoretical prediction of potential jump across shock ramp
Curves: $\Delta \phi$ satisfies $J_x(\Delta \phi) = 0$
where

$$J_x(\Delta \phi) = e\left\{ [F_{IF}(\Delta \phi) - F_{IB}(\Delta \phi)] - [F_{eF}(\Delta \phi) - F_{eB}(\Delta \phi)] \right\}$$

$$F_{eF} = \frac{n_i}{\sqrt{2\pi} \sigma_{\text{rel}}} \int_{-\infty}^{\infty} v_x \exp\left[-(v_x - V_{ex1})^2/(2\sigma_{\text{rel}})^2\right] dv_x$$
\[ F_e B = \frac{n_2}{\sqrt{2\pi} \sigma_{the2}} \int_{-\infty}^{V_{e0}(\Delta\phi)} v_x \exp\left[-\left(v_x - V_{ex2}\right)^2 / (2\sigma_{the2})^2\right] dv_x \]

\[ F_i F = \frac{n_i}{\sqrt{2\pi} \sigma_{hi1}} \int_{0}^{V_{i0}(\Delta\phi)} v_x \exp\left[-\left(v_x - V_{ix1}\right)^2 / (2\sigma_{hi1})^2\right] dv_x \]

\[ F_i B = \frac{n_i}{\sqrt{2\pi} \sigma_{hi2}} \int_{0}^{V_{i0}(\Delta\phi)} v_x \exp\left[-\left(v_x - V_{ix2}\right)^2 / (2\sigma_{hi2})^2\right] dv_x \]

\[ V_{i0}(\Delta\phi) = \sqrt{\frac{2e\Delta\phi}{m_i}} \quad V_{e0}(\Delta\phi) = \sqrt{\frac{2e\Delta\phi}{m_e}} \]

**Summary**

- Electrostatic potential jump is established due to the presence of electron thermal pressure gradient.
- Upstream electrons are accelerated by the cross-shock potential jump and form a cold electron beam in the downstream region.
- The electron-electron two-stream instability plays an important role in the generation of nonlinear electrostatic waves in the upstream and downstream regions of the collisionless shocks.
- The low-density cold electron beam and the hot dense downstream electrons in region (A) can result in short wavelength high frequency nonlinear Langmuir waves.
- The upstream incident electrons and the low-density leakage electrons in regions (B) and (C) can result in longer wavelength nonlinear ES waves.
- The large-amplitude ES waves generated in region (C) have a chance to be blown back to the downstream side. These nonlinear ES waves can catch up the pre-exist Langmuir waves and destroy the pre-exist Langmuir waves in the downstream region.
- The non-uniform leakage electrons can lead to non-uniform development of electron-electron two-stream instabilities in the upstream region.
- The interference of these ES waves can result in modulate nonlinear ES wave structures in both upstream and downstream regions.
Discussion

\[ E_0 = 1.2(V/m) \sqrt{T(M^oK)n(#/c.c)} \]

For \( T = 1M^oK, \ n = 5#/c.c., \ E_0 = 2.68 \text{ V/m} \)

The wave amplitude \( 0.1E_0 = 268 \text{ mV/m} \).

- The wave amplitude of the ES waves observed upstream from the Earth bow shock\(^1\) are a few 100s mV/m.
- Our simulation results are consistent with the observations.

\[ \phi_0 = 862(V)71(M^oK) \]

For \( T = 1M^oK, \) we have \( \phi_0 \approx 86V \)

Simulation results :
- low Mach number shock \( \Delta \phi = 5\phi_0 \approx 0.43kV \)
- high Mach number shock \( \Delta \phi = 50\phi_0 \approx 4.3kV \)

- The interplanetary shocks are low Mach number shocks. The energy of the observed counterstreaming electrons\(^2\) are of a few 100s eV.
- The potential jump obtained in the low-Mach-number shock simulation is consistent with the observed electron beam energy in the upstream and downstream regions.

References


**Acknowledgements**

This work is supported by NSC grants NSC93-2111-M-008-012 and NSC94-211-M-008-026-AP5 to the National Central University.