On the DHT-based Multicarrier Tranceiver over Multipath Fading Channel

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Abstract—In this paper, an Multicarrier modulation (MCM) tranceiver based on discrete Hartley transform (DHT) is investigated. Instead of conventional discrete Fourier transform (DFT), DHT is an alternative orthogonal basis with real-valued operation. However, the DHT cannot directly diagonalize the multipath fading channel due to its inherent property. To deal with this problem, we devise a DHT-based OFDM architecture that can perfectly diagonalize the channel matrix based on the complementary property of DHT matrix and channel gain symmetric features. In addition, two-dimensional complex signaling, quadrature amplitude modulations (QAM), can also be applied to the proposed structure for bandwidth efficient transmission. Finally, numerical simulations have shown the validity of the proposed DHT-based OFDM system.

I. INTRODUCTION

Multicarrier modulation (MCM), has been widely adopted as the modulation/demodulation technique in many wired or wireless communication standards due to the robustness to deal with severe multipath channel fading [1]. Orthogonal frequency-division multiplexing (OFDM) is a kind of MCM technique where multiple data streams are modulated with mutually orthogonal subcarriers. An OFDM system which realizes digital basedband modulation and demodulation with inverse DFT (IDFT) and DFT is denoted as DFT-OFDM. The main advantage of DFT-OFDM is that IDFT/DFT has fast computation algorithm, (inverse) fast Fourier transform (IFFT/FFT), and can be easily implemented by digital circuit. However DFT is not the only orthogonal basis for OFDM systems to perform multicarrier modulation, several trigonometric functions, such as sinusoidal and cosinusoidal functions, have been proposed to be an alternative orthogonal basis for OFDM systems [2]–[7]. Discrete Hartley transform (DHT) can synthesize these orthogonal functions to be an OFDM signal, and such system are denoted as DHT-OFDM. As well as DFT owns fast implementation algorithms, FFT, [9] and [10] showed that fast Hartley transform (FHT) has better computational complexity than FFT.

A DHT-based OFDM system was developed initially to substitute for DFT-OFDM in wireline systems [6], [7]. However, there are two main problems for DHT-based OFDM to apply to wireless communication. First, only one-dimensional (1D) modulation schemes such as PAM and binary phase-shift keying (BPSK) are considered since DHT is a real-valued transformation. Second, DHT-OFDM can not directly diagonalize the multipath channel impulse response (CIR) due to the inherent property of DHT. In this paper, we propose a generalized DHT-OFDM structure that can not only apply two-dimensional (2D) modulation formats but also diagonalize the passband complex-valued CIR perfectly. In other words, the coupling ICI effects is eliminated and one-tap equalizer can be employed. We use matrix algebra framework to verify that the frequency-selective channel matrix is diagonalizable by DHT by applying the complementary property of DHT [11].

The organization of this paper is as follows. The signal model is introduced in Section II. In Section III, we analyze the DHT matrix properties and present the problems of DHT-OFDM system. Section IV contains the proposed DHT-OFDM systems that can perfectly diagonalize the multipath fading channel. Finally, the simulation results are given in Section V, and the paper is concluded in Section VI.

II. SIGNAL MODEL

Fig. 1 shows the block diagram of DFT-OFDM system. Here we assume CP is longer than CIR (i.e., $G \geq L$), hence the equivalent channel matrix $\tilde{A}$ is a circulant matrix. Then the received signal on each subcarrier can be written as

$$\vec{Y} = \mathbf{F}^H \tilde{A} \mathbf{F} \times \vec{X} + \vec{W}$$

(1)

where $\vec{Y}$, $\vec{X}$, and $\vec{W}$ are $N \times 1$ vectors containing the DFT outputs of $\vec{y}$, $\vec{x}$, and $\vec{w}$, respectively. $\mathbf{F}$ and $\mathbf{F}^H$ are the IDFT and DFT matrices, and $[,]^H$ denotes the conjugate transpose operation. The $(l,m)$ entry of $\mathbf{F}$ is defined as

$$\mathbf{F}(l,m) = \frac{1}{\sqrt{N}} \exp\left(\frac{j2\pi lm}{N}\right), \quad 0 \leq l, m \leq N - 1.$$
According to the property of circulant matrices in [12], \( \tilde{A} \) can be diagonalized by the DFT matrix. We will have
\[
\Lambda = \mathbf{F}^H \tilde{A} \mathbf{F} = \text{diag}\{\lambda_i\} \tag{2}
\]
where
\[
\lambda_i = \sqrt{N} \mathbf{F}^H \tilde{A} \vec{e}^i = [\lambda_0, \lambda_1, \ldots, \lambda_{N-1}]^T.
\]
\( \vec{e} = [1, 0, \ldots, 0]^T \) is an \( N \times 1 \) vector used to select the first column of \( \tilde{A} \).

Before mentioning the meaning of \( \lambda_k \), we introduce the flip matrix \( \mathbf{J}_N \) to describe the retrograde indexing relations of a vector or matrix. Let the \( \mathbf{J}_N \) matrix, i.e.,
\[
\mathbf{J}_N = \begin{bmatrix}
1 & 0 & \cdots & 0 \\
0 & 1 & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & 1
\end{bmatrix}.
\]
\( \mathbf{J}_N \) is not only a symmetric matrix but also an orthogonal matrix, i.e.,
\[
\mathbf{J}_N^H \mathbf{J}_N = \mathbf{J}_N \mathbf{J}_N = \mathbf{I}_N.
\]
An \( N \times 1 \) vector \( \vec{v} \) is called even if \( \mathbf{J}_N \vec{v} = \vec{v} \) and odd if \( \mathbf{J}_N \vec{v} = -\vec{v} \).

Actually, \( \lambda_k \) is channel gain on the \( k \)th subcarrier, which is given by the \( N \)-point DFT of the CIR \( \alpha_k \). According to the symmetry property of DFT [13], if the CIR \( \alpha_k \) are real-valued (i.e., wireline system), the following relations exist:
\[
\Re\{\lambda_k\} = \Re\{\lambda_{(N-k)}\} \Rightarrow \Re\{\tilde{\lambda}\} = \mathbf{J}_N \Re\{\tilde{\lambda}\}
\]
\[
\Im\{\lambda_k\} = -\Im\{\lambda_{(N-k)}\} \Rightarrow \Im\{\tilde{\lambda}\} = -\mathbf{J}_N \Im\{\tilde{\lambda}\},
\]
where \( k = 0, 1, \ldots, N - 1 \). \( \Re\{\} \) and \( \Im\{\} \) denote the real and imaginary parts of a complex number. For a real-valued CIR, Eq. (3) shows that channel gain \( \lambda \) is a conjugate symmetric sequence, i.e., the real and imaginary parts of \( \lambda \) are even and odd sequences respectively. This implies that just only \( N/2 \) samples of channel gain are known \( (\lambda_k; k = 0, \ldots, N/2) \), then the overall channel gain can be gotten due to the symmetry properties. The relations in (3) are important for us to design the structure of DHT-OFDM system.

### III. DHT Property Analysis and Problem Definition

Since DHT and IDHT have the same transform definition, the DHT of a real sequence \( \vec{d} = [d_0, d_1, \ldots, d_{N-1}]^T \) and its inverse are given by
\[
x_n = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} d_k \cos\left(\frac{2\pi kn}{N}\right), \quad 0 \leq n \leq N - 1
\]
\[
d_k = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} x_n \cos\left(\frac{2\pi kn}{N}\right), \quad 0 \leq k \leq N - 1 \tag{4}
\]
where
\[
\cos t \equiv \cos t + \sin t.
\]
In order to decompose the DFT and DHT with cosine and sine matrix, let us define the \((l, m)\) entries of \( N \times N \) matrices \( \mathbf{C} \) and \( \mathbf{S} \) as follows:
\[
\mathbf{C}(l, m) = \cos\left(\frac{2\pi lm}{N}\right)
\]
\[
\mathbf{S}(l, m) = \sin\left(\frac{2\pi lm}{N}\right), \quad 0 \leq l, m \leq N - 1.
\]
Thus the DFT and DHT matrices can be given by
\[
\mathbf{F} = \frac{1}{\sqrt{N}} (\mathbf{C} + j\mathbf{S}) \tag{5}
\]
\[
\mathbf{H} = \frac{1}{\sqrt{N}} (\mathbf{C} + \mathbf{S}). \tag{6}
\]
From the properties of trigonometric functions, each row or column of \( \mathbf{S} \) is odd symmetric, we obtain \( \mathbf{J}_N \mathbf{S} = -\mathbf{J}_N = -\mathbf{S} \). On the other hand, \( \mathbf{C} \) is even symmetric on the row and column, i.e., \( \mathbf{J}_N \mathbf{C} = \mathbf{C} \mathbf{J}_N = \mathbf{C} \). Due to the orthogonality between sine and cosine functions, we have \( \mathbf{C} \mathbf{S} = \mathbf{S} \mathbf{C} = 0 \). On account of the trigonometric identities, we have \( \mathbf{C}^2 + \mathbf{S}^2 = \mathbf{N} \mathbf{I}_N \). Therefore the inverse of DHT (IDHT) can be easily checked to have the same definition as DHT:
\[
\mathbf{H} = \frac{1}{N} (\mathbf{C} + \mathbf{S})(\mathbf{C} + \mathbf{S}) \Rightarrow \mathbf{H} = \mathbf{J}_N \mathbf{H} = \mathbf{H} \mathbf{J}_N. \tag{7}
\]
Eq. (7) presents an important property of DHT that the complementary DHT outputs of a vector can be accomplished with DHT in two ways. One is to retrograde the output samples of DHT and the other is to retrograde the input vector before DHT.

Since the circulant matrices can be diagonalized by DFT matrix in (2), we are interested in whether the DHT matrix has such property. Substituting (5) into (2), we will have
\[
\mathbf{F}^H \tilde{A} \mathbf{F} = \frac{1}{N} (\mathbf{S} \mathbf{C} \mathbf{C} \mathbf{S} + \mathbf{S} \mathbf{C} \mathbf{A} \mathbf{C} \mathbf{S} + j \frac{1}{N} (\mathbf{C} \mathbf{S} \mathbf{C} \mathbf{C} \mathbf{S} + \mathbf{S} \mathbf{C} \mathbf{A} \mathbf{C} \mathbf{S})), \tag{8}
\]
hence
\[
\Re\{\Lambda\} = \frac{1}{N} (\mathbf{S} \mathbf{C} \mathbf{C} \mathbf{S} + \mathbf{S} \mathbf{C} \mathbf{A} \mathbf{C} \mathbf{S}),
\]
\[
\Im\{\Lambda\} = \frac{1}{N} (\mathbf{C} \mathbf{S} \mathbf{C} - \mathbf{S} \mathbf{C} \mathbf{A} \mathbf{C} \mathbf{S}). \tag{9}
\]
Now let the DFT matrix $F$ in (2) be replaced with DHT matrix $H$ to have the following:

$$H \bar{A} H = \frac{1}{N} (C + S) \bar{A} (C + S) = \frac{1}{N} (SAS + CAC + C\bar{A}S + \bar{S}AC). \quad (10)$$

We can apply the trigonometric properties for (10), that results $CAS + SAC = J_N(CAS - SAC)$, Therefore, Eq. (10) can be rewritten as

$$\bar{H} \bar{A} H = \begin{bmatrix} \Re\{\lambda_0\} & 0 & \cdots & 0 \\ \Re\{\lambda_1\} & \Re\{\lambda_1\} & \cdots & \Im\{\lambda_{N-1}\} \\ \vdots & \vdots & \ddots & \vdots \\ 0 & \Im\{\lambda_1\} & \cdots & \Re\{\lambda_{N-1}\} \end{bmatrix} \quad (11)$$

Eq. (11) clearly shows that DHT matrix can not diagonalize the circulant matrix. In fact, It has been proved in [15] that a circulant symmetric matrix can be diagonalized by DHT. Matrix $\bar{A}$ only meets the circular condition, hence the entries $\Im\{\lambda_1, \ldots, \lambda_{N-1}\}$ exist on the anti-diagonal direction of matrix $HAH$. That is to say for a baseband OFDM system with real-valued CIR, the transmitted data on mirror-symmetric subcarriers will couple together and induce ICI if the multicarrier modulator uses DHT instead of DFT. In order to avoid such problem, one previous work [14] arranged the same data symbol on the mirror-symmetric subcarriers. This technique does not solve the coupling effects but loss half of the data rate.

IV. PROPOSED DHT-BASED OFDM SYSTEM

In this section, we propose a DHT-OFDM transceiver with two real-valued DHT, which can apply 2D QAM symbol format. In addition, the mirror-symmetric coupling effects can also be eliminated based on the diagonal properties of DHT.

A. Transmitter

To modulate 2D QAM symbols into $N$ orthogonal subcarriers with DHT, the multicarrier passband modulator is represented in a pre-enveloped format $H + j\tilde{H}$, where $\tilde{H}$ is the Hilbert transform of $H$. From the relation of (7), $\tilde{H}$ can be rewritten as:

$$\tilde{H} = \frac{1}{\sqrt{N}} (S - C) = -J_N H = -H'. \quad (12)$$

The passband DHT-OFDM transceiver architecture is depicted in Fig. 2. In Fig. 2, the in-phase and quadrature vector sequences after IDHT multicarrier modulator is given by

$$\tilde{x}R + j\tilde{x}I = (H + j\tilde{H})(\tilde{x}R + j\tilde{x}I) = (H\tilde{x}R + J_N H\tilde{x}I) + j(H\tilde{x}I - J_N H\tilde{x}R) \quad (13)$$

After inserting guard interval to prevent ISI, the I/Q discrete data sequences are fed into D/A converters to be continuous waveforms. The I-phase and quadrature waveforms of one DHT-OFDM symbol can respectively be written as

$$x_I(t) = \sum_{n=0}^{N+Q-1} x_{IR} g(t - nT) \quad (14)$$

$$x_Q(t) = \sum_{n=0}^{N+Q-1} x_{IQ} g(t - nT), \quad (15)$$

where $g(t)$ is the lowpass reconstruction filter of D/A converter. Then the I/Q baseband waveforms are translated to passband by quadrature mixers in RF front end. The transmitted passband signal with a carrier frequency $f_c$ is represented by

$$x(t) = \Re \{ x_I(t) + jx_Q(t) \cdot e^{j2\pi f_c t} \} \quad (16)$$

B. Receiver

At the receiver as shown in Fig. 2, the received signal is down-converted to the baseband and then sampled with A/D converters. Since the multipath channel for passband communication can be expressed by a complex equivalent channel model $\tilde{A}_R + j\tilde{A}_I$, the discrete vector sequence after discarding CP can be given by

$$y(t) = (\tilde{A}_R + j\tilde{A}_I)(H - jH') \times \tilde{X} + \tilde{W}, \quad (17)$$

where both $\tilde{A}_R$ and $\tilde{A}_I$ are real-valued circulant matrices, and $\tilde{W}$ is complex white Gaussian noise. The real and imaginary parts of $y(t)$ are fed into DHT to demodulate multicarrier signal, then the DHT output $\tilde{Y}$ are given by

$$\tilde{Y} = \Re \{ \tilde{Y}R + j\tilde{Y}I \} = D\tilde{X} + \tilde{W}, \quad (18)$$

where

$$D = (\tilde{H}\tilde{A}_R H + \tilde{H}\tilde{A}_I H') + j(\tilde{H}\tilde{A}_R H - \tilde{H}\tilde{A}_R H')$$

is not a diagonal matrix. Therefore, our goal is to develop a DHT-OFDM transceiver that diagonalizes the frequency-selective channel into decoupled subcarriers. To design the receiver architecture, we will use the following two facts in [15].

**Fact 1:** All $N \times N$ symmetric circulant matrices can be diagonalized by DHT matrix.
**Fact 2**: All $N \times N$ skew-symmetric circulant matrices multiplied by flip matrix $J_N$ can be diagonalized by DHT matrix.

Let $\tilde{A}$ be a real-valued circulant matrix and it can be readily checked $\tilde{A}^T = J_N \tilde{A} J_N$. In addition, symmetric and skew-symmetric circulant matrices can be expressed respectively as $\tilde{A} + \tilde{A}^T$ and $\tilde{A} - \tilde{A}^T$. To investigate the diagonalization properties of DHT, we can describe the fact 1 and 2 as the following relations:

\[
H(\tilde{A} + J_N \tilde{A} J_N)H = H\tilde{A}H + H\tilde{A}H^T = 2\Re\{\Lambda\} \tag{16}
\]

\[
HJ_N(\tilde{A} - J_N \tilde{A} J_N)H = H\tilde{A}H - H\tilde{A}H^T = 2\Im\{\Lambda\} \tag{17}
\]

where $\Lambda$ was defined the same as (2). In (16), we can see $H\tilde{A}H$ and $H\tilde{A}H^T$ as a complementary pair since the sum of them make the circulant matrix $\tilde{A}$ meet symmetric condition and diagonalizable by DHT. By the same results, $H\tilde{A}H$ and $H\tilde{A}H^T$ can also be seen as a complementary pair. Let us now focus on $D$, we can find the complementary pair of the real or imaginary part of $D$ can respectively be obtained by the imaginary or real part of $D$ itself multiplied by a flip matrix $J_N$. Accordingly, we can derive a complex diagonal matrix $D'$ from $D$ with the following real and imaginary parts with cross interacting relationship:

\[
D' = (I + jJ_N)D = (H\tilde{A}_R H + H'\tilde{A}_R H' + H\tilde{A}_I H' - H'\tilde{A}_I H) + j(2\Re\{\Lambda_{\tilde{A}_R}\} - \Im\{\Lambda_{\tilde{A}_I}\}) + j2\Re\{\Re\{\Lambda_{\tilde{A}_R}\} + \Im\{\Lambda_{\tilde{A}_I}\} \}
\]

\[
= 2(\Re\{\Lambda_{\tilde{A}_R}\} - \Im\{\Lambda_{\tilde{A}_I}\}) + j2\Re\{\Re\{\Lambda_{\tilde{A}_R}\} + \Im\{\Lambda_{\tilde{A}_I}\} \}
\]

where $\Lambda_{\tilde{A}_R} = F^H\tilde{A}_R F$ and $\Lambda_{\tilde{A}_I} = F^H\tilde{A}_I F$ are both diagonal matrices. Substituting (18) for $D$ in (15), we obtain the relation between the decoupled $\tilde{Y}'$ vector and $\tilde{X}$:

\[
\tilde{Y}' = D'\tilde{X} + \tilde{W}. \tag{19}
\]

Fig. 2 shows the I/Q cross interacting architecture in the passband DHT-OFDM receiver. Due to the cross operations with I/Q components after DHTs, it is obvious $D'$ becomes a diagonal matrix. Accordingly, one-tap complex FEQ can also applied in DHT-OFDM system with the zero forcing (ZF) or minimum mean square error (MMSE) coefficients as follows:

\[
\begin{align*}
E_{ZF} &= D'^{-1} \\
E_{MMSE} &= D'^H \left(D'D'^H + \frac{\sigma^2}{\sigma_x^2}I_N\right)^{-1}. \tag{20}
\end{align*}
\]

**TABLE I**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Passband System</th>
</tr>
</thead>
<tbody>
<tr>
<td>Effective bandwidth (MHz)</td>
<td>7.61</td>
</tr>
<tr>
<td>Modulation type</td>
<td>64-QAM</td>
</tr>
<tr>
<td>DFT or DHT size ($N$)</td>
<td>8192</td>
</tr>
<tr>
<td>OFDM block duration ($\mu$ sec)</td>
<td>924</td>
</tr>
<tr>
<td>Guard interval ($\mu$ sec)</td>
<td>28</td>
</tr>
</tbody>
</table>

Fig. 3. Real and imaginary parts of ETSI DVB-T F1 channel impulse response.

**V. SIMULATION RESULTS**

The European digital video broadcasting (DVB) system is considered to demonstrate the proposed DHT-OFDM architecture. We would like to verify the diagonalized channel characteristic for both DFT-OFDM and DHT-OFDM by observing the channel estimation results. The DVB-T standard [16] has defined various modes and parameters for the flexible transmission. We choose 8MHz channel bandwidth and 8K subcarriers as simulation environment. The corresponding parameters of the OFDM systems investigated here are shown in Table I. Two complex channel models defined in DVB standard are considered here. The first channel model F1, as shown in Fig. 3, exists a line of sight (LOS) direct path and it is suitable for fixed reception environment. The second channel P1 is suitable for portable reception environment, and its power delay profile is similar to F1 with the difference that P1 does not exist the LOS direct path.

Shown in Figs. 4 and 5 are the estimated F1 channel magnitude and phase on each subcarrier at SNR = 30dB. The least squares (LS) estimator is applied to estimate channel state information with the scattered pilots. Since the maximum delay spread of channel F1 (5.4$\mu$s) is smaller than the guard interval length, we could observe the optimally diagonalized channel gains by the channel estimates. Fig. 4 shows the estimated channel magnitude for hybrid DHT-OFDM system is 2 times of the DFT-OFDM system. These results are because of the inherent definitions differences between DFT and DHT. Fig. 5 shows the estimated channel phase of DHT-OFDM coincides with that of DFT-OFDM system. These numerical results agree with the analysis in Section IV. Fig. 6 compares the performances of DFT-OFDM and DHT-OFDM systems over F1 and P1 channel. We adopt the MMSE FEQ coefficients.
in this simulation. It can be seen the bit error rate (BER) curves of DHT-OFDM are identical with DFT-OFDM.

VI. CONCLUSION

In this paper, we have investigated the data coupling problems on mirror-symmetric for OFDM system that uses the DHT orthogonal basis over the multipath fading channel. According to the DHT diagonal properties, we presented the DHT-based OFDM structure that can optimally diagonalize the frequency-selective channel. Moreover, the 2D complex QAM data symbol can be applied in the proposed structure that provides the same data throughput with DFT-OFDM. The numerical simulation results show that the proposed DHT-OFDM has the same BER performance compared with the conventional DFT-OFDM.

REFERENCES